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# Output Feedback Control of Jet Engine Stall and Surge Using Pressure Measurements*

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**Abstract**

The problem of controlling surge and stall in jet engine compressors is of fundamental importance in preventing damage and lengthening the life of these components. In this paper, we use the Moore-Greitzer mathematical model to develop an output feedback controller for these two instabilities (only one of the three states is measurable). This problem is particularly challenging since the system is not completely observable and, hence, none of the output feedback control techniques found in the literature can be applied to recover the performance of a full state feedback controller. However, we show how to successfully solve it by using a novel output feedback approach for the stabilization of general stabilizable and incompletely observable systems.

## 1 Introduction and Problem Description

In this paper we consider the problem of controlling two instabilities which occur in jet engine compressors, namely rotating stall and surge. Rotating stall develops when there is a region of stagnant flow rotating around the circumference of the compressor causing undesired vibrations in the blades and reduced

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pressure rise of the compressor. Surge is an axisymmetric oscillation of the flow through the compressor that can cause undesired vibrations in other components of the compression system and damage to the engine. In [18], Moore and Greitzer developed a three-state finite dimensional Galerkin approximation of a nonlinear PDE model describing the compression system. Since its development, several researchers have used the Moore-Greitzer three state model (MG3) to design stabilizing controllers for stall and surge. The available control approaches may be divided into three main categories: 1) Linearization and linear perturbation models (e.g., [23, 19, 4] among others); 2) Bifurcation analysis (e.g., [10, 11, 6, 17, 1]); and 3) Lyapunov based methods (e.g., [8, 3, 22]). Most existing results focus on the development of state feedback controllers, thus complicating their practical implementation as in [6], where the authors use sensor arrays (2D sensing) to implement a state feedback control law depending on the squared amplitude of the first harmonic of asymmetric flow and the derivative of the air flow through the compressor. In [8], a partial state feedback controller simplifies practical implementation by only requiring measurements of the mass flow and plenum pressure rise (hence 2D sensing is not needed). On the other hand, the limitation of this partial state feedback controller lies in the fact that it cannot globally stabilize a unique equilibrium point.

To the best of our knowledge, no attempt has been made to design a stabilizing output feedback controller (using only plenum pressure rise feedback) based on a full-state feedback control law. This is probably due to the fact that MG3 becomes unobservable when there is no mass flow through the compressor, i.e., the system is not uniformly completely observable (UCO), and none of the techniques found in the output feedback control literature (e.g., [5, 21, 20, 7, 15, 16, 2]) can be employed for the solution of this problem. In this paper we introduce a new globally stabilizing full state feedback control law for MG3, and we employ the theory developed in [14, 12] for the output feedback control of incompletely observable nonlinear systems to regulate stall and surge by using only pressure measurements. The MG3 model is described by (see [9, 8] for an analogous exposition)

$$
\begin{aligned}
\dot{\Phi} &= -\Psi + \Psi_C(\Phi) - 3\Phi R \\
\dot{\Psi} &= \frac{1}{\beta^2}(\Phi - \Phi_T) \qquad (1)\\
\dot{R} &= \sigma R(1 - \Phi^2 - R),\ R(0) \geq 0
\end{aligned}
$$

where $\Phi$ represents the mass flow, $\Psi$ is the plenum pressure rise, $R \geq 0$ is the normalized stall cell squared amplitude, $\Phi_T$ is the mass flow through the throttle, $\sigma = 7$, and $\beta = 1/\sqrt{2}$. The functions $\Psi_c(\Phi)$ and $\Phi_T(\Psi)$ are the compressor and throttle characteristics, respectively, and are defined as $\Psi_C(\Phi) = \Psi_{C_0} + 1 + 3/2\Phi - 1/2\Phi^3$, $\Psi = \frac{1}{\gamma}(1 + \Phi_T(\Psi))^2$, where $\Psi_{C_0}$ is a constant and $\gamma$ is the throttle opening, the control input. Given the static relationship existing between $\Phi_T$ and $\gamma$, without loss of generality, in what follows we will design a controller assuming that $\Phi_T$ is our control input. Our control objective is to stabilize system (1) around the critical equilibrium $R^e = 0, \Phi^e = 1, \Psi^e = \Psi_C(\Phi^e) = \Psi_{C_0} + 2$, which achieves the peak operation on the compressor characteristic. We shift the origin to the desired equilibrium with the change of variables $\phi = \Phi - 1, \psi = \Psi - \Psi_{C_0} - 2$. System (1) then becomes

$$
\begin{aligned}
\dot{R} &= -\sigma R^2 - \sigma R(2\phi + \phi^2) \\
\dot{\phi} &= -\psi - 3/2\phi^2 - 1/2\phi^3 - 3R\phi - 3R \qquad (2)\\
\dot{\psi} &= -\frac{1}{\beta^2}(\Phi_T - 1 - \phi)
\end{aligned}
$$

