AUTOREGRESSIVE MODELING OF RADAR DATA WITH APPLICATION TO TARGET IDENTIFICATION

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Abstract

This paper is concerned with the classification and identification of radar targets from frequency domain data. The approach taken is to form an autoregressive (AR) model of the downrange and Doppler profiles of the target from a set of coherent, stepped-frequency radar measurements. Simple, effective methods for motion compensation of the data and for averaging of the profile estimates are derived. This composite algorithm is applied to X-band radar measurements of aircraft in flight to illustrate the effectiveness of the modeling procedure. The results indicate that the AR method can be used for scattering center identification, and for discrimination between two targets.

I. Introduction

Traditional radar systems have been designed to detect and track targets. More recently, radar systems which not only detect targets, but also classify or identify them are of interest [1,2]. Such radar systems attempt to exploit information obtained from radar measurements. This information is often in the form of amplitude and phase of the radar return at different frequencies and different times [2,3].

There are various ways in which the information in these radar measurements can be used for classification. One popular method involves transforming the frequency-domain measurements into downrange and Doppler profiles via a Fourier Transform operation. Radar measurements at a fixed frequency but at different times can be transformed to obtain a Doppler profile of the target; similarly, measurements at a set of stepped frequencies can be transformed to obtain a downrange profile of the target.

Traditionally, the Discrete Fourier Transform (DFT) is used to implement the transform operation [2]. However, there are some drawbacks in using the DFT for target identification. The most important of these is that the DFT produces a nonparametric estimate of the downrange or Doppler profile of a target; in fact, the number of "output" data points is the same as the original number of measurements. Any classification algorithm must then operate on these output points (or "features"). This presents difficulties for two reasons. First, classification algorithms become computationally burdensome as the number of features increases. For many algorithms, tractable implementations can be obtained only if the number of features is kept small. On the other hand, the resolution of DFT methods is proportional to the number of measurements; thus, if DFT processing is used, there is a tradeoff between resolution in the downrange or Doppler profile and computational burden of the subsequent classification algorithm.

In this paper, we apply autoregressive (AR) modeling techniques to obtain downrange and Doppler profiles of the target. Autoregressive modeling techniques have been applied to a wide range of problems [4,5,6,7]. These techniques are appealing for target identification for two reasons. First, the AR methods produce a small number of parameters which describe the downrange or Doppler profile of the target; these parameters can be used directly in a classification scheme. Since this number of parameters is small, the resulting classification algorithm will require fewer computations. Second, AR methods are not limited in resolution as DFT methods are. In fact, theoretically infinite resolution of scattering centers if possible.

The outline of the paper is as follows. In the next section, we outline the AR modeling procedure. In Section III we address the problem of motion compensation and data averaging for the present application. We obtain computationally simple, effective methods for performing both tasks. In Section IV we apply this modeling procedure to actual X-band radar data measurements of two aircraft in flight. We also present guidelines for selecting the order of the AR model. Section V summarizes the results.

II. THE AUTOREGRESSIVE MODELING PROCEDURE

A. *Description of the Data Measurements*

The radar data available for our application was taken at the Naval Ocean Systems Center. This data consists of sets of 64×64 arrays of measurements. For each array, the measurement frequency is stepped from 9.01 GHz to 2.262 GHz in 4 MHz steps, for a "burst" of 64 measurements. Each measurement in the burst is taken in 0.1453 msec, for a total of 9.3 msec/burst. The system repeats the above measurement process until 64 bursts

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are measured. Figure 1 shows the format of the resulting 64×64 measurement array.

Note that each row of the array is a set of stepped-frequency measurements which can be processed to give downrange profiles. Similarly, each column of the array gives a set of measurements at a single frequency and at a set of equally-spaced time samples; these data can be processed to give Doppler profiles. Two-dimensional transforms (using either DFT or AR techniques) can be applied to this data to produce a two-dimensional ISAR image of the target [2]; in this paper, we restrict attention to the one-dimensional processing problem.

B. *The Autoregressive Model*

Let $\{x_n\}_{n=1}^N$ denote a single row or column of the radar data array. The AR model assumes that $\{x_n\}$ satisfies the following linear difference equation:

$$x_n = -\sum_{i=1}^{P} a_i x_{n-i} + e_n \tag{1}$$

where $\{e_n\}$ is a white noise sequence. The a_i coefficients are called the autoregressive coefficients.

