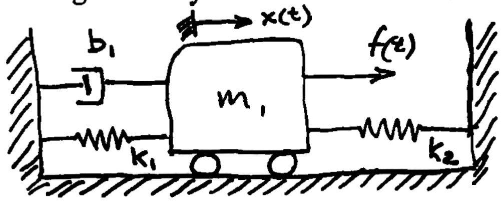
Name: Solution

The Ohio State University
Department of Electrical and Computer Engineering

ECE 551 — Quiz #1 — April 11, 2008

Suppose that you are given the single-cart system shown below:



f(t) is a force

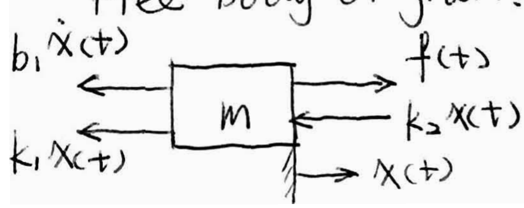
$k_1 > 0, \; k_2 > 0$

$b_1 > 0, \; m_1 > 0$

Assume that there are zero initial conditions (i.e., that the cart is initially at x=0 and is not moving). Assume that the cart is traveling on a frictionless surface.

(1) Find the ordinary differential equation that models the single-cart system shown.

Free body diagram:



ODE: $m\ddot{x}(t) = f(t) - b_1\dot{x}(t) - k_1 x(t) - k_2 x(t)$

$$m\ddot{x} + b_1\dot{x} + (k_1+k_2)x = f(t)$$

(2) Find the transfer function from (F(s) to X(s) - i.e., viewing X(s) as the output).

Assume zero initial condition, use Laplace transform.

$$ms^2 X(s) + b_1 s X(s) + (k_1+k_2)X(s) = F(s)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + b_1 s + k_1 + k_2}$$

(3) Let f(t) be a unit step. What is the final value of x(t) (i.e., find x(∞), the final position of the cart).

poles of $sX(s)$: $S_{1,2} = \dfrac{-b_1 \pm \sqrt{b_1^2 - 4m(k_1+k_2)}}{2m}$

$k_1 > 0, k_2 > 0, b_1 > 0, m_1 > 0$. s.t., $S_{1,2}$ are in left half plane. we can use final value theorem.

$$x(\infty) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} sF(s)\frac{1}{ms^2+b_1 s+k_1+k_2}$$

$$= \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{1}{ms^2+b_1 s+k_1+k_2} \text{ (unit step input)} = \frac{1}{k_1+k_2}$$