The pressure rise (and hence $\psi$) is the only measurable state variable. It is readily seen that this system is input output feedback linearizable with relative degree one (the first derivative of $\psi$ contains the input $\gamma$), and its zero-dynamics are nonminimum phase.

## 2 State Feedback Control Design

For convenience, in the remainder of the paper we will redefine the control input to be $u = \Phi_T - 1$. Next, notice that Assumption A2 in [14, 12] is satisfied since, for example, a stabilizing control law for (2) is given in [9] by means of backstepping design. However, the control law proposed in [9] turns out to be quite complex. In [8], it is shown that a linear partial state feedback control law of the type $u = d_1\psi - d_2\phi$ achieves either a unique asymptotically stable equilibrium point with domain of attraction $\{(R, \phi, \psi) \in \mathbb{R}^3 | R \geq 0\}$ or two equilibria on the axisymmetric and stall characteristic, with domains of attraction $\{(R, \phi, \psi) \in \mathbb{R}^3 | R = 0\}$ and $\{(R, \phi, \psi) \in \mathbb{R}^3 | R > 0\}$, respectively (see Theorem 3.1 in [8]). Here, this problem is overcome by viewing system (2) as an interconnection of two subsystems, namely the $R$-subsystem and the $(\phi, \psi)$-subsystem, and then building a full state feedback controller which makes the origin of (2) an asymptotically stable equilibrium point with domain of attraction $\{(R, \phi, \psi) \in \mathbb{R}^3 | R \geq 0\}$, as seen in the next theorem.

**Theorem 1** *For system (2), with the choice of the control law*

$$\bar{u} = (1 - \beta^2 k_1 k_2)\phi + \beta^2 k_2 \psi + 3\beta^2 k_1 R\phi \tag{3}$$

*where $k_1$ and $k_2$ are positive scalars satisfying the inequalities,*

$$k_1 > \frac{17}{8} + \frac{(2C\sigma + 3)^2}{2} \tag{4}$$

$$\left(C\sigma - \frac{105}{64}\right) k_1^2 + \frac{3}{4}\left(-\frac{1}{2}C\sigma + \frac{21}{4}\right) k_1 - (C\sigma + 3)^2 > 0 \tag{5}$$

$$k_2 > k_1 + \frac{9}{4}k_1^2 + \frac{9k_1}{4k_1 - 9/2} + \frac{(k_1^2 - 1)^2}{4} \tag{6}$$

$$C > \frac{3}{2\sigma} \tag{7}$$

*the origin is an asymptotically stable equilibrium point with domain of attraction $\mathcal{A} = \{(R, \phi, \psi) \in \mathbb{R}^3 | R \geq 0\}$.*

**Proof.** For the sake of simplicity, redefine the control input to be $u' = -\frac{1}{\beta^2}(u - \phi)$, so that the last equation in (2) becomes $\dot{\psi} = u'$. Next, notice that system (2) can be viewed as the interconnection of two subsystems:

$$[S_1]\ \dot{R} = -\sigma R^2, \quad [S_2] \begin{cases} \dot{\phi} = -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 \\ \dot{\psi} = -u' \end{cases}$$

A Lyapunov function for $[S_1]$, defined on the domain $\{R \in \mathbb{R} |\, R \geq 0\}$, is $V_1 = R$, and its time derivative is readily found to be $\dot{V}_1 = -\sigma R^2$ thus showing that the origin of $[S_1]$ is an asymptotically stable equilibrium point of $[S_1]$, and its domain of attraction is $\{R \in \mathbb{R} |\, R \geq 0\}$. As for subsystem $[S_2]$ the analysis found in Section 2.4.3 in [9] suggests using $V_2 = \frac{1}{2}\phi^2 + \frac{k_1}{8}\phi^4 + \frac{1}{2}(\phi - k_1\psi)^2$, where $k_1$ is a positive design constant. Furthermore, in [9], a stabilizing control law for $[S_2]$ is found to be $u' = -c_1\phi + c_2\psi$, where $c_1$ and $c_2$ are two appropriate positive constants. In the following we will show that, in order to stabilize the interconnection of systems $[S_1]$ and $[S_2]$, one needs to add to $u' = -c_1\phi + c_2\psi$ a term which is proportional to the product $R\phi$. Based on these considerations, consider the following candidate Lyapunov function for system (2),

