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Adaptive Control for a Class of Nonlinear Time-Varying Systems *

Yixin Diao and Kevin M. Passino

Department of Electrical Engineering
The Ohio State University
2015 Neil Avenue, Columbus, OH 43210

Abstract

Adaptive control for nonlinear time-varying systems is of both theoretical and practical importance. In this paper we present an adaptive control methodology for a class of nonlinear systems with a time-varying structure where radial basis function neural networks are used as on-line approximators. This class of systems is composed of interpolations of nonlinear subsystems which are input-output feedback linearizable. Without assumptions on rate of change of system dynamics, a stable indirect adaptive control method is presented with analysis of stability for all signals in the closed-loop as well as asymptotic tracking. The performance of the controller is demonstrated using a jet engine control problem.

1 Introduction

Adaptive control has been employed in situations where little *a priori* knowledge of the plant is known. Adaptive control has also been used to compensate for on-line system parameter variations, which may arise due to changes in operating points, component faults, plant deterioration, etc. The general methodology of adaptive control for time-varying systems is to treat the effects of parameter variations as un-modeled perturbations so that it turns into a robustness problem [1]. This methodology has been applied in linear time-varying systems, where the parameters vary slowly and smoothly, or discontinuously (i.e., jumps) but the discontinuities occur over large intervals of time [2, 3, 4, 5]. Adaptive control for linear time-varying systems using model reference adaptive control or adaptive pole placement control schemes has been extensively studied [6, 7, 8, 9, 10]. The assumption of slow parameter variations may also be relaxed if some information about the fast varying parameters is available *a priori* [11, 12].

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Adaptive control for nonlinear time varying systems has also been studied by some researchers, but only restricted classes of systems are considered and only limited results exist so far. In [13] a high-order nonlinear plant is remodeled to be a lower-order linear time-invariant plant with an uncertainty which is bounded by a bounding signal, and global stability and small tracking error were guaranteed using reduced-order adaptive feedback controllers with regular or variable structure. In [14] the authors study a robust state feedback control for a class of nonlinear time-varying systems in the strict feedback form with unknown unmodeled time-varying parameters or disturbances (whose bounds are known) and use the backstepping design method. This result has been extended to adaptive control in [15] (and [16] by adaptive output feedback control), and similar work has been done in [17] and [18]. Besides controlling the nonlinear time-varying system as a whole, another control methodology is to exploit the internal time-varying structure of nonlinear systems, for instance, the class of systems consists of an interpolation of nonlinear dynamic equations in the strict feedback form, and construct backstepping control laws tailored to each of the dynamic components of the nonlinear system [19, 20, 21, 22]. Our work is most similar to this philosophy.

Note that for the adaptive control problem of nonlinear time-varying systems, only a restricted class of systems in the strict feedback form are considered and only limited results exist so far. In this paper we consider a more general class of nonlinear time-varying systems, which is input-output feedback linearizable, and present stable adaptive control approaches using the on-line learning capabilities of radial basis function neural networks. This class of systems is large enough so that it is not only of theoretical interest but also of practical applicability. The idea of using function approximation structures with universal approximation properties (such as neural networks or fuzzy systems) to deal with arbitrary continuous nonlinearities has been widely used in adaptive control for nonlinear systems [23]. In fact, on-line approximation based stable adaptive neural/fuzzy methods have been significantly impacted by the work in [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37] using neural networks as approximators of nonlinear functions, the work in [38, 39, 40, 41, 42, 43, 44] using fuzzy systems for the same purpose, and the work in [26, 32] using dynamical neural networks. The neural and fuzzy approaches are most of the time equivalent, differing between each other mainly in the structure of the approximator chosen. Indeed, to try to bridge the gap between the neural and fuzzy approaches several researchers (e.g., in [44]) introduce adaptive schemes using a class of parameterized functions that include both neural networks and fuzzy systems. As to the approximator structure, linear in the parameter approximators are used in [38, 39, 24, 25, 45, 27, 28, 42, 33, 36, 44] and non-linear in [29, 30, 31, 32, 34, 35, 37]. Finally, most of the papers [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43] deal with indirect adaptive control (trying first to identify the dynamics of the systems and then generating a control input according to the certainty equivalence principle), whereas very few authors (e.g., [44, 46]) face the direct approach (directly generating the control input to guarantee stability), because it is not always clear how to construct the control law without knowledge of the system dynamics.

In this paper we present an adaptive control methodology for a class of nonlinear systems that depends on exogenous scheduling variables. This class of systems consists of interpolations of nonlinear dynamic equations in feedback linearizable form and it may represent systems with a time-varying nonlinear structure, which is, indeed, a generalization of the class of feedback linearizable systems traditionally considered in nonlinear adaptive control literature [47, 48, 49, 44]. To generalize stable adaptive fuzzy/neural control [44], following the general approach in [19, 20, 21, 22], the adaptive laws applied here are localized in the sense that only the part of the approximator parameters is updated, which corresponds to the region of the scheduling space.

To approach the nonlinear control problem by studying simplified, localized approximations of the plant, the control methodology studied here shares some common views of gain scheduling control, which deals with nonlinear systems that are linearized along reference trajectories or operating points [50, 51, 52]. Gain scheduling control is widely used in industrial applications, but so far only local stability results exist due to the difficulty of stability analysis. Other related results exist in the parallel distributed compensation literature [53, 54, 55, 56], where authors assume the existence of a model consisting of interpolations of linearized system dynamics which are valid within a subset of the state space or other relevant operating space, and linear controllers are designed within these regions and then interpolated with the same interpolation structure of the system. The class of time-varying systems studied in this paper is related to the system studied in the parallel distributed compensation literature. However, instead of interpolating controllable linear systems (obtained by local linearization within divided sub-spaces), in the spirit of [19, 20, 21, 22], here we focus on using feedback linearizable nonlinear systems as the “pieces” to form the “global” nonlinear system by interpolation. Note that to achieve global stability of parallel distributed compensation approach, a single positive definite matrix that simultaneously stabilizes all combinations of linear subsystems needs to be found (which is not trivial), usually, relying on linear matrix inequalities (LMI) optimization methods. By interpolating feedback linearizable nonlinear systems, not only can LMI optimization be avoided, but also the adaptation mechanism could be incorporated so as to guarantee asymptotic stability and deal with model uncertainty. Furthermore, this class of system is large enough (compared to the nonlinear system in the strict feedback form as studied in [19, 20, 21, 22]) so that it may have more practical applicability.

