AUTOREGRESSIVE MOVING AVERAGE MODELING OF RADAR TARGET SIGNATURES

Rob Carrière and Randolph L. Moses
Department of Electrical Engineering
and ElectroScience Laboratory
The Ohio State University
2015 Neil Avenue
Columbus, Ohio 43210

Abstract

A method for characterizing radar target signatures with Autoregressive Moving Average (ARMA) models is developed. A parameterization of the model that corresponds directly to the geometric properties of the target is chosen, and an efficient algorithm for estimating these parameters is presented. Procedures for minimizing the effects of unmodeled dynamics are also developed. Experiments on radar measurements obtained from a compact range are presented to test the effectiveness of the ARMA modeling procedure.

I. Introduction

The response of a target to a radar signal (the radar return) contains a considerable amount of information about the target. It is possible to extract information about the shape and orientation of the target from the radar return; this information can then be compared with a catalog database to classify or identify an unknown target. This process is known as radar target identification (RTI).

The radar target identification system typically consists of a signal processing stage followed by a feature classification stage. The signal processing step involves operating on the raw radar data to extract salient *features* of that data which can be readily used for target classification or identification. Traditionally, target identification methods have used direct frequency domain data from the radar. The features used for classification consist of raw frequency domain data [1]. This method requires little or no signal processing, but has the disadvantage that one obtains no geometric characterization of the target. (Geometric features of an unknown target are useful because they can provide information about the target even when identification is not possible.) More recently, Discrete Fourier Transform (DFT) methods have been applied [2]; these methods first convert the frequency domain radar data into an estimate of the downrange impulse response of the target. Classification then proceeds based on some extracted features from this downrange impulse response (such as the ranges of strong scattering centers). While this concept is potentially very useful for RTI applications, the use of DFT methods presents some problems. First, DFT methods are resolution limited, so closely spaced scattering centers may not be resolved. Second, DFT methods are nonparametric; thus, there is no data reduction from the raw radar data to the downrange impulse response. Data reduction is desirable because classification algorithms are computationally burdensome if the number of features is large.

In this paper we develop and test an alternate radar signal processing approach. This approach applies AutoRegressive Moving Average (ARMA) modeling techniques to radar target data. Like DFT methods, the ARMA modeling technique produces an estimate of the downrange impulse response of the target. However, unlike the DFT model the ARMA model is parametric; the output of the signal processing stage consists of a small number of parameters which can be directly used for classification. ARMA methods are also not resolution limited by the bandwidth of the radar data as are DFT methods. Thus, these ARMA modeling methods possess some potentially useful properties for radar target identification.

In this paper we first develop a signal processing method for the ARMA modeling of radar target signatures from stepped frequency measurements. We then apply this modeling procedure to compact range measurements of scale models of several commercial aircraft.

II. The ARMA Modeling Method

Assume that we are given a set of N coherent stepped frequency response measurements of a target:

$$y(f_k) \qquad 0 \le k \le N-1 \tag{1}$$

$$f_k = f_0 + k\delta_f \tag{2}$$

Here, f_k is the k^{th} interrogation frequency and δ_f is the frequency step. From these measurements, we wish to obtain an accurate estimate of the impulse response of the target.

We can model the target impulse response using the partial fraction form of an ARMA model:

$$Y(r) = \sum_{i=1}^{na} \frac{d_i}{e^{j\pi(1-2\frac{r}{R})}} \qquad 0 \le r \le R \tag{3}$$

where

This work was supported in part by the Office of Naval Research under contract number N00014-85-K-0321

88 CH 2572-6/88/0000-0225 $01.00 1988 IEEE

$$R = \frac{c}{2\delta_f}$$

is the maximum unambiguous range. Here r is the range relative to a zero reference.

Each "pole" p_i corresponds to a scattering center on the target. The argument of p_i ($\arg p_i$) relates to the range r_i of this scattering center by

$$r_i = \pi(1 - 2\frac{\arg p_i}{R})$$

It can be seen from (3) that r_i is the range at which the i^{th} component of the impulse response achieves maximum amplitude. The magnitude of the pole p_i relates to the distribution in range of the energy received from this scattering center; as $|p_i| \to 1$, the scattered energy becomes more tightly concentrated at range r_i; this corresponds to an ideal point scatterer. For $|p_i| \neq 1$, the energy is spread over some range centered at r_i. The d_i parameter gives the amplitude of the i^{th} scattering center return.