$$V = CV_1 + V_2 = CR + \frac{1}{2}\phi^2 + \frac{k_1}{8}\phi^4 + \frac{1}{2}\left(\psi - k_1\phi\right)^2 \tag{8}$$

where $C > 0$ is a scalar. After noticing that $V$ is positive definite on the domain $\mathcal{A}$, and letting $\tilde{\psi} = \psi - k_1\phi$, we calculate the time derivative of V as follows,

$$\begin{aligned}\dot{V} =& -C\sigma R^2 - C\sigma R(2\phi + \phi^2) + \left(\phi + \frac{k_1}{2}\phi^3\right)\left(-\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R\right) + \\ &+ \tilde{\psi}\left(u' + k_1\psi + \frac{3}{2}k_1\phi^2 + \frac{1}{2}k_1\phi^3 + 3k_1R\phi + 3k_1R\right)\end{aligned} \tag{9}$$

Here, as in [9], we use the identity $-\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 = -\frac{1}{2}\left(\phi + \frac{3}{2}\right)^2\phi + \frac{9}{8}\phi$ to eliminate the potentially destabilizing term $-\left(\phi + k_1/2\phi^3\right)3/2\phi^2$. Next, substituting (3) into (9) (after taking in account the definition of $u'$), letting $\bar{k}_1 = k_1 - 9/8$, and using the definition of $\tilde{\psi}$, we get

$$\begin{aligned}\dot{V} =& -C\sigma R^2 - C\sigma R(2\phi + \phi^2) + \left(\phi + \frac{k_1}{2}\phi^3\right)\left(-\tilde{\psi} - \bar{k}_1\phi - \frac{1}{2}\left(\phi + \frac{3}{2}\right)^2\phi - 3R\phi - 3R\right) + \\ &+ \tilde{\psi}\left(-(k_2 - k_1)\tilde{\psi} + k_1^2\phi + \frac{3}{2}k_1\phi^2 + \frac{1}{2}k_1\phi^3 + 3k_1R\right)\end{aligned} \tag{10}$$

Now notice that the expression $-\left(\phi + \frac{k_1}{2}\phi^3\right)\frac{1}{2}\left(\phi + \frac{3}{2}\right)^2$ can be discarded since it is negative definite, and that the term $\frac{k_1}{2}\phi^3\tilde{\psi}$ cancels out. After collecting the remaining terms, we get

$$\begin{aligned}\dot{V} \leq & -C\sigma R^2 - (2C\sigma+3)R\phi - (C\sigma+3)R\phi^2 - \bar{k}_1\phi^2 - \left(\frac{k_1\bar{k}_1}{2} + \frac{3k_1}{2}R\right)\phi^4 - \frac{3k_1}{2}R\phi^3 + \\ & + \tilde{\psi}\left(-(k_2-k_1)\tilde{\psi} + (k_1^2-1)\phi + \frac{3}{2}k_1\phi^2 + 3k_1 R\right)\end{aligned} \tag{11}$$

By using Young's inequality five times we have

$$\begin{gathered}-(2C\sigma+3)R\phi \leq \frac{1}{2}R^2 + \frac{(2C\sigma+3)^2}{2}\phi^2, \qquad -\frac{3k_1}{2}R\phi^3 \leq \frac{3k_1}{2}\left(\frac{R\phi^2}{4} + R\phi^4\right), \\ (k_1^2-1)\phi\tilde{\psi} \leq \phi^2 + \frac{(k_1^2-1)^2}{4}\tilde{\psi}^2, \qquad 3k_1 R\tilde{\psi} \leq R^2 + \frac{9}{4}k_1^2\tilde{\psi}^2, \qquad \frac{3}{2}k_1\phi^2\tilde{\psi} \leq \frac{k_1\bar{k}_1}{4}\phi^4 + \frac{9k_1}{4\bar{k}_1}\tilde{\psi}^2\end{gathered}$$