This paper is organized as follows. In Section 2 the details of the problem formulation for a class of input-output feedback linearizable time-varying nonlinear systems are given. The adaptive algorithms and system stability analysis are presented in Sections 3. Section 4 contains descriptions of simulations used to illustrate the performance of the methods

2 A Class of Nonlinear Systems with a Time-Varying Structure

Consider a class of nonlinear systems consisting of an interpolation of nonlinear subsystems

$$\dot{x} = \left(\sum_{i=1}^{N} f^i(x,v)\xi^i(v)\right) + \left(\sum_{i=1}^{N} g^i(x,v)\xi^i(v)\right) u \tag{1}$$

$$y = \sum_{i=1}^{N} h^i(x,v)\xi^i(v) \tag{2}$$

where $x = [x_1, x_2, \ldots, x_n]^\top$ is the (measurable) state vector, u is the (scalar) input and y is the (scalar) output of the system. The functions $f^i(x,v)$, $g^i(x,v)$, and $h^i(x,v)$ represent local nonlinear dynamics, $\xi^i(v)$ ($\xi^i(v) > 0$) are piecewise continuous interpolation functions, $i = 1, 2, \ldots, N$ that we think of as splitting the domain of v into N different nonlinear sub-regions, where v is a vector of exogenous scheduling variables, which are measurable and bounded.

Let $L_g^d h^i(x,v)$ be the d^{th} Lie derivative of $h^i(x,v)$ with respect to $g(x,v) = \sum_{j=1}^{N} g^j(x,v)\xi^j(v)$, that is, for example

$$L_g h^i(x,v) = \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top g(x,v) = \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \sum_{j=1}^{N} g^j(x,v)\xi^j(v) \tag{3}$$

$$L_g^2 h^i(x,v) = L_g\left[L_g h^i(x,v)\right] \tag{4}$$

and so on. For convenience of derivation, we define $\bar{L}_f^d h^i(x,v)$ be the d^{th} *modified* Lie derivative of $h^i(x,v)$ with respect to $f(x,v) = \sum_{j=1}^{N} f^j(x,v)\xi^j(v)$, that is,

$$\bar{L}_f h^i(x,v) = \frac{h^i(x,v)\dot{\xi}^i(v)}{\xi^i(v)} + \left(\frac{\partial h^i(x,v)}{\partial v}\right)^\top \dot{v} + \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \sum_{j=1}^{N} f^j(x,v)\xi^j(v) \tag{5}$$

and, for example,

$$\bar{L}_f^2 h^i(x,v) = \bar{L}_f\left[\bar{L}_f h^i(x,v)\right] \tag{6}$$

Next, we give the definition of the "strong relative degree," that is, a system is said to have strong relative degree d ($1 \le d \le n$) if

$$\sum_{i=1}^{N} L_g h^i(x,v)\xi^i(v) = \sum_{i=1}^{N} L_g \bar{L}_f h^i(x,v)\xi^i(v) = \cdots = \sum_{i=1}^{N} L_g \bar{L}_f^{d-2} h^i(x,v)\xi^i(v) = 0 \tag{7}$$

and

$$\sum_{i=1}^{N} L_g \bar{L}_f^{d-1} h^i(x,v)\xi^i(v) \neq 0 \tag{8}$$

for all x and v. Note that we use both the standard and *modified* Lie derivatives to provide a compact representation here.

Under the above definitions, if system (1) and (2) have strong relative degree d, then

$$
\begin{aligned}
\dot{y} &= \sum_{i=1}^{N}\left[h^i(x,v)\dot{\xi}^i(v) + \left(\frac{\partial h^i(x,v)}{\partial x}\right)^{\top}\dot{x}\xi^i(v) + \left(\frac{\partial h^i(x,v)}{\partial v}\right)^{\top}\dot{v}\xi^i(v)\right] \\
&= \sum_{i=1}^{N}\left[h^i(x,v)\dot{\xi}^i(v) + \left(\frac{\partial h^i(x,v)}{\partial v}\right)^{\top}\dot{v}\xi^i(v) + \left(\frac{\partial h^i(x,v)}{\partial x}\right)^{\top}\sum_{j=1}^{N} f^j(x,v)\xi^j(v)\xi^i(v)\right] \\
&\quad + \sum_{i=1}^{N}\left[\left(\frac{\partial h^i(x,v)}{\partial x}\right)^{\top}\sum_{j=1}^{N} g^j(x,v)\xi^j(v)\xi^i(v)\right]u \\
&= \sum_{i=1}^{N}\bar{L}_f h^i(x,v)\xi^i(v)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{y} &= \sum_{i=1}^{N}\left[\bar{L}_f h^i(x,v)\dot{\xi}^i(v) + \left(\frac{\partial \bar{L}_f h^i(x,v)}{\partial x}\right)^{\top}\dot{x}\xi^i(v) + \left(\frac{\partial \bar{L}_f h^i(x,v)}{\partial v}\right)^{\top}\dot{v}\xi^i(v)\right] \\
&= \sum_{i=1}^{N}\left[\bar{L}_f h^i(x,v)\dot{\xi}^i(v) + \left(\frac{\partial \bar{L}_f h^i(x,v)}{\partial v}\right)^{\top}\dot{v}\xi^i(v) + \left(\frac{\partial \bar{L}_f h^i(x,v)}{\partial x}\right)^{\top}\sum_{j=1}^{N} f^j(x,v)\xi^j(v)\xi^i(v)\right] \\
&\quad + \sum_{i=1}^{N}\left[\left(\frac{\partial \bar{L}_f h^i(x,v)}{\partial x}\right)^{\top}\sum_{j=1}^{N} g^j(x,v)\xi^j(v)\xi^i(v)\right]u \\
&= \sum_{i=1}^{N}\bar{L}_f^2 h^i(x,v)\xi^i(v)
\end{aligned}
$$