The inverse Fourier transform of (3) yields:

$$y(f_k) = \sum_{i=1}^{na} d_i p_i^k \tag{4}$$

In other words, the ARMA representation assumes that the radar data can be modeled in the frequency domain as the sum of a number (na) of damped exponentials. Each exponential term corresponds to a scattering center. The energy P_i associated with the i^{th} scatterer can be found from (4)

$$P_i = \sum_{k=0}^{N-1} d_i d_i^* (p_i p_i^*)^k = |d_i|^2 \frac{1 - |p_i|^{2N}}{1 - |p_i|^2} \tag{5}$$

Note that equation (4) represents the data model for all k (i.e. for all frequencies), not only the measurement range. Thus, the ARMA model implicitly extrapolates the given measurement data $\{y(f_k)\}_{k=0}^{N-1}$ using the exponential rule (4). On the other hand, DFT methods implicitly assume that $y(f_k) \equiv 0$ outside the measurement range. As a result, DFT methods are inherently resolution limited. ARMA methods can theoretically have infinite resolution [3].

III. Formulation of a Pole-Residue Estimator

The pole-residue model formulated in the previous section can be computed directly from the discrete frequency measurements, as detailed below. This procedure is adapted from the time series analysis literature [4,5].

The first step involves estimating the coefficients a_i of the polynomial

$$\begin{aligned} A(z) &= z^{na} + a_1 z^{na-1} + \cdots + a_{na} \\ &= \Pi_{k=1}^{na}(z - p_k) \end{aligned} \tag{6}$$

and then finding the roots p_k. First a standard estimate for the autocorrelation sequence corresponding to the data is found; here we use the standard unbiased autocorrelation estimates:

$$\tilde{r}_k = \frac{1}{N-k} \sum_{t=0}^{N-1-k} y(f_t)^* y(f_{t+k}) \quad k = 0, 1, \ldots, N-1 \tag{7}$$

$$\tilde{r}_{-k} = \tilde{r}_k^* \tag{8}$$

Next, the coefficients of $A(z)$ are estimated from the autocorrelations parameters by solving the well-known overdetermined Yule Walker equation [6]:

$$\begin{bmatrix} \tilde{r}_{nc} & \cdots & \tilde{r}_{nc+1-na} \\ \vdots & & \vdots \\ \tilde{r}_{K-1} & \cdots & \tilde{r}_{K-na} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_{na} \end{bmatrix} = - \begin{bmatrix} \tilde{r}_{nc+1} \\ \vdots \\ \tilde{r}_K \end{bmatrix} \tag{9}$$

Where $K \geq na + nc$. From the a_k coefficients, we can find the poles $\{p_i\}_{i=1}^{na}$ by solving $A(z) = 0$ using standard complex polynomial root finding techniques.

Once the poles are obtained, the amplitudes d_i can be estimated using a least squares technique. From equation (4), such an estimator is given by minimizing $\| e \|$ in

$$\begin{bmatrix} p_1^0 & \cdots & p_{na}^0 \\ \vdots & & \vdots \\ p_1^{N-1} & & p_{na}^{N-1} \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_{na} \end{bmatrix} - \begin{bmatrix} y(f_0) \\ \vdots \\ y(f_{N-1}) \end{bmatrix} = \begin{bmatrix} e_0 \\ \vdots \\ e_{N-1} \end{bmatrix} \tag{10}$$

or

$$Pd - r = e$$

This gives the formula

$$d = (P^* P)^{-1} P^* r \tag{11}$$

where P^* is the complex conjugate transpose of the matrix P. Equations (7), (9), and (11) comprise the ARMA estimation procedure.

IV. Removal of Dispersive Scattering Centers

While the set of ARMA functions can model any continuous function arbitrarily well [7], we may still experience difficulties from the fact that some types of scatterers cannot be modeled exactly by the ARMA model, and by the problem that the radar data is not perfectly stationary with respect to frequency. Our experiments with the compact range data has shown that this effect may be significant over the frequency ranges considered. These effects typically show up in the estimates as spurious scatterers that are spatially very widely distributed; that is, one obtains poles with magnitudes which are not close to one. One way to counter this effect is to deliberately use a model order na that is larger than the expected number of scattering centers, then eliminate from the model any pole whose magnitude is not near one.