Applying the inequalities above to (11) we get

$$\begin{aligned}\dot{V} \leq & -\left(C\sigma - \frac{3}{2}\right)R^2 - \left(\bar{k}_1 - \frac{(2C\sigma+3)^2}{2} - 1\right)\phi^2 - \left(k_2 - k_1 - \frac{9}{4}k_1^2 - \frac{9k_1}{4\bar{k}_1} - \frac{(k_1^2-1)^2}{4}\right)\tilde{\psi}^2 + \\ & - \left(C\sigma + 3 - \frac{3}{8}k_1\right)R\phi^2 - \frac{k_1\bar{k}_1}{4}\phi^4, \\ \leq & -\begin{bmatrix} R \\ \phi^2 \end{bmatrix}^\top \begin{bmatrix} C\sigma - \frac{3}{2} & \frac{1}{2}\left(C\sigma + 3 - \frac{3}{8}k_1\right) \\ \frac{1}{2}\left(C\sigma + 3 - \frac{3}{8}k_1\right) & \frac{1}{4}k_1\bar{k}_1 \end{bmatrix}\begin{bmatrix} R \\ \phi^2 \end{bmatrix} - \left(\bar{k}_1 - \frac{(2C\sigma+3)^2}{2} - 1\right)\phi^2 + \\ & - \left(k_2 - k_1 - \frac{9}{4}k_1^2 - \frac{9k_1}{4\bar{k}_1} - \frac{(k_1^2-1)^2}{4}\right)\tilde{\psi}^2\end{aligned} \tag{12}$$

Hence, $\dot{V}$ is negative definite on the domain $\mathcal{A}$, provided that the quadratic form above is positive definite and that the coefficients multiplying $\phi^2$ and $\tilde{\psi}^2$ be positive. By imposing the positive definiteness of the quadratic form we obtain $C\sigma - \frac{3}{2} > 0$, $\left(C\sigma - \frac{3}{2}\right)\frac{1}{4}k_1\bar{k}_1 - \frac{1}{4}\left(C\sigma + 3 - \frac{3}{8}k_1\right)^2 > 0$, while by imposing the positivity of the coefficients of the remaining two terms we get $\bar{k}_1 > \frac{(2C\sigma+3)^2}{2} + 1$, $k_2 > k_1 + \frac{9}{4}k_1^2 + \frac{9k_1}{4\bar{k}_1} + \frac{(k_1^2-1)^2}{4}$. By using the definition of $\bar{k}_1$, inequalities (4), (5), (6), and (7) follow. In conclusion, if $k_1$, $k_2$, and $C$ are chosen so that (4)-(7) hold, we have that $\dot{V}$ is negative definite on $\mathcal{A}$ which contains the origin. This leads to the conclusion that $\{R = 0, \phi = 0, \tilde{\psi} = 0\}$ is an asymptotically stable equilibrium point, which in turn implies that $\{R = 0, \phi = 0, \psi = 0\}$ is an asymptotically stable equilibrium point.

Our next objective is to show that $\mathcal{A}$ is a region of attraction for the origin. This, however, is not immediately evident from our result, since the set $\{[R,\phi,\psi]^\top \in \mathbb{R}^3 \,|\, V \le K, K > 0\}$ is unbounded and, due to the presence of the term $CR$ in $V$, it is not completely contained in $\mathcal{A}$. In other words, it may happen that, while the Lyapunov function is decreasing, $R$ becomes negative, and thus the state trajectory exits the set $\mathcal{A}$, where $\dot{V}$ is guaranteed to be negative definite. Therefore, in order to complete our analysis, we need to show that $\mathcal{A}$ is invariant, which, together with $\dot{V} < 0$, implies that the set $\{[R,\phi,\psi]^\top \in \mathbb{R}^3 \,|\, V \le K, K > 0\} \cap \mathcal{A}$ is a region of attraction of the origin for any $K > 0$. This is readily seen by noticing that, on the boundary of $\mathcal{A}$, $R = 0$. From (2), $R = 0$ implies $\dot{R} = 0$, thus proving that no trajectory of the system can cross the boundary of $\mathcal{A}$, and therefore $\mathcal{A}$ is invariant. In conclusion, given any initial condition $[R(0), \phi(0), \psi(0)]^\top$ in $\mathcal{A}$, there exists a constant $K > 0$ such that the initial condition is contained in the set $\{[R,\phi,\psi]^\top \in \mathbb{R}^3 \,|\, V \le K, K > 0\} \cap \mathcal{A}$, thus proving that the origin of system (2) is an asymptotically stable equilibrium point with domain of attraction $\mathcal{A}$.

