ECE 551 HW4 Solution

1. P6.1 b, c (5 pts)

(b) s^3+4s^2+8s+4

s^3	1	8
s^2	4	4
s^1	7	0
s^0	4	

stable

(c) $s^3+2s^2-4s+20$

s^3	1	-4
s^2	2	20
s^1	-14	0
s^0	20	

unstable

2. P6.1e (5 pts)

$s^4+s^3+3s^2+2s+K$

s^4	1	3	K
s^3	1	2	0
s^2	1	K	
s^1	2-K	0	
s^0	K		

For a stable system

$2-K>0,\ K>0 \Rightarrow 0<K<2$

If $K=0, 2$, marginally stable.

3. P6.4a (10 pts)

Closed-loop transfer function:

$$T_{CL}(s)=\frac{G_c(s)G(s)}{1+G_c(s)G(s)H(s)}=\frac{K(s+20)(s+40)}{s^3+30s^2+(200+K)s+40K}$$

s^3	1	200+K
s^2	30	40K
s^1	$200-\frac{K}{3}$	0
s^0	40K	0

For a stable system.

$200-\frac{K}{3}>0,\quad 40K>0$

$\Rightarrow\ 0<K<600$

If $K=600 \Rightarrow$ marginally stable

(4) PI.1 a.b.c (20 pts)

a. $G_c(s)G(s) = \dfrac{k}{s(s+8)(s+10)}$. Poles: $p_1 = 0$. $p_2 = -8$. $p_3 = -10$

① asymptotes: $n-m=3$, $\sigma_a = \dfrac{\sum p_i - \sum z_j}{n-m} = \dfrac{-8-10}{3} = -6$

$\varphi_a = \dfrac{(2k+1)\pi}{n-m} = \dfrac{(2k+1)\pi}{3}$, $k = 0, 1, 2$. : $\dfrac{\pi}{3}, \pi, \dfrac{5}{3}\pi$.

② d: $\displaystyle\sum_{j=1}^{m} \frac{1}{d-z_j} = \sum_{i=1}^{n} \frac{1}{d-p_i} \Rightarrow \frac{1}{d} + \frac{1}{d+8} + \frac{1}{d+10} = 0$. $d = -2.9$

angle: $\dfrac{(2k+1)\pi}{l}$ $l = 2$. $\Rightarrow \dfrac{\pi}{2}$, $\dfrac{3}{2}\pi$.

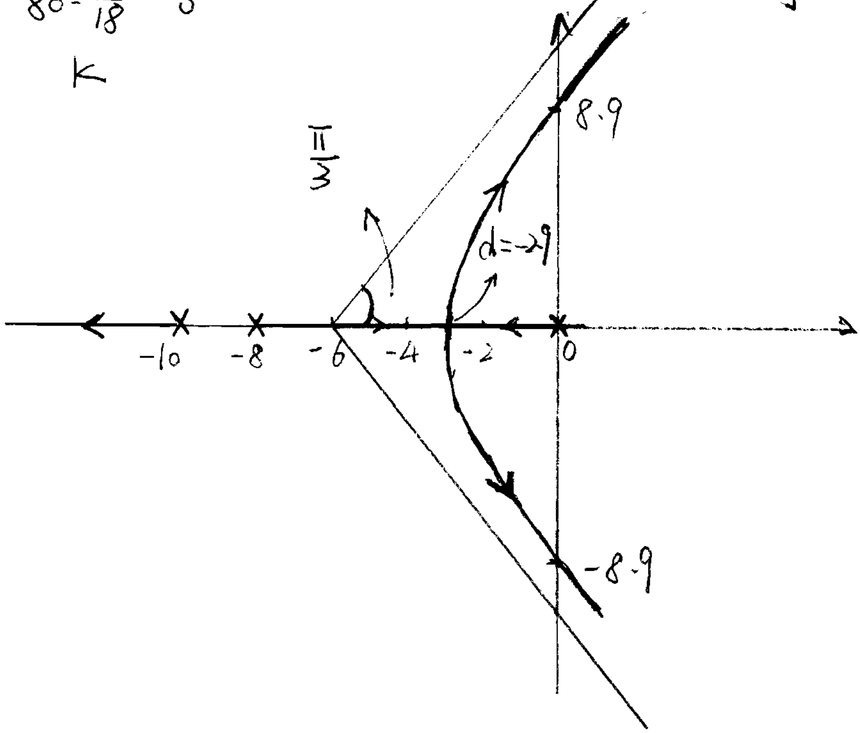
③ points on $j\omega$-axis: characteristic eq. $s(s+8)(s+10)+k=0$.