The spatial distribution of a scattering center is related to the magnitude of the pole estimated for that center; the contribution of the center to the frequency response at frequency f_i is (*c.f.* (4))

$$d_i p_i^k \qquad 0 \leq k < N$$

If $|p_i|$ is too far from unity (i.e. if the scattering center's energy is widely spread in range), then its contribution to the frequency model is greatly different at frequency f_0 than at frequency f_{N-1}. The ratio of these contributions is:

$$\begin{aligned} D &= \left| \frac{p_i^{N-1}}{p_i^0} \right| = |p_i|^{N-1} \quad , \quad |p_i| > 1 \\ D &= \left| \frac{p_i^0}{p_i^{N-1}} \right| = |p_i|^{-N+1} \quad , \quad |p_i| < 1 \end{aligned} \tag{12}$$

In the modified algorithm, if $|p_i|$ is such that D is greater than some threshold, that pole is discarded from the impulse response model. Empirical tests have shown that $D = 100$ provides good removal of highly dispersive scatterers without removing sharp scatterers. Moreover, it was found that estimation results are much less sensitive to model order selection if these poles are discarded.

V. Application of the ARMA Techniques to the ESL Compact Range Data

In this section we describe the results of applying the techniques derived above to OSU ESL Compact Range data. This is data taken from five aircraft models: the Boeing 707, 727 and 747, the DC10 and the Concorde. Measurements are taken at scaled frequencies that correspond to 5-80 MHz for a full-size aircraft.

All Figures below give response power (in dB meter^2) as a function of range (in meters). The range axis is scaled to correspond to actual aircraft dimensions; also, the physical center is located at the center of the range axis.

In every case, the estimation results are obtained from 20 radar measurements taken at evenly spaced frequencies in the 40–80 MHz band using horizontal polarization in both transmitter and receiver.

The ARMA results are shown using two plots, namely the estimated response and the estimated scattering centers (labelled "response" and "scatterers", respectively). The "response" plots are target impulse responses and can be compared with the DFT figures. The "scatterers" plot is a graphical presentation of the pole and residue coefficients: each horizontal line represents a scattering center; the vertical tick mark gives the estimated range of the scattering center, and the width of the horizontal line represents the 3dB spatial dispersion of the scattering center (analogous to 3 dB bandwidth). The height of each horizontal line gives the energy associated with the scattering center.

Figure 1 gives the downrange response profile from an ARMA model of order $na = 9$ for the Concorde model at 10° azimuth. Note that although 9 scattering centers were originally modeled, the algorithm retained only five scattering centers. For comparison, Figure 2 gives the downrange profile of this target using DFT methods. Comparing Figures 1 and 2, the strong peaks of the two responses coincide.

The "scatterers" graph in Figure 1 represents the actual coefficients in the ARMA model. Note that the scattering centers depicted on the graph coincide with the peaks in the "response" curve. Moreover, these peaks seem to correspond to locations of geometrically important features on the aircraft (such as cockpit windows, leading and trailing edges of wings and engines, etc.)

Figures 3,1 and 4 show the estimated ARMA responses for the Concorde at azimuth angles of 0°, 10°, 20° respectively. Some of the scattering centers move smoothly in range as the azimuth angle changes, while other scattering centers seem to appear at some azimuths but not at others. The mechanism behind this effect is not well understood at this point, and is currently being studied. However, it appears that some scattering centers reliably appear for several aspect angles, and these could be used for target classification.

Figure 5 shows the ARMA downrange profile estimate from a Boeing 707 at 10° azimuth. Comparing with Figure 1, it can be seen that the responses of the two aircraft are markedly different.

While no formal classification studies have yet been carried out, it is apparent from Figures 1 – 5 that there are significant differences in the estimated models, which can be exploited for use in target identification.

VI. Conclusions

From our simulations and analysis, we can conclude the following:

- ARMA models can be used to estimate the impulse response of a radar target, given radar cross-section measurements at a number of stepped frequencies. Our simulations show that about 20 frequencies is enough to get satisfactory results for the five models tested.
- The resulting ARMA estimates have advantages over estimates obtained using Discrete Fourier Transform techniques:
  - ARMA models describe the target as a set of scattering centers. The model is parameterized by a small number of coefficients which directly relate to physical attributes of scattering centers.
  - ARMA models are not resolution limited by the bandwidth of the measurements.

Spacing of the frequency samples should be chosen such that the maximum unambiguous range is at least as large as the target. If the spacing of the frequency samples is chosen such that the maximum unambiguous range is larger than the target, modeling quality can be improved by applying data decimation techniques before the actual estimation. In a noisy environment, such a strategy could be used to improve the signal to noise ratio.

The total bandwidth of the measurements should be restricted so as to avoid violating the implicit stationarity assumption of the ARMA model. For the cases we have studied, we found that a bandwidth of 40–80 MHz works well.