and so on, so that the system dynamics may be written in the normal form as

$$
\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 = \sum_{i=1}^{N}\bar{L}_f h^i(x,v)\xi^i(v) \\
\dot{\zeta}_2 &= \zeta_3 = \sum_{i=1}^{N}\bar{L}_f^2 h^i(x,v)\xi^i(v) \\
&\vdots \\
\dot{\zeta}_{d-1} &= \zeta_d = \sum_{i=1}^{N}\bar{L}_f^{d-1} h^i(x,v)\xi^i(v) \\
\dot{\zeta}_d &= \sum_{i=1}^{N}\bar{L}_f^{d} h^i(x,v)\xi^i(v) + \sum_{i=1}^{N} L_g\bar{L}_f^{d-1} h^i(x,v)\xi^i(v)u \\
\dot{\pi} &= f_0(\zeta,\pi,v)
\end{aligned}
$$

with $\zeta \in \Re^d$, $\pi \in \Re^{n-d}$, and $\zeta_1 = y$. Note that the class of nonlinear systems we consider here is actually a special case of a general class of nonlinear time-varying systems

$$\begin{aligned} \dot{x} &= f(x,t) + g(x,t)u & (9) \\ y &= h(x,t) & (10) \end{aligned}$$

and there exists certain kind of equivalence between the Lie derivatives defined above and the ones for the time-varying systems [57, 58]. In particular

$$\begin{aligned}
L_g h(x,t) &= \left(\frac{\partial h}{\partial x}\right)^{\top} g(x,t) \\
&= \sum_{i=1}^{N} \left(\frac{\partial h^i(x,v)}{\partial x}\right)^{\top} g(x,t)\xi^i(v) \\
&= \sum_{i=1}^{N} L_g h^i(x,v)\xi^i(v) \\
\bar{L}_f h(x,t) &= \frac{\partial h}{\partial t} + \left(\frac{\partial h}{\partial x}\right)^{\top} f(x,t) \\
&= \left(\frac{\partial h}{\partial v}\right)^{\top} \dot{v} + \left(\frac{\partial h}{\partial x}\right)^{\top} f(x,t) \\
&= \sum_{i=1}^{N} \left(\frac{\partial \left[h^i(x,v)\xi^i(v)\right]}{\partial v}\right)^{\top} \dot{v} + \sum_{i=1}^{N} \left(\frac{\partial h^i(x,v)}{\partial x}\right)^{\top} \sum_{j=1}^{N} f^j(x,v)\xi^j(v)\xi^i(v) \\
&= \sum_{i=1}^{N} \frac{h^i(x,v)\dot{\xi}^i(v)}{\xi^i(v)}\xi^i(v) + \sum_{i=1}^{N} \left(\frac{\partial h^i(x,v)}{\partial v}\right)^{\top} \dot{v}\xi^i(v) \\
&\quad + \sum_{i=1}^{N} \left(\frac{\partial h^i(x,v)}{\partial x}\right)^{\top} \sum_{j=1}^{N} f^j(x,v)\xi^j(v)\xi^i(v) \\
&= \sum_{i=1}^{N} \bar{L}_f h^i(x,v)\xi^i(v)
\end{aligned}$$

Therefore, the above normal form can be rewritten as

$$\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 = \bar{L}_f h(x,t) \\
\dot{\zeta}_2 &= \zeta_3 = \bar{L}_f^2 h(x,t) \\
&\vdots \\
\dot{\zeta}_{d-1} &= \zeta_d = \bar{L}_f^{d-1} h(x,t) \\
\dot{\zeta}_d &= \bar{L}_f^d h(x,t) + L_g \bar{L}_f^{d-1} h(x,t)u \\
\dot{\pi} &= f_0(\zeta,\pi,t)
\end{aligned}$$

which is the same as the normal form of the time-varying systems [57, 58]. Therefore, there exist functions

$$
\begin{aligned}
\psi_1(x,t) &= h(x,t) \\
&\vdots \\
\psi_d(x,t) &= \frac{\partial \psi_{d-1}}{\partial t} + \left(\frac{\partial \psi_{d-1}}{\partial x}\right)^\top f(x,t) \\
\left(\frac{\partial \phi_1}{\partial x}\right)^\top g(x,t) &= 0 \\
&\vdots \\
\left(\frac{\partial \phi_{n-d}}{\partial x}\right)^\top g(x,t) &= 0
\end{aligned}
$$

and the diffeomorphism

$$
z = T(x,t) = \begin{bmatrix} \psi_1(x,t) \\ \vdots \\ \psi_d(x,t) \\ \cdots\cdots \\ \phi_1(x,t) \\ \vdots \\ \phi_{n-d}(x,t) \end{bmatrix} = \begin{bmatrix} \psi(x,t) \\ \cdots\cdots \\ \phi(x,t) \end{bmatrix} = \begin{bmatrix} \zeta \\ \cdots \\ \pi \end{bmatrix} \tag{11}
$$

so that the normal form can be obtained from the system (1) and (2) by a change of variables, as shown in [57] for the time-varying systems.

Although the class of nonlinear systems studied here is a special case of the general time-varying systems, the advantage of using this interpolated form is to exploit the internal structure of the time-varying dynamics so that it can be separated into known scheduling functions $\xi^i(v)$ and unknown local nonlinear dynamics consisting of $f^i(x,v)$, $g^i(x,v)$, and $h^i(x,v)$. In this way we take advantage of our *a priori* knowledge on part of time-varying dynamics so as to reduce the burden of on-line learning. By using an interpolated on-line approximation based adaptive control strategy, we expect that the local nonlinear dynamics may be approximated more accurately by corresponding on-line approximators so that the performance of adaptive control may be improved, of course, at the cost of the increase of computational complexity.