$\blacksquare$

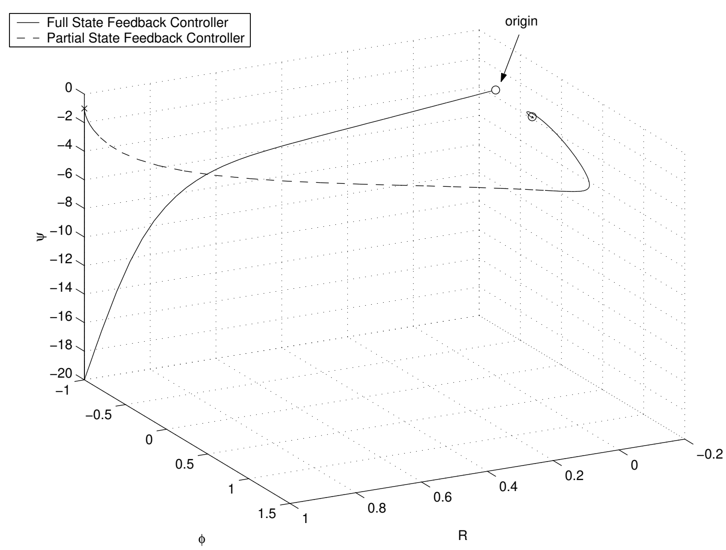



Figure 1: Comparison between the partial state feedback controller developed in [8] and the full state feedback controller (3).

**Remark 1:** By using inequalities (4)-(7), it is easy to show that the only equilibrium point of the closed-loop system on the set $\mathcal{A}$ is the origin, as predicted by Theorem 1. Figure 1 shows the evolution of the closed-loop trajectories under the partial state feedback controller developed in [8] and the controller (3) for a particular choice of the coefficients $d_1, d_2, k_1, k_2$. The partial state feedback controller stabilizes an equilibrium point different from the origin $(R, \phi, \psi) = (0, 0, 0)$.

**Remark 2:** Inequalities (4)-(7) represent conservative bounds on $k_1$ and $k_2$. In practical implementation, these parameters may be chosen significantly smaller after some tuning.

In order to complete the state feedback design, we have to add an appropriate number of integrators at the input side of the system (see [14, 12]). Following the procedure outlined in [14, 12], we form the observability mapping $\mathcal{H}$

$$y_e = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \mathcal{H}\left([R, \phi, \psi]^\top, u, \dot{u}\right) = \begin{bmatrix} \psi \\ -1/\beta^2(u - \phi) \\ 1/\beta^2\left(-\dot{u} - \psi - 3/2\phi^2 - 1/2\phi^3 - 3R\phi - 3R\right) \end{bmatrix} \tag{13}$$

Notice that the observability assumption A1 in [14, 12] is satisfied, for all $\phi \neq -1$, with $n_u = 2$ in that given $y_e, u$, and $\dot{u}$, one can uniquely find $R, \phi, \psi$. The operating point $\phi = -1$ corresponds to $\Phi = 0$, i.e., no mass flow through the compressor which is a condition we would like to avoid during normal engine operation. Since $n_u = 2$, we extend the system with two integrators $\dot{z}_1 = z_2$, $\dot{z}_2 = v$, $u = z_1$. To simplify the notation in the following, define $x = [R, \phi, \psi]^\top$, and rewrite (2) as $\dot{x} = f(x) + g(x)z_1$. Next, we find a stabilizing control law for the extended system by using the integrator backstepping lemma: $v = \dot{\alpha} - \tilde{z}_1 - k_4\tilde{z}_2 \triangleq \varphi(x, z)$, where $\tilde{z}_1 = z_1 - \bar{u}$, $\alpha = -k_3\tilde{z}_1 - \dfrac{\partial V}{\partial x}g(x) + \dfrac{\partial \bar{u}}{\partial x}[f(x) + g(x)\, z_1]$, $\tilde{z}_2 = z_2 - \alpha$, and $k_3, k_4$ are arbitrary positive constants. This completes the design of a stabilizing state feedback for the extended system. The Lyapunov function of the closed-loop extended system is $\bar{V} = V + \frac{1}{2}\tilde{z}_1^2 + \frac{1}{2}\tilde{z}_2^2$. Notice that, following the same reasoning as in the proof of Theorem 1 the set $\{[R, \phi, \psi, z_1, z_2]^\top \in \mathbb{R}^5 \mid R \geq 0\}$ is invariant; hence by applying the backstepping lemma we guarantee that the origin of the extended system is asymptotically stable with domain of attraction $\mathcal{D} = \mathcal{A} \times \mathbb{R}^2$.

