ECE 551 HW #3 Solution.

E5.2 (10 pts)

(a) $E(s) = R(s) - G(s)$ $G(s) = E(s) \cdot \frac{100}{(s+2)(s+5)}$ such that.

$$E(s) = \frac{1}{1 + \frac{100}{(s+2)(s+5)}} R(s) = \frac{(s+2)(s+5)}{(s+2)(s+5)+100} \cdot R(s)$$

$$\text{FVT} \Rightarrow e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{(s+2)(s+5)}{(s+2)(s+5)+100} = \frac{1}{11}$$

(Assuming unity step input)

(b) overshoot = 32.6%

E5.5 (10 pts)

$$E(s) = R(s) - Y(s) \qquad Y(s) = (R(s) - Y(s)ks - Y(s)) \cdot \frac{100}{s^2}$$

s.t. $$\frac{E(s)}{R(s)} = \frac{s^2 + 100ks}{s^2 + 100ks + 100}$$

step input: $e(\infty) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{s^2+100ks}{s^2+100ks+100} = 0$

ramp input: $e(\infty) = \lim_{s\to 0} s \cdot \frac{1}{s^2} \cdot \frac{s^2+100ks}{s^2+100ks+100} = k$

parabola input: $e(\infty) = \lim_{s\to 0} s \cdot \frac{1}{s^3} \cdot \frac{s^2+100ks}{s^2+100ks+100} = \infty$

type I.

E5.8 (10 pts)

$$T_{CL}(s) = \frac{G(s)}{1+G(s)} = \frac{k}{s^2 + \sqrt{2k}s + k}$$ second order system

$\omega_n = \sqrt{k}$, $\zeta = \frac{\sqrt{2}}{2}$

(a). overshoot $= e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\% = 4.32\%$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{8}{\sqrt{2k}}.$$

(b) $T_s < 1 \Rightarrow k > 32$.