Name: Solution

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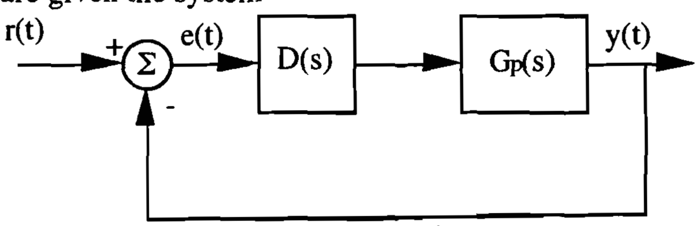
ECE 551 | Quiz #2 | April 18, 2008

Suppose that you are given the system



where $D(s) = \frac{1}{s}$ is the controller and $G_p(s) = \frac{1}{(s+1)(s+2)}$ is the plant.

(a) Find the closed-loop transfer function, Y(s)/R(s). The denominator of this transfer function is the "pole polynomial" and it can be used to study the stability of the closed-loop system. Use the Routh Criterion (you must show all steps) to show that the closed-loop system is stable.

50 pts

Open-loop transfer function: $T_{OL}(s) = D(s) \cdot G_p(s) = \frac{1}{s(s+1)(s+2)}$

Closed-loop: $T_{CL}(s) = \frac{T_{OL}(s)}{1+T_{OL}(s)} = \frac{1}{s^3+3s^2+2s+1}$.

$P(s) = s^3+3s^2+2s+1$

Routh:

s^3	1	2	0
s^2	3	1	0
s^1	$\frac{5}{3}$	0	
s^0	1		

Since there is no change of sign in the first column the closed-loop system is stable.

(b) If r(t) is a unit step input, what is e(∞)? If r(t) is a ramp input, what is e(∞)? Show all your work.

50 pts

$E(s) = R(s) - Y(s)$
$= R(s) - D(s)G_p(s) \cdot E(s)$

s.t. $E(s) = \frac{1}{1+D(s)G_p(s)} R(s) = R(s) \cdot \frac{s(s+1)(s+2)}{1+s(s+1)(s+2)}$ → stable

We can use FVT.

<1> unit step input

$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{s(s+1)(s+2)}{1+s(s+1)(s+2)} = 0$

<2> ramp input

$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{1}{s^2} \cdot \frac{s(s+1)(s+2)}{1+s(s+1)(s+2)} = 2$