The normal form decomposes the system states into an external part ζ and an internal part π. For the external part, if we let $y^{(d)}$ denote the d^{th} derivative of y, it may be rewritten as

$$
y^{(d)} = \sum_{i=1}^{N} \left(\alpha_k^i(t) + \alpha^i(x,v)\right) \xi^i(v) + \sum_{i=1}^{N} \left(\beta_k^i(t) + \beta^i(x,v)\right) \xi^i(v) u \tag{12}
$$

where $\alpha_k^i(t)$ and $\beta_k^i(t)$ are "known" local dynamics of the system (which are assumed to be bounded if x is bounded), and $\alpha^i(x,v)$ and $\beta^i(x,v)$ represent nonlinear time-varying local dynamics of the plant that are unknown. We assume that for some known $\beta_0^i > 0$, we have $|\beta_k^i(t) + \beta^i(x,v)| \geq \beta_0^i$ so that it is always bounded away from zero (for convenience we further assume that $\beta_k^i(t) + \beta^i(x,v) > 0$, however, the following analysis may easily be modified for systems which are defined with $\beta_k^i(t) + \beta^i(x,v) < 0$ as well). The external part may be stabilized by the control u (which we will show later), while the internal part is made unobservable by the same control. By having $\zeta = 0$ in the inner part, the "zero dynamics" of the system are given by

$$\dot{\pi} = f_0(0, \pi, t) \tag{13}$$

If the plant is of relative degree $d = n$, then there are no zero dynamics (i.e., no internal part π). Alternatively, if the relative degree $d < n$, we assume that the zero dynamics are exponentially attractive so that if we have some control u to let ζ bounded, this also ensures boundedness of π.

3 Indirect Adaptive Control

The on-line learning abilities of neural networks are considered here to approximate the local time-varying dynamics of the nonlinear system. In particular, the *linear in the parameter* radial basis function networks (multilayer perceptrons, or Takagi-Sugeno fuzzy systems) may take the form of

$$\hat{\alpha}^i(x,v) = \theta_\alpha^{i\top}(t)\phi_\alpha^i(x,v) \tag{14}$$
$$\hat{\beta}^i(x,v) = \theta_\beta^{i\top}(t)\phi_\beta^i(x,v) \tag{15}$$

where the vectors $\theta_\alpha^i(t)$ and $\theta_\beta^i(t)$, $i = 1, 2, \ldots, N$, are updated on-line and are assumed to be defined within the compact parameter sets Ω_α and Ω_β, respectively (an explanation of how neural networks or fuzzy systems can be put into this form is contained in [59]). In addition, we define the subspace $S_x \subseteq \Re^n$ as the space through which the state trajectory may travel under closed-loop control (we are making no *a priori* assumptions here about the size of S_x). We also define the actual system as

$$\alpha^i(x,v) = \theta_\alpha^{i*\top}\phi_\alpha^i(x,v) + \omega_\alpha^i(x,v) \tag{16}$$
$$\beta^i(x,v) = \theta_\beta^{i*\top}\phi_\beta^i(x,v) + \omega_\beta^i(x,v) \tag{17}$$

where

$$\theta_\alpha^{i*} = \arg\min_{\theta_\alpha^i \in \Omega_\alpha} \left(\sup_{x \in S_x} |\theta_\alpha^{i\top}\phi_\alpha^i(x,v) - \alpha^i(x,v)| \right) \tag{18}$$

$$\theta_\beta^{i*} = \arg\min_{\theta_\beta^i \in \Omega_\beta} \left(\sup_{x \in S_x} |\theta_\beta^{i\top}\phi_\beta^i(x,v) - \beta^i(x,v)| \right) \tag{19}$$

are the optimal parameters and $\omega_\alpha^i(x,v)$ and $\omega_\beta^i(x,v)$ are approximation errors which arise when $\alpha^i(x,v)$ and $\beta^i(x,v)$ are represented by finite size approximators. We assume that

$$
\begin{aligned}
|\omega_\alpha^i(x,v)| &\leq W_\alpha^i(x) && (20)\\
|\omega_\beta^i(x,v)| &\leq W_\beta^i(x) && (21)
\end{aligned}
$$

where $W_\alpha^i(x)$ and $W_\beta^i(x)$ are known state dependent bounds on the error in representing the actual system with approximators. We also define parameter errors to be

$$
\begin{aligned}
\tilde{\theta}_\alpha^i(t) &= \theta_\alpha^i(t) - \theta_\alpha^{i*} && (22)\\
\tilde{\theta}_\beta^i(t) &= \theta_\beta^i(t) - \theta_\beta^{i*} && (23)
\end{aligned}
$$

We want to design a control system which will cause the output $y(t)$ and its derivatives $\dot{y}(t),\ldots,y^{(d)}(t)$ to track a desired reference trajectory $y_m(t)$ and its derivatives $\dot{y}_m(t),\ldots,y_m^{(d)}(t)$, respectively, which we assume to be bounded. The reference trajectory may be defined by a reference signal whose first d derivatives may be measured, or by any reference input $r(t)$ passing through a reference model, with relative degree equal to or greater than d. In particular, a linear reference model may be

$$\frac{Y_m(s)}{R(s)} = \frac{q(s)}{p(s)} = \frac{q_0}{s^d + p_{d-1}s^{d-1} + \cdots + p_0} \tag{24}$$

where $p(s)$ is the pole polynomial with stable roots.

The indirect adaptive control law is designed as

$$u = u_{ce} + u_{si} \tag{25}$$

which is comprised of a "certainty equivalence" control term u_{ce} and a "sliding mode" control term u_{si}

$$
\begin{aligned}
u_{ce} &= \frac{1}{\sum_{i=1}^N \left[\beta_k^i(t) + \hat{\beta}^i(x,v)\right]\xi^i(v)}\left(-\sum_{i=1}^N \left[\alpha_k^i(t) + \hat{\alpha}^i(x,v)\right]\xi^i(v) + \nu(t)\right) && (26)\\
u_{si} &= \frac{\sum_{i=1}^N \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right]\xi^i(v)}{\sum_{i=1}^N \beta_0^i\xi^i(v)} sgn(e_s) && (27)
\end{aligned}
$$

Let the tracking error be $e(t) = y_m(t) - y(t)$ and a measure of the tracking error be $e_s(t) = e^{(d-1)}(t) + k_{d-2}e^{(d-2)}(t) + \cdots + k_1\dot{e}(t) + k_0 e(t)$, that is, in the frequency domain, $e_s(s) = L(s)e(s)$ with $L(s) = s^{(d-1)} + k_{d-2}s^{(d-2)} + \cdots + k_1 s + k_0$ whose roots are chosen to be in the (open) left half plane. Also, for convenience below we let $\bar{e}_s(t) = \dot{e}_s(t) - e^{(d)}(t)$. Notice that our control goal is to drive $e_s(t) \to 0$ as $t \to \infty$ and the shape of the error dynamics is dictated by the choice of the design parameters in $L(s)$. We define

$$\nu(t) = y_m^{(d)} + \eta e_s + \bar{e}_s \tag{28}$$

with $\eta > 0$ as a design parameter.