There are several types of data which are well approximated by an AR model [9]. For the radar application, the following propoerty is of importance. Assume that the data sequence $\{x_n\}$ consists of a sum of P complex exponential signals

$$x_n = \sum_{i=1}^{P} K_i\,(\rho_i)^n \tag{2}$$

Then this sequence is exactly modeled by equation (1) with $e_n \equiv 0$. In this case, the AR coefficients and the exponential coefficients are related by

$$A(z) \triangleq 1 + a_1 z^{-1} + \cdots + a_P z^{-P} = \prod_{i=1}^{P}(1-\rho_i z^{-1}) \tag{3}$$

That is, the zeros of $A(z)$ give the exponential weights ρ_i.

The use of an AR model for the radar data can be justified for short wavelength radars. For such frequencies, radar targets are often well approximated by a small number of specular scattering centers [2]. Each specular scattering center produces an undamped complex exponential response ρ in the frequency domain along any row of the data array. The range of this scattering center is linearly related to the argument of ρ [2,8]. Similarly, if a scattering center is moving at a constant velocity, the radar data will contain an undamped exponential component along any column of the data array. If P such scattering centers exist, there will be a sum of P undamped exponentials in the data. In either case, the AR model of order P exactly fits this type of data.

In practice, scattering centers are not exactly specular, and motion is not exactly linear; thus, we would expect the exponential responses to deviate slightly from being undamped; that is, we expect $|\rho_i|$ to deviate from its nominal value of one.

C. *The AR Estimation Algorithm*

There are several methods for obtaining estimates of the AR coefficients from a set of data; see, e.g. [9]. We use a modified forward-backward prediction error algorithm [7] because it is computationally efficient, and because it is insensitive to initial phases of complex exponential signals in the data. Given a set of data $\{x_n\}_{n=1}^N$, the AR estimation algorithm entails solving the following system of equations:

$$\begin{bmatrix} r_P(0,0) & r_P(0,1) & \cdots & r_P(0,P) \\ r_P(1,0) & r_P(1,1) & \cdots & r_P(1,P) \\ \vdots & \vdots & \ddots & \vdots \\ r_P(P,0) & r_P(P,1) & \cdots & r_P(P,P) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_P \end{bmatrix} = \begin{bmatrix} \epsilon_P \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

where

$$r_P(i,j) = \sum_{k=1}^{N-P} x_{k+P-j}\, x^*_{k+P-i} + x_{k+i}\, x^*_{k+j} \tag{5}$$

The quantity ϵ_P represents the error energy for a P^{th} order AR model [7]; thus, the ratio ϵ_P/ϵ_0 indicates the percent error between the model and the original data. We note that a computationally efficient, order recursive implementation of the above algorithm is derived in [7].

Once the autoregressive coefficients have been estimated, the squared magnitude of the downrange or Doppler profile is given by

$$S_x(z) = \frac{\epsilon_P}{\left|1 + \sum_{i=1}^{P} a_i z^{-i}\right|^2} \tag{6}$$

where $z = e^{j4\pi f\delta T}$ for the Doppler profile (δT is the time between measurements), and where $z = e^{j4\pi c/r\delta f}$ for the downrange profile (δf is the frequency step). For target classification, however, the actual profile need not be computed; this profile is completely specified by the coefficients $\{a_1, a_2, \ldots, a_P, \epsilon_P\}$, and classification algorithms can operate directly on these coefficients.

III. Motion Compensation and Averaging Techniques

In the previous section we discussed the estimation of a single downrange or Doppler profile from a row or column of the data array. In this section we consider ways of combining the estimates from several rows or columns. Specifically, the problems of motion compensation and estimate averaging are discussed.

A. *Motion Compensation*

As described in Section II, the radar measurements are made at equal increments of time; since the target is in motion, some compensation is needed to align these measurements. Assume that the first measurement $x(1,1)$ is taken at the zero time reference. In order to compensate for the motion of the target, $x(i,j)$ must be multiplied by a phase factor to shift the target to the zero time reference. Thus, we can form a motion compensated data array $\{x_c(i,j)\}$ as

$$x_c(i,j) = x(i,j)e^{2\pi j\alpha(i,j)} \tag{7}$$

where $\alpha(i,j)$ is proportional to the difference in target range between the times of the measurement of $x(1,1)$ and $x(i,j)$.