While no formal quantative studies have been made, our simulations indicate that the ARMA modeling technique is capable of distinguishing between different aircraft, and between different aspect angles for the same aircraft. In other words, the ARMA modeling technique seems to be a suitable signal processing method for preparing the radar data for classification. Future research will focus on a more formal, quantative evaluation of classification performance using ARMA parameters.

References

[1] A. Kamis, F. D. Garber, and E. K. Walton, "Radar target classification studies – software development and documentation," Technical Report 716559-1, The Ohio State University, Department of Electrical Engineering, ElectroScience Laboratory, September 1985.

[2] D. L. Mensa, *High Resolution Radar Imaging.* Dedham, MA: Artech House, 1981.

[3] L. Marple, *Digital Spectral Analysis with Applications.* Englewood Cliffs: Prentice-Hall, 1987.

[4] A. A. Beex and L. L. Scharf, "Covariance sequence approximation for parametric spectrum modeling," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. ASSP-29, no. 5, pp. 1042–1052, October 1981.

[5] J. Li, "On the estimation of damped and undamped sinusoids with application to speech analysis," M.S. Thesis, The Ohio State University, 1987.

[6] G. Box and G. Jenkins, *Time Series Analysis: Forecasting and Control.* San Fransisco: Holden-day, 1976.

[7] L. H. Koopmans, *The Spectral Analysis of Time Series.* New York: Academic Press, 1974.

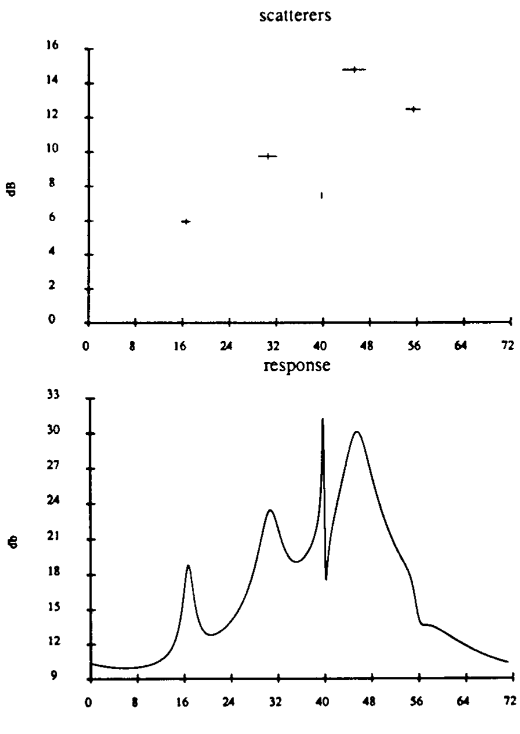

Figure 1: ARMA response of the Concorde at 10° azimuth, using 20 frequency measurements from 40–80 MHz, 9th order model.

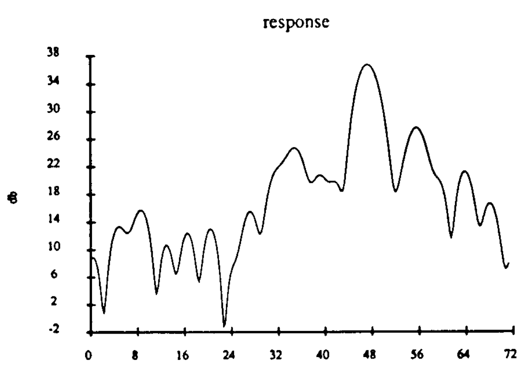

Figure 2: FFT of the Concorde at 10° azimuth, using 20 frequency measurements from 40–80 MHz, zero padded to 256 points.

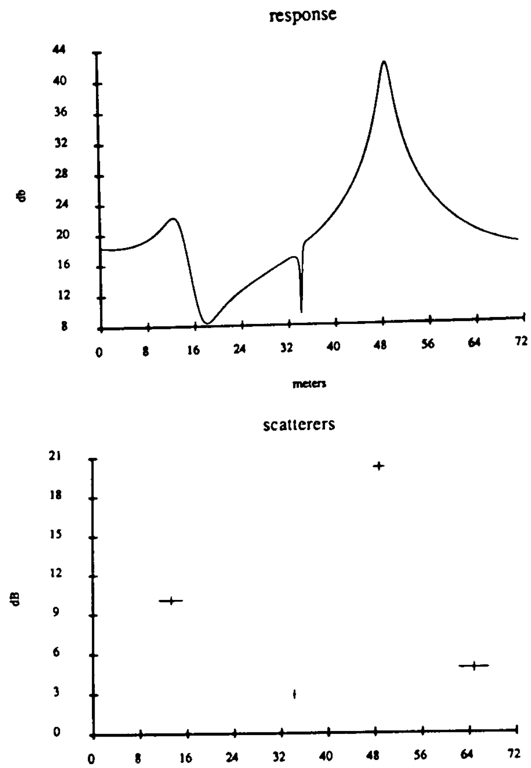

Figure 3: ARMA response of the Concorde at 0° azimuth, using 20 frequency measurements from 40–80 MHz, 9th order model.