## 3 Output Feedback Design

The validity of the observability assumption A1 in [14, 12] allows us to design a stable observer. As already pointed out, Assumption A1 in [14, 12] is satisfied on the domain $\mathcal{X} \times \mathcal{U} = \left\{[R, \phi, \psi] \in \mathbb{R}^3 \mid \phi > -1\right\} \times \mathbb{R}^2$.

We first design the observer developed in [14, 12],

$$\begin{aligned}
\dot{\hat{R}} &= -\sigma\hat{R}^2 - \sigma R(2\hat{\phi} + \hat{\phi}^2) - \frac{(l_1/\rho) + \beta^2(3\hat{\phi} + 3\hat{R} + (3/2)\hat{\phi}^2)(l_2/\rho^2) + \beta^2(l_3/\rho^3)}{3(1+\hat{\phi})}(\psi - \hat{\psi}) \\
\dot{\hat{\phi}} &= -\hat{\psi} - 3/2\,\hat{\phi}^2 - 1/2\,\hat{\phi}^3 - 3\hat{R}\hat{\phi} - 3\hat{R} + \beta^2(l_2/\rho^2)(\psi - \hat{\psi}) \\
\dot{\hat{\psi}} &= -\frac{z_1 - \hat{\phi}}{\beta^2} + (l_1/\rho)(\psi - \hat{\psi})
\end{aligned} \tag{14}$$

where $\rho$ is a positive design parameter and the vector $L = [l_1, l_2, l_3]^\top \in \mathbb{R}^3$ is chosen to be Hurwitz. Next, we calculate the solution $P$ of the Lyapunov equation $P(A_c - LC_c) + (A_c - LC_c)^\top P = -I$, where $(A_c, C_c)$ is a canonical observable pair. In order to confine the observer estimates to within the observable space, we implement the following projection,

$$\dot{\hat{x}}^P = \left[\frac{\partial \mathcal{H}}{\partial \hat{x}}\right]^{-1} \left\{\mathcal{P}\left(\hat{\xi}, \dot{\hat{\xi}}, z, \dot{z}\right) - \frac{\partial \mathcal{H}}{\partial z}\dot{z}\right\}$$

$$\mathcal{P}(\hat{\xi}, \dot{\hat{\xi}}, z, \dot{z}) = \begin{cases} \dot{\hat{\xi}} - \Gamma\dfrac{N(\hat{\xi})\left(N(\hat{\xi}, z)^\top\dot{\hat{\xi}} + N_z(\hat{\xi}, z)^\top\dot{z}\right)}{N(\hat{\xi}, z)^\top\Gamma N(\hat{\xi}, z)} & \text{if } N(\hat{\xi}, z)^\top\dot{\hat{\xi}} + N_z(\hat{\xi}, z)^\top\dot{z} \geq 0 \text{ and } \hat{\xi} \in \partial C_\xi(z) \\ \dot{\hat{\xi}} & \text{otherwise} \end{cases}$$

where $\Gamma = (S\mathcal{E}')^{-1}(S\mathcal{E}')^{-1}$, $S = S^\top$ denotes the matrix square root of $P$, $\hat{\xi} = \mathcal{H}(\hat{x}, z)$, $\dot{\hat{\xi}} = \left\{\frac{\partial \mathcal{H}}{\partial \hat{x}}\dot{\hat{x}} + \frac{\partial \mathcal{H}}{\partial z}\dot{z}\right\}$, and $C_\xi(z)$ is the cube

$$C_\xi(z) = \left\{\xi \in \mathbb{R}^3 \mid \xi_1 \in [a_1, b_1], \xi_2 \in \left[-\frac{1}{\beta^2}(z_1 + a_2), -\frac{1}{\beta^2}(z_1 - b_2)\right], \xi_3 \in \left[\frac{1}{\beta^2}(-z_2 - a_3), \frac{1}{\beta^2}(-z_2 + b_3)\right]\right\}$$