$s^3 + 18s^2 + 80s + k = 0$ let $80 - \dfrac{k}{18} = 0$. $k = 1440$.

s^3	1	80
s^2	18	k
s^1	$80 - \frac{k}{18}$	0
s^0	k	

auxilliary equation: $18s^2 + 1440 = 0$

$s_{1,2} = \pm 8.94j$



b. $G_c(s)G(s) = \dfrac{k}{(s^2+2s+2)(s+1)}$. poles: $p_1 = -1,\ p_{2,3} = -1 \pm j$

① asymptotes: $n-m=3$. $\sigma_a = \dfrac{\sum p_i - \sum z_j}{n-m} = -1$ $\varphi_a = \dfrac{(2k+1)\pi}{3} : \dfrac{\pi}{3}, \pi, \dfrac{5}{3}\pi$.

② departure angle: $\theta_i = (2k+1)\pi + \sum_{j=1}^{m} \varphi_{z_j p_i} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \theta_{p_j p_i}$, $k=0, 1$.

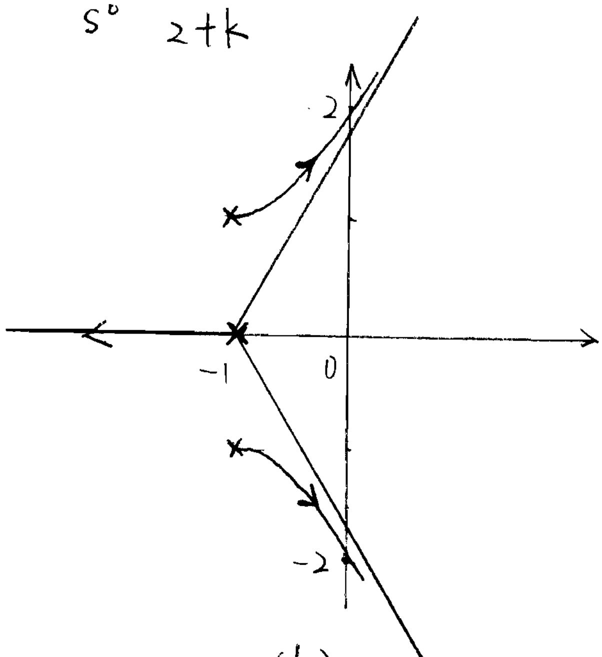
$\theta_1 = 0,\ \theta_2 = 2\pi$

③ points on $j\omega$-axis $s^3 + 3s^2 + 4s + 2 + k = 0$

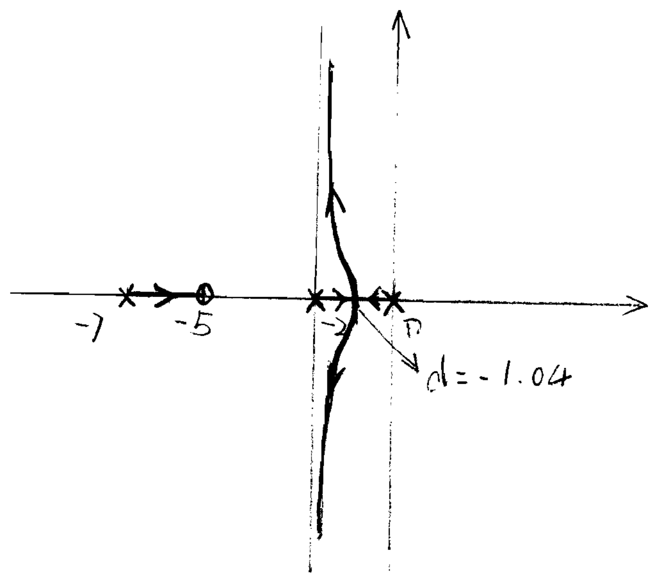
s^3	1	4
s^2	3	$2+k$
s^1	$\frac{10-k}{3}$	0
s^0	$2+k$	

let $\dfrac{10-k}{3} = 0$, $k = 10$

auxilliary eq. $3s^2 + 12 = 0$, $s_{1,2} = \pm j2$.



(b)



(c)

c. $G_c(s)G(s) = \dfrac{k(s+5)}{s(s+2)(s+7)}$ zeros: $z_1 = -5$. poles: $p_1 = 0, p_2 = -2, p_3 = -7$

① asymptotes: $n-m=2$. $\sigma = \dfrac{-7-2-(-5)}{2} = -2$, $\varphi = \dfrac{(2k+1)\pi}{2}$ $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$.

② $\dfrac{1}{d+5} = \dfrac{1}{d} + \dfrac{1}{d+2} + \dfrac{1}{d+7}$. $\Rightarrow d = -1.04$. angle: $\dfrac{(2k+1)\pi}{l} \Rightarrow \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

③ points on $j\omega$-axis : no points on $j\omega$-axis.