Consider the update laws

$$\begin{aligned}\dot{\theta}^i_\alpha(t) &= -{Q^i_\alpha}^{-1}\phi^i_\alpha(x,v)e_s\xi^i(v) &(29)\\ \dot{\theta}^i_\beta(t) &= -{Q^i_\beta}^{-1}\phi^i_\beta(x,v)e_s u_{ce}\xi^i(v) &(30)\end{aligned}$$

where Q^i_α and Q^i_β are positive definite and diagonal and serve as adaptation gains for the parameter updates, and $\xi^i(v)$ represent the effects of interpolated adaptive laws, that is, the degree of parameter adaptation of the local approximator is up to the extent of involvement of that subregion indicated by $\xi^i(v)$. Note that the above adaptation laws do not guarantee that $\theta^i_\alpha \in \Omega_\alpha$ and $\theta^i_\beta \in \Omega_\beta$ so that we will use a projection method to ensure this, in particular, to make sure that $\beta^i_k(t) + \hat{\beta}^i(x,v) \geq \beta^i_0$.

Theorem 1: Consider the nonlinear system (1) and (2) with strong relative degree d. Assume that (i) $\alpha^i_k(t)$ and $\beta^i_k(t)$ in (12) are bounded if x is bounded, (ii) $\beta^i_k(t) + \beta^i(x,v) \geq \beta^i_0$ for some known $\beta^i_0 > 0$, (iii) $|\omega^i_\alpha(x,v)| \leq W^i_\alpha(x)$ and $|\omega^i_\beta(x,v)| \leq W^i_\beta(x)$ with known $W^i_\alpha(x)$ and $W^i_\beta(x)$, (iv) $y_m(t), \dot{y}_m(t), \ldots, y_m^{(d)}(t)$ are measurable and bounded, (v) $x(t), y(t), \dot{y}(t), \ldots, y^{(d-1)}(t)$ are measurable, and (vi) $1 \leq d < n$ with the zero dynamics exponentially attractive or $d = n$. Under these conditions there exist indirect adaptive control law (25, 26, 27) and update law (29, 30) such that all internal signals are bounded and the tracking error e is asymptotically stable.

Proof: Consider the Lyapunov function candidate

$$V_i = \frac{1}{2}e_s^2 + \frac{1}{2}\sum_{i=1}^{N}\tilde{\theta}^{i\top}_\alpha Q^i_\alpha \tilde{\theta}^i_\alpha + \frac{1}{2}\sum_{i=1}^{N}\tilde{\theta}^{i\top}_\beta Q^i_\beta \tilde{\theta}^i_\beta \tag{31}$$

Using vector derivatives and following [44, 60], the time derivative of (31) is

$$\dot{V}_i = e_s\dot{e}_s + \sum_{i=1}^{N}\tilde{\theta}^{i\top}_\alpha Q^i_\alpha \dot{\tilde{\theta}}^i_\alpha + \sum_{i=1}^{N}\tilde{\theta}^{i\top}_\beta Q^i_\beta \dot{\tilde{\theta}}^i_\beta \tag{32}$$

Note that $\bar{e}_s(t) = \dot{e}_s(t) - e^{(d)}(t)$ and the d^{th} derivative of the output error is $e^{(d)} = y_m^{(d)} - y^{(d)}$ so that

$$\dot{e}_s(t) = \bar{e}_s(t) + y_m^{(d)} - y^{(d)} \tag{33}$$

and from (12), (25), (28) and (26)

$$\begin{aligned}
\dot{e}_s(t) &= \bar{e}_s(t) + (\nu(t) - \eta e_s - \bar{e}_s) \\
&\quad - \left(\sum_{i=1}^{N} \left[\alpha_k^i(t) + \alpha^i(x,v)\right] \xi^i(v) + \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v)(u_{ce} + u_{si}) \right) \\
&= -\eta e_s + \left(\nu(t) - \sum_{i=1}^{N} \alpha_k^i(t)\xi^i(v) - \sum_{i=1}^{N} \left[\beta_k^i(t) + \hat{\beta}^i(x,v)\right] \xi^i(v) u_{ce} \right) - \sum_{i=1}^{N} \alpha^i(x,v)\xi^i(v) \\
&\quad + \sum_{i=1}^{N} \left[\hat{\beta}^i(x,v) - \beta^i(x,v)\right] \xi^i(v) u_{ce} - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) u_{si} \\
&= -\eta e_s + \left(\nu(t) - \sum_{i=1}^{N} \alpha_k^i(t)\xi^i(v) + \sum_{i=1}^{N} \left[\alpha_k^i(t) + \hat{\alpha}^i(x,v)\right] \xi^i(v) - \nu(t) \right) - \sum_{i=1}^{N} \alpha^i(x,v)\xi^i(v) \\
&\quad + \sum_{i=1}^{N} \left[\hat{\beta}^i(x,v) - \beta^i(x,v)\right] \xi^i(v) u_{ce} - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) u_{si} \\
&= -\eta e_s + \sum_{i=1}^{N} \left[\hat{\alpha}^i(x,v) - \alpha^i(x,v)\right] \xi^i(v) + \sum_{i=1}^{N} \left[\hat{\beta}^i(x,v) - \beta^i(x,v)\right] \xi^i(v) u_{ce} \\
&\quad - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) u_{si}
\end{aligned}$$

also from (14,15), (16,17) and (22,23) we have

$$\begin{aligned}
\dot{e}_s &= -\eta e_s + \sum_{i=1}^{N} \left[\tilde{\theta}_\alpha^{i\top} \phi_\alpha^i(x,v) - \omega_\alpha^i(x,v)\right] \xi^i(v) + \sum_{i=1}^{N} \left[\tilde{\theta}_\beta^{i\top} \phi_\beta^i(x,v) - \omega_\beta^i(x,v)\right] \xi^i(v) u_{ce} \\
&\quad - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) u_{si}
\end{aligned}$$