which, when $a_2 < 1$, is contained in $\mathcal{H}(\mathcal{X}, z)$, for all $z$ (the scalars $a_i$, $b_i$, $i = 1, 2, 3$ have to be chosen to satisfy Assumption A3 in [14, 12]). Finally, $N(\hat{\xi}, z)$ and $N_z(\hat{\xi}, z)$ are the normal vectors to the boundary of $C_\xi(z)$ with respect to $\xi$ and $z$, respectively, and are given by

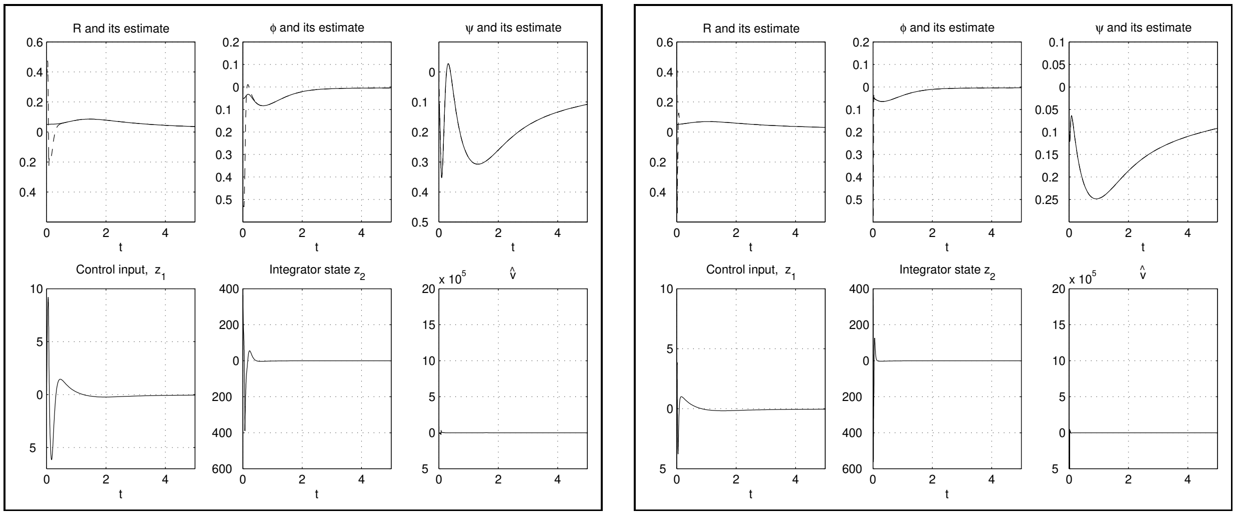
$$N(\hat{\xi}, z) = \begin{cases} [1, 0, 0]^\top \text{ if } \hat{\xi}_1 = b_1 & [-1, 0, 0]^\top \text{ if } \hat{\xi}_1 = a_1 \\ [0, 1, 0]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 - b_2) & [0, -1, 0]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 + a_2) \\ [0, 0, 1]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 + b_3) & [0, 0, -1]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 - a_3) \end{cases}$$

$$N_z(\hat{\xi}, z) = \begin{cases} [0,0]^\top \text{ if } \hat{\xi}_1 = b_1 \text{ or } \hat{\xi}_1 = a_1 \\ \left[\frac{1}{\beta^2}, 0\right]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 - b_2) \quad \left[-\frac{1}{\beta^2}, 0\right]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 + a_2) \\ \left[0, \frac{1}{\beta^2}\right]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 + b_3) \quad \left[0, -\frac{1}{\beta^2}\right]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 - a_3) \end{cases}$$

Thus, the output feedback controller design is completed by letting $\hat{v} = \varphi(\hat{x}^P, z)$, and Theorem 2 in [14, 12] guarantees that the origin of the closed-loop system, controlled by $\hat{v}$, is asymptotically stable with domain of attraction $\mathcal{D}' \times \Omega^x_{c_2}$, where $\Omega^x_{c_2} \stackrel{\triangle}{=} \{[R, \phi, \psi]^\top \mid \bar{V} \leq c_2, \text{ and } R \geq 0\}$, $c_2 > 0$ is the largest scalar such that $\Omega^x_{c_2} \subset \{[R, \phi, \psi]^\top \in \mathbb{R}^3 \mid \phi > -1\}$, and $\mathcal{D}' \subset \Omega^x_{c_2}$ can be made arbitrarily close to $\Omega^x_{c_2}$ by choosing $\rho$ in (14) small enough (see Theorem 2 in [14, 12]).