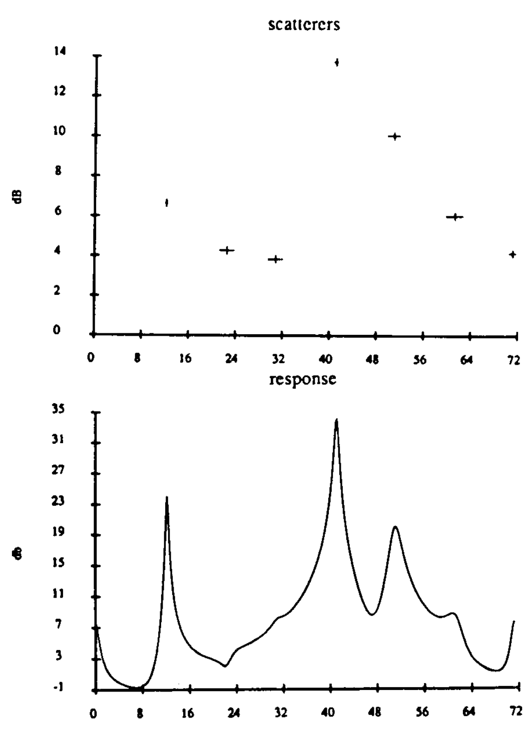

Figure 4: ARMA response of the Concorde at 20° azimuth, using 20 frequency measurements from 40–80 MHz, 9th order model.

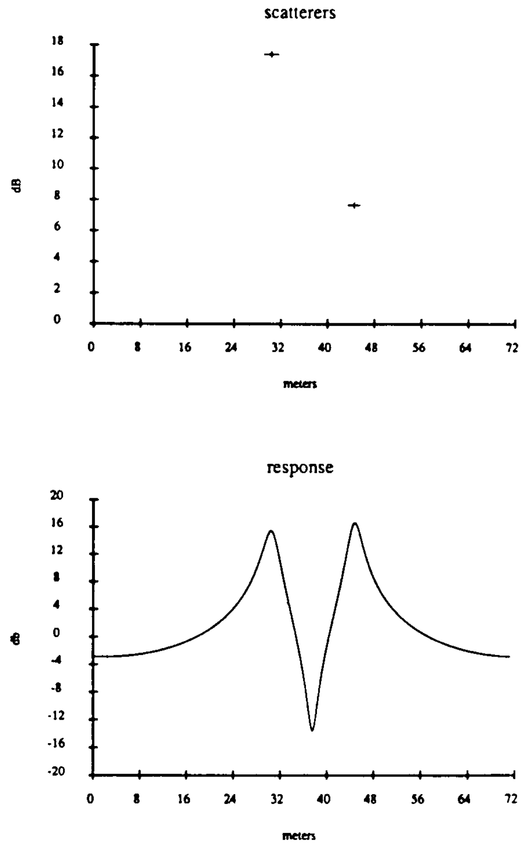

Figure 5: ARMA response of the Boeing 707 at 10° azimuth, using 20 frequency measurements from 40–80 MHz, 9th order model.

Rob Carrière (S'86) received the ingenieur degree in electrical engineering from the Eindhoven University of Technology in 1987.

He is currently working towards a PhD in electrical engineering at The Ohio State University. His research interests are in digital signal processing, especially parameter estimation techniques.

Randolph L. Moses (S'78–M'85) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Virginia Polytechnic Institute and State University, Blacksburg, in 1979, 1980, and 1984, respectively.

During the summer of 1983 he was a SCEEE Summer Faculty Research Fellow at Rome Air Development Center, Rome, NY. From 1985 to 1986 he was with the Eindhoven University of Technology, Eindhoven, The Netherlands, as a NATO Postdoctoral Fellow. Since 1985 he has been an Assistant Professor in the Department of Electrical Engineering, The Ohio State University. His research interests are in digital signal processing, and include parametric time series analysis, system identification, and model reduction.

Dr. Moses is a member of Eta Kappa Nu, Tau Beta Pi, Phi Kappa Phi, and Sigma Xi.