There are several possible assumptions which can be made about target motion. If we assume negligible motion for elements along the rows of the data array, then $\alpha(i,j)$ has the form

$$\alpha(i,j) = \sum_{m=2}^{i} \Delta\alpha_m \quad (8)$$

where $\Delta\alpha_m$ is proportional to the the radial distance that the target moves between the $x(m-1,1)$ and the $x(m,1)$ measurement times. If the target is moving at a constant velocity during the data array measurement time, then these $\Delta\alpha_m$ terms are equal, so

$$\alpha(i,j) = (i-1)\Delta\alpha \quad (9)$$

If the motion along the rows of the data array are not negligible, then equations (8) and (9) can be modified accordingly. However, for the radar system considered in this paper, it was found that motion compensation along the rows of the data array is negligible [8].

If equation (8) or (9) is used to compensate for motion, then the parameters $\Delta\alpha_m$ or $\Delta\alpha$ must be estimated from the data. Below we discuss a simple, effective estimation scheme. The procedure is to model each row of the data array by a first order AR model (*i.e.* $P = 1$ in equation (4)). This estimate is equivalent to assuming that the target consists of only one scattering center. Each first order AR model produces a single pole estimate ρ_1 (which is equal to $-a_1$ in this case). The change in the argument of this pole from row to row in the data array gives the range $\Delta\alpha_m$:

$$\Delta\alpha_m = \arg(\rho_{m+1}) - \arg(\rho_m) \quad (10)$$

If these phase changes are assumed to be equal as in equation (9), then an estimate of α can be found by averaging the α_m estimates; from equation (10), this estimate is given by:

$$\Delta\alpha = \frac{\arg(\rho_N) - \arg(\rho_1)}{N-1} \quad (11)$$

In either case, the estimate involves performing first order AR estimates and using equation (10) or (11). Since an order-recursive implementation of the AR estimation algorithm is available [7], the first order AR estimate is computed as an intermediate step in the downrange impulse response AR estimates; thus, the above motion compensation method requires almost no additional computations to implement.

As a side note, if the AR estimate for a row of uncompensated data has been found, then the corresponding AR estimate for the motion compensated data can be easily determined. Let $\{\rho_i\}_{i=1}^{P}$ denote the poles from uncompensated AR estimates. Then the poles $\{\rho_i^c\}_{i=1}^{P}$ for data compensated by equation (7) are related to the uncompensated poles by a phase shift:

$$\rho_i^c = \rho_i e^{2\pi j\alpha(i,j)} \quad 1 \le i \le P \quad (12)$$

The above equation is valid only if motion compensation along the elements of a row is neglected, that is, if $\alpha(i,j)$ is not a function of j.

B. AR Estimate Averaging

From each 64×64 data array, we obtain 64 downrange profiles and 64 Doppler profiles. Since each estimate is noisy, it is desirable to apply some type of averaging technique to these estimates.

In [?], a number of averaging methods were considered for a slightly different AR coefficient estimation algorithm. It was found that averaging of the autoregressive coefficients and the reflection coefficients gave the best results. Autoregressive coefficient averaging is computed by:

$$\hat{a}_p = \frac{1}{N} \sum_{m=1}^{N} a_{m,p} \quad 1 \le p \le P \quad (13)$$

where N is the number of individual AR model estimates.

Both AR coefficient and reflection coefficient averaging were implemented and tested on X-band radar data. It was found that AR coefficient averaging gave decidedly better results. See [8] for details.

IV. Examples

In this section we present some examples which illustrate the performance of the AR estimation algorithm. For these estimates, two data sets were used, corresponding to two different aircraft. Both aircraft are flying directly toward the radar.

The estimamtes shown below are the poles ρ_i corresponding to the AR coefficient estimates. These poles are plotted in the complex plane; the unit circle is also shown in the plots. The plots show 64 overlapped estimates using motion compensated data as found from equations (7), (9), and (11).

First, we address the issue of model order selection. Figures 2–4 show the pole estimates corresponding to the downrange profile estimates for Aircraft 1 for AR model orders of 3, 5, and 9, respectively. It can be seen that in these cases, the poles form distinct clusters. As the order is increased, some of these clusters split into two clusters. For excessively high model orders, there are many pole estimates which are not clustered; this occurs because for high orders the AR method attempts to model noise.

Figure 5 shows the 64 pole estimates obtained by using a different order for each estimate; in this case, the model order was chosen automatically such that the relative modeling error ϵ_P/ϵ_0 was less than 25%. Note that while the average model order is ≈ 5 in this case, the poles are much less tightly clustered than in fixed order estimates (Figure 3). Moreover, variable order estimates are less amenable to averaging, especially if AR coefficient averaging is used. Thus, fixed order estimates seem to be the preferred order selection method for this application.