Substitute the above equation into (32) and assume that the ideal parameters are constant (which is achieved by having $\omega_\alpha^i(x,v)$ and $\omega_\beta^i(x,v)$ in (16) and (17) represent approximation errors from the time-varying part of system dynamics) so that $\dot{\tilde{\theta}}_\alpha^i = \dot{\theta}_\alpha^i$ and $\dot{\tilde{\theta}}_\beta^i = \dot{\theta}_\beta^i$, and substitute (29,30) into (32)

$$\begin{aligned}
\dot{V}_i &= e_s \dot{e}_s + \sum_{i=1}^{N} \tilde{\theta}_\alpha^{i\top} Q_\alpha^i \left[-{Q_\alpha^i}^{-1} \phi_\alpha^i(x,v) e_s \xi^i(v) \right] + \sum_{i=1}^{N} \tilde{\theta}_\beta^{i\top} Q_\beta^i \left[-{Q_\beta^i}^{-1} \phi_\beta^i(x,v) e_s u_{ce} \xi^i(v) \right] \\
&= e_s \dot{e}_s - \sum_{i=1}^{N} \tilde{\theta}_\alpha^{i\top} \phi_\alpha^i(x,v) e_s \xi^i(v) - \sum_{i=1}^{N} \tilde{\theta}_\beta^{i\top} \phi_\beta^i(x,v) e_s \xi^i(v) u_{ce} \\
&= -\eta e_s^2 - \sum_{i=1}^{N} \left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v) u_{ce}\right] \xi^i(v) e_s - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) u_{si} e_s
\end{aligned}$$

Notice that we did not consider a projection modification to the update laws above. Clearly, since $\theta_\alpha^{i*} \in \Omega_\alpha$ and $\theta_\beta^{i*} \in \Omega_\beta$, when the projection is in effect it always results in smaller parameter errors that will decrease $\dot{V}_i$ so that

$$\dot{V}_i \leq -\eta e_s^2 - \sum_{i=1}^{N} \left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v)u_{ce}\right] \xi^i(v) e_s - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) u_{si} e_s \tag{34}$$

Note that

$$-\left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v)u_{ce}\right] e_s \leq \left[|\omega_\alpha^i(x,v)| + |\omega_\beta^i(x,v)u_{ce}|\right] |e_s| \leq \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] |e_s|$$

and $\xi^i(v) > 0$, we have

$$-\sum_{i=1}^{N} \left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v)u_{ce}\right] \xi^i(v) e_s \leq \sum_{i=1}^{N} \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] \xi^i(v)|e_s|$$

Also note that $\beta_k^i(t) + \beta^i(x,v) \geq \beta_0^i > 0$ so that $\sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) \geq \sum_{i=1}^{N} \beta_0^i \xi^i(v) > 0$ and consider (27), we have

$$\begin{aligned}
& -\sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) u_{si} e_s \\
= \; & \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) \frac{\sum_{i=1}^{N} \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] \xi^i(v)}{\sum_{i=1}^{N} \beta_0^i \xi^i(v)} |e_s| \\
\leq \; & -\sum_{i=1}^{N} \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] \xi^i(v)|e_s|
\end{aligned}$$

Thus, by substituting the above equation into (34) we have $\dot{V}_i \leq -\eta e_s^2$ which means V_i is a nonincreasing function of time, so that the measure of the tracking error e_s is bounded. Notice that $e_s(s) = L(s)e(s)$ and $L(s)$ is a stable function with the degree of $d-1$ we known that the tracking error and its derivatives $e, \dot{e}, \ldots, e^{(d-1)}$ are bounded. Since the reference trajectory y_m and its derivatives $\dot{y}_m, \ldots, y_m^{(d-1)}$ are assumed to be bounded, the system output y and its derivatives $\dot{y}, \ldots, y^{(d-1)}$ are bounded. Hence, ξ is bounded and thus x is bounded. Besides, the fact that $\dot{V}_i$ is negative semidefinite also implies that parameter estimations θ_α^i and θ_β^i are bounded. Therefore, the boundedness of $\hat{\alpha}^i(x,v)$, $\hat{\beta}^i(x,v)$, $\alpha_k^i(t)$, and $\beta_k^i(t)$ assures that u_{ce} and u_{si} and hence u are bounded.

To show asymptotic stability of the output, note that

$$\int_0^\infty \eta e_s^2 dt \leq -\int_0^\infty \dot{V}_i dt = V_i(0) - V_i(\infty) \tag{35}$$

this establishes that $e_s \in L_2$ ($L_2 = \{z(t) : \int_0^\infty z^2(t) < \infty\}$) since $V_i(0)$ and $V_i(\infty)$ are bounded. Since e_s and $\dot{e}_s$ are bounded and $e_s \in L_2$, by Barbalat's Lemma we have asymptotic stability of e_s, which implies asymptotic stability of the tracking error e (i.e., $\lim_{t\to\infty} e = 0$). $\square$

4 Simulation Examples: Jet Engine Control

To study the effectiveness of the proposed adaptive control methods, we apply them to the component level model simulation of an aircraft jet engine (General Electric XTE46). The General Electric XTE46 engine is a simplified, unclassified version of the original IHPTET engine [61]. The component level engine cycle model of the XTE46 engine is a thermodynamic simulation package, where each engine component is simulated. The CLM executes one pass within the digital control's sampling time and thermodynamic states are assumed to be in equilibrium after each pass through the simulation. The operating condition of the engine is defined by the altitude (ALT), mach number (XM), difference of temperature (DTAMB), and throttle setting represented by power code (PC). The health of the engine is described by ten "quality parameters" which include the flows and efficiencies of the fan, the compressor, and turbines. The model has three state variables, including the fan rotor speed (XNL), the core rotor speed (XNH), and the temperature at combustor inlet (TMPC). There are six actuators, but the major control variables are the combustor fuel flow (WF36), the exhaust nozzle area (A8), and the variable area bypass injector area (A16). For simplicity, we assume that the fundamental dynamic characteristics of the CLM can be represented by a single-input single-output system in the form

$$\dot{x} = f(x, u, c, p) \tag{36}$$

$$y = h(x, u, c, p) \tag{37}$$

where $x = [\text{XNL, XNH}]^\top$ is the state vector, $u = \text{WF36}$ is the input variable, $y = \text{XN2}$ is the output of the engine, $c = [\text{ALT, XM, DTAMB, PC}]^\top$ represents the operating condition of the engine, $p = [\text{ZSW2, SEDM2, ZSW7D, SEDM7D, ZSW27, SEDM27, ZSW41, ZSE41, ZSW49, ZSE49}]^\top$ represents the quality parameter vector, $f(\cdot)$ denotes the unknown function representing the nonlinear characteristics of the engine, and $h(\cdot) = \text{XNL}$ because the output variable XN2 is just the measurement of the state variable XNL.