## 4 Simulation Results

Here we present the simulation results when the output feedback controller developed in the previous section is applied to system (2). We choose $k_1 = 25$ and $k_2 = 1.1 \cdot 10^5$ to fulfill inequalities (4)-(7) in Theorem 1. In order to choose the size of the compact set $C_\xi(z)$ so that Assumption A3 in [14, 12] is satisfied, we may use the Lyapunov function $\bar{V}$ to calculate $\Omega^x_{c_2}$, choose $c_2$ small enough to guarantee that $\Omega^x_{c_2} \subset \mathcal{X}$, and use $\mathcal{H}$ to calculate bounds on $\xi$ when $x \in \Omega^x_{c_2}$. However, a more practical way to address the design of $C_\xi(z)$ consists of running a number of simulations for the closed-loop system under state feedback corresponding to several initial conditions $[R(0), \phi(0), \psi(0)]^\top$, and calculating upper and lower bounds for $\psi$, $\phi$, and $-\psi - 3/2\phi^2 - 1/2\phi^3 - 3R\phi - 3R$: these will provide the values of $a_i, b_i$, $i = 1, 2, 3$, respectively. By doing that, we found that whenever $[R(0), \phi(0), \psi(0)]^\top \in \Omega_0 \stackrel{\triangle}{=} \{[R, \phi, \psi]^\top \in \mathbb{R}^3 \mid R \in [0, 0.1], \phi \in [-0.1, 0.1], \psi \in [-0.5, 0.5]\}$, we have that $a_1 = -2$, $b_1 = 1$, $a_2 = -0.5$, $b_2 = 1$, $a_3 = -0.5$, $b_3 = 0.3$ satisfy Assumption A3 in [14, 12]. We must point out that our choice of $\Omega_0$ is rather conservative and is made primarily for the sake of illustration. The actual domain of attraction $\mathcal{D}'$ under output feedback control is larger that $\Omega_0$. In Figure 2 system and controller states, together with the control input, are plotted for two decreasing values of $\rho$ confirming the theoretical predictions about the arbitrary fast rate of convergence of the observer found in Theorem 1 in [14]. Furthermore, the figures




$\rho = 0.1$. $\rho = 0.02$.

Figure 2: Output feedback trajectories.

also show the operation of the projection which prevents the observer from peaking and guarantees that $\hat{\phi} > -0.5$. Finally, note that the output feedback trajectories approach the state feedback ones, as showed in Figure 3.

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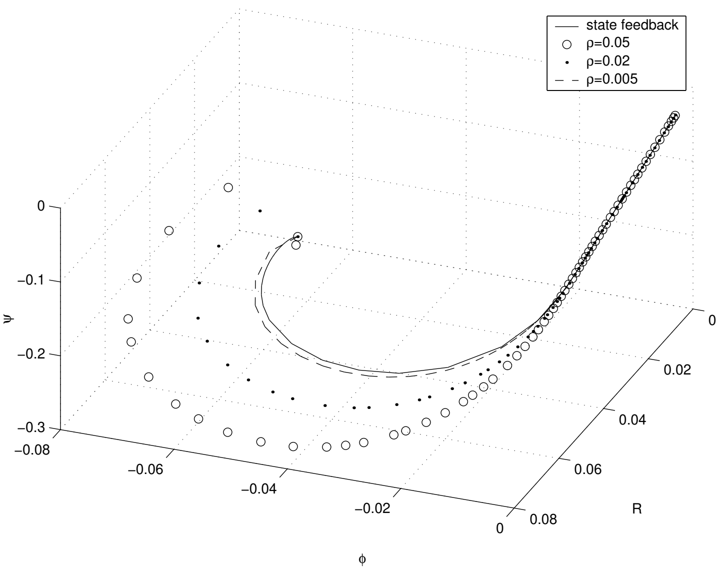


Figure 3: State feedback trajectories and output feedback trajectories for $\rho = 0.05$, $\rho = 0.02$, and $\rho = 0.005$.

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