Next, the result of AR coefficient averaging is shown. Figure 6 shows the pole location of the 5^{th} order AR model obtained by averaging the AR coefficients corresponding to the estimate in Figure 3. It can be seen that the poles of the averaged model lie near the centroids of the pole clusters in Figure 3. For comparison, Figures 7 and 8 show the downrange profile estimates corresponding to the pole estimates in Figures 3 and 6, respectively.

Finally, Figure 9 shows the 5^{th} order pole estimates corresponding to the downrange profile estimates of Aircraft 2. It is clear that these pole clusters are substantially different from those of Aircraft 1. While formal classification experiments have not been carried out, it is apparent that the averaged pole locations for the two targets would be effective features to use for target identification.

V. Conclusions

We have studied the use of autoregressive modeling of radar data. An algorithm for obtaining AR estimates of the downrange and Doppler profiles of a target was developed. A simple, effective method of motion compensation was also presented; this method uses first order AR models to estimate the target motion. Issues of model order selection and estimate averaging were also considered.

The AR algorithm was applied to actual radar data obtained at the NOSC stepped-frequency radar facility. It was shown that using a fixed AR model order provides pole estimates which form tighter clusters than if an error energy order selection method is used. Also, when the AR coefficients of fixed order estimates are averaged, the resulting model has poles which lie near the centroids of the pole clusters from the individual AR estimates.

Finally, AR estimates from two different aircraft were shown. While no formal classification procedure was applied to the data, it is clear from the pole clusters that the centroids of these clusters can be used as features for target classification. Future work in this area will focus on development and analysis of classification algorithms which use AR coefficients (or their corresponding pole locations) as classification features.

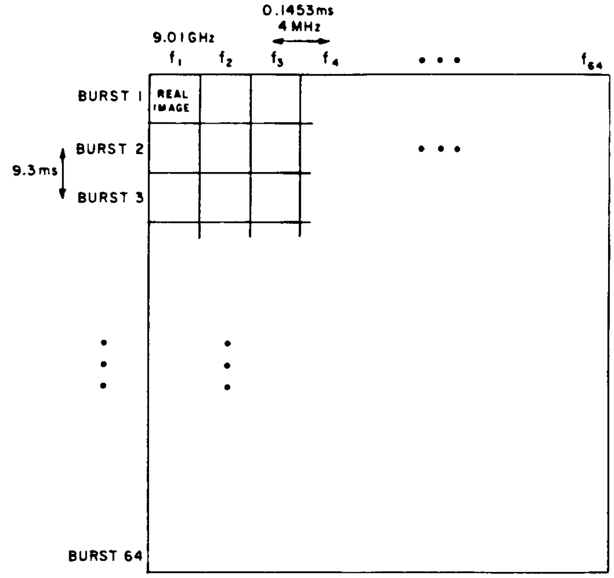

Figure 1: Data Array Timing and Frequency Information

VI. References

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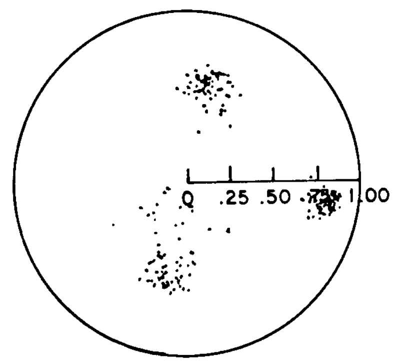

Figure 2: Downrange pole estimates for Aircraft 1; 3^{rd} order AR model.

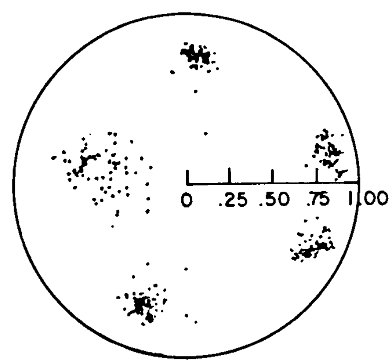

Figure 3: Downrange pole estimates for Aircraft 1; 5^{th} order AR model.