To develop a computational model for the XTE46 engine, we performed nonlinear system identification to approximate local engine dynamics (specified by fixed values of operating conditions and quality parameters), followed by interpolating these local models to generate the "global" model (actually, it is a "regional" model valid in the "climb" region) [60, 62]. The general form of the model can be described as

$$\dot{x} = f(x, c, p) + g(x, c, p)u \tag{38}$$

$$y = x_1 \tag{39}$$

where

$$
\begin{aligned}
f(x,c,p) &= \frac{\sum_{i=1}^{N} \underline{f}(x,c_i,p_i)\mu_i(c,p)}{\sum_{i=1}^{N} \mu_i(c,p)} && (40)\\
g(x,c,p) &= \frac{\sum_{i=1}^{N} \underline{g}(x,c_i,p_i)\mu_i(c,p)}{\sum_{i=1}^{N} \mu_i(c,p)} && (41)\\
\underline{f}(x,c_i,p_i) &= \frac{\sum_{j=1}^{R} [a_{j,0}(c_i,p_i) + a_{j,1}(c_i,p_i)x_1 + a_{j,2}(c_i,p_i)x_2]\underline{\mu}_j(x_1)}{\sum_{j=1}^{R} \underline{\mu}_j(x_1)} && (42)\\
\underline{g}(x,c_i,p_i) &= \frac{\sum_{j=1}^{R} a_{j,3}(c_i,p_i)\underline{\mu}_j(x_1)}{\sum_{j=1}^{R} \underline{\mu}_j(x_1)} && (43)
\end{aligned}
$$

where $u = \mathrm{WF36}$ and $x = [x_1, x_2]^\top = [\widehat{\mathrm{XNL}}, \widehat{\mathrm{XNH}}]^\top$ which is positive since the fan speed and the core speed cannot be negative and $x \in S_x$ (a valid speed region). The value of c is the known operating condition vector, p is unknown quality parameter vector, and c_i, p_i are specified nodes where we establish node models. Also, $f = [f_1, f_2]^\top$, $g = [g_1, g_2]^\top$, $\underline{f} = [\underline{f}_1, \underline{f}_2]^\top$, and $\underline{g} = [\underline{g}_1, \underline{g}_2]^\top$ are 2×1 function vectors, $a_{j,0}, a_{j,1}, a_{j,2}, a_{j,3}$ are 2×1 vectors of linear parameters of Takagi-Sugeno fuzzy systems, $\mu_i(c,p)$ are membership functions of fuzzy interpolation between different operating conditions and quality parameters, and $\underline{\mu}_j(x_1)$ are membership functions describing the nonlinearity with respect to x_1. By inspecting the parameters that result from the identification process we found that $a_{j,3}^1(c_i,p_i) > a_{j,3}^2(c_i,p_i) > 0$ and $a_{j,2}^2(c_i,p_i) < a_{j,2}^1(c_i,p_i) < 0$ for any $i = 1, 2, \ldots, N, j = 1, 2, \ldots, R$. Thus, we know that the "relative degree" of the engine is 1 and the engine zero dynamics are uniformly exponentially attractive [60].

We develop an adaptive controller for the engine control problem using the indirect method. We choose to implement the indirect controller because we could facilitate the indirect adaptive controller by using our developed models to represent nominal system dynamics in different regions. Consider the engine in the form of

$$
\begin{aligned}
\dot{y} &= f_1(x,c,p) + g_1(x,c,p)u && (44)\\
&= \sum_{i=1}^{N} \left(\alpha_k^i(c,p_0,t) + \alpha^i(x,c,p)\right)\xi^i(c) + \sum_{i=1}^{N} \left(\beta_k^i(c,p_0,t) + \beta^i(x,c,p)\right)\xi^i(c)u && (45)
\end{aligned}
$$

where $x(t)$ and $y(t)$ are measurable according to the properties of the component level engine model. By studying dynamics of the developed nonlinear model we know that $g_1(x,c,p) > 0.32$ so that we can set $\beta_0 = 0.32$. We use our developed engine model to represent the nominal model dynamics $\alpha_k^i(c,p_0,t)$ and $\beta_k^i(c,p_0,t)$ by setting the quality parameters to be the nominal value p_0, and they are bounded if x is bounded since the model is in the form of a Takagi-Sugeno fuzzy system. The unknown dynamics $\alpha^i(x,c,p)$ and $\beta^i(x,c,p)$ describe the model uncertainties caused by nominal model inaccuracy and system changes (time-varying characteristics). They will be approximated by radial basis function networks $\hat{\alpha}^i$ and $\hat{\beta}^i$ with 11 receptive field units for each. The inputs to the neural networks include two state variables and the parameters are updated on-line to capture the unknown time-varying dynamics. Note that the stable adaptive