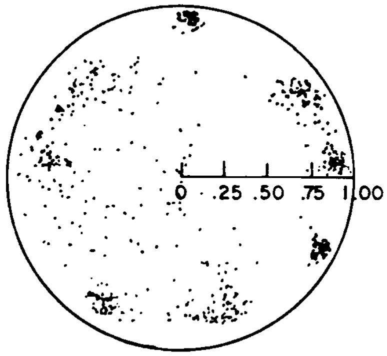

Figure 4: Downrange pole estimates for Aircraft 1; 9^{th} order AR model.

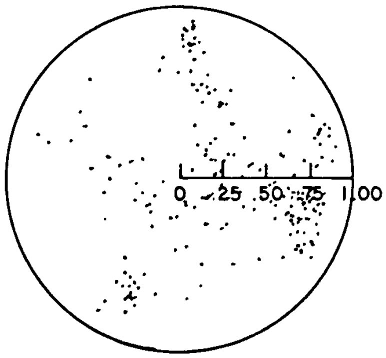

Figure 5: Downrange pole estimates for Aircraft 1 using a 25% model error criterion for order selection.

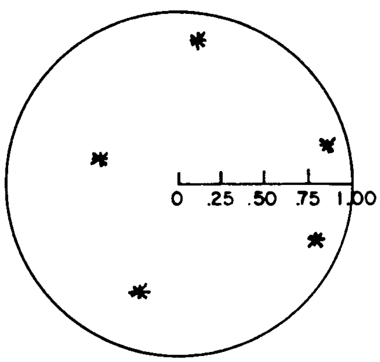

Figure 6: Pole locations for AR coefficient averaging of estimates in Figure 3.

Randolph L. Moses For a photograph and biography, see the paper *"Autoregressive Moving Average Modeling of Compact Radar Range Data"* which appears elsewhere in this proceedings.

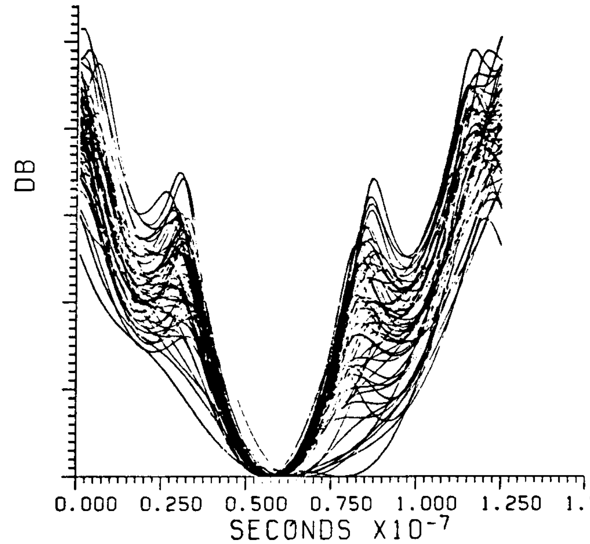

Figure 7: Downrange profile corresponding to Figure 3.

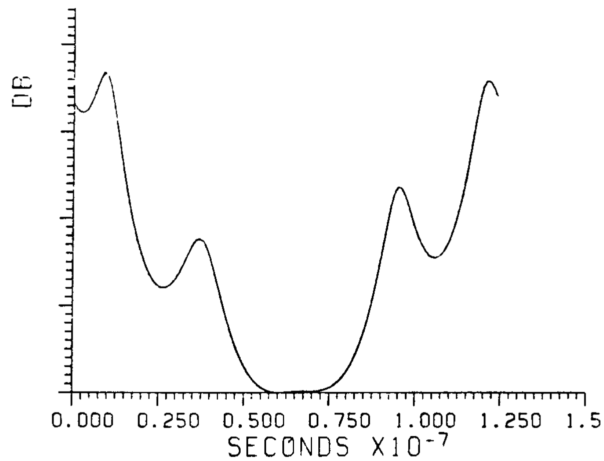

Figure 8: Downrange profile corresponding to Figure 6.

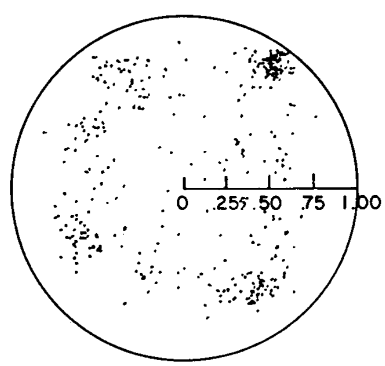

Figure 9: Downrange pole estimates for Aircraft 2; 5^{th} order AR model.