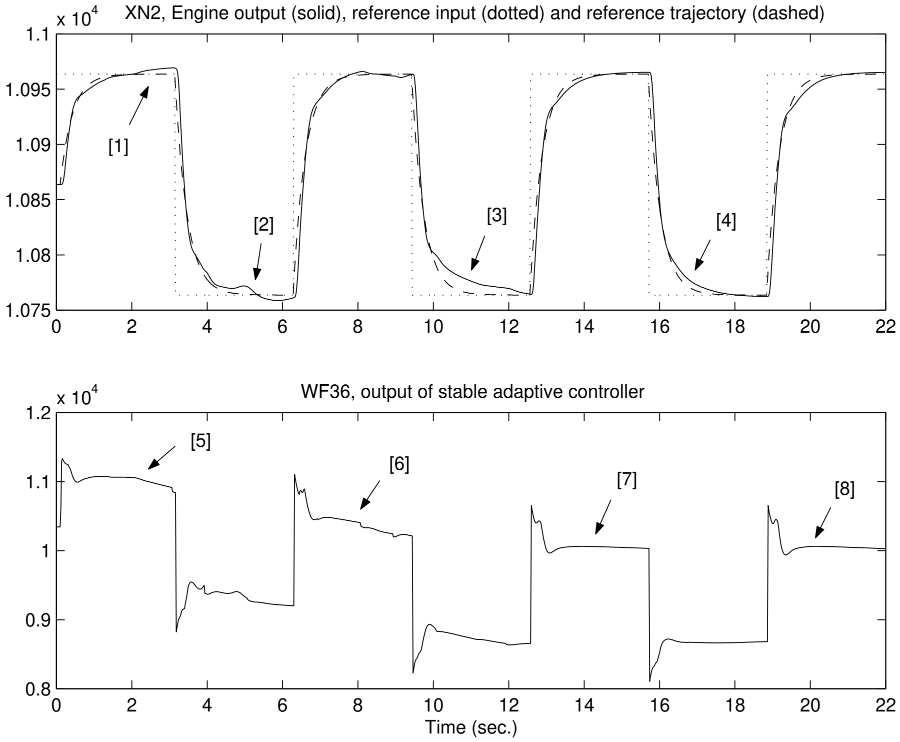

Figure 1: Performance of the interpolated adaptive controller.

controller will ensure the stability of x_1, and the uniform exponential attractivity of the engine zero dynamics will ensure the stability of the unobservable state x_2. Since the relative degree of the system is 1, the error dynamics are simple ($e_s(t) = e(t)$ and $\bar{e}_s(t) = 0$). Taking into account of the engine dynamics, the model uncertainty is described by $W_\alpha^i = 0.005$ and $W_\beta^i = 0.005$. Note that we cannot explicitly know the model uncertainty so that the parameters W_α^i and W_β^i are treated as design parameters and tuned by trial and error to achieve good control performance. In addition, the adaptation gains are tuned to be ${Q_\alpha^i}^{-1} = 5e-8$ and ${Q_\beta^i}^{-1} = 1e-17$, and the design parameter $\eta = 1$. The reference trajectory is defined by passing a reference signal through a linear reference model $\frac{Y_m(s)}{R(s)} = \frac{3}{s+3}$ so that $y_m(t)$ and $\dot{y}_m(t)$ are measurable and bounded.

We let the component level engine model run at a scenario where the altitude (ALT) varies from 12500 to 17500 (i.e., ALT = 12500 for first 2 seconds; then ALT is linearly increased from 12500 to 17500 for 10 seconds; after that ALT keeps constant at 17500) and other operating conditions are kept constant, that is, XM = 0.7, DTAMB = 0, and PC = 46. As shown in Figure 1, although the altitude changes significantly during the time period ($T = 2 \sim 12$ sec.), the performance does not deteriorate too much (as indicted by arrows 1, 2, and 3). Moreover, the adaptation scheme is used to improve the performance by compensating for model uncertainties (as indicted by arrows 3 and 4). The output of the adaptive controller also indicates the smooth changes of control laws (as pointed by arrows 5, 6, 7, and 8).

The effectiveness of the proposed "interpolated" adaptive controller can be further demon-

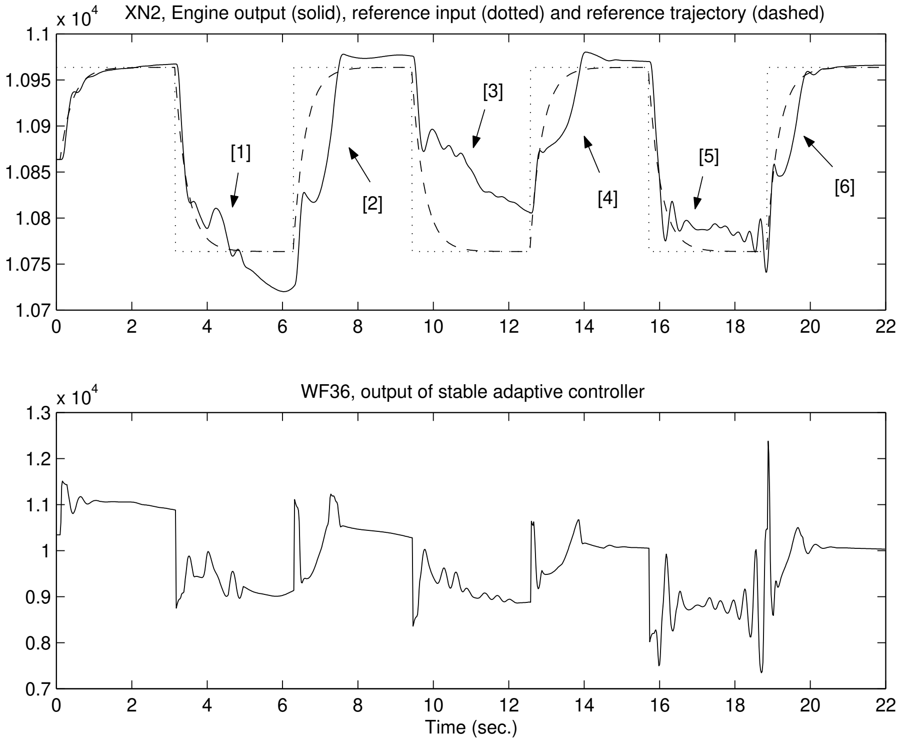

Figure 2: Performance of the single adaptive controller.

strated by comparing its performance with that of a "single" adaptive controller (where the adaptive control law is not in the interpolated form but is driven by one set of on-line approximators) [58]. As shown in Figure 2, the fast time-varying nature of altitude change makes it difficult for the on-line approximators to adapt fast enough, which results in deteriorated performance (as indicted by arrows 1, 2, and 3), even though the adaptation scheme does have apparent effects when system dynamics are not fast time-varying (as indicted by arrows 4, 5, and 6).

5 Conclusion

Adaptive control for nonlinear time-varying systems is of both theoretical and practical importance. In this paper we present an adaptive control methodology for a class of nonlinear systems with a time-varying structure where linear in the parameter approximators are used. This class of systems is composed of interpolations of nonlinear subsystems which are input-output feedback linearizable. Without assumptions on rate of change of system dynamics, a stable indirect adaptive control methods are presented with analysis of stability for all signals in the closed-loop as well as asymptotic tracking. The performance of the adaptive controller is demonstrated using a jet engine control problem.

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