HW #2 Solution.

(1) E4.2 (10 pts).

(a) $G(s) = K_2$, $T_d(s) = 0$. $\quad T(s) = \dfrac{V_o(s)}{V_{in}(s)} = \dfrac{k_1K_2}{1+k_1K_2}$

$$S_G^T = S_{K_2}^T = \frac{k_2}{T}\cdot\frac{\partial T}{\partial k_2} = \frac{1}{1+k_1k_2}$$

(b) $$V_o(s) = \frac{k_1k_2}{1+k_1k_2}V_{in}(s) + \frac{k_2}{1+k_1k_2}T_d(s)$$

effect of disturbance: assume $V_{in}(s)=0$.

$$V_o(s) = \frac{k_2}{1+k_1k_2}T_d(s)$$

(c) $V_o(s) = \dfrac{1}{\frac{1}{k_2}+k_1}T_d(s)$. Pick large k_1 to minimize the effect of disturbance.

(2) P4.2 (10 pts)

(a) Open-loop: $T_d(s)=0$ $\quad T_{OL}(s) = \dfrac{\theta(s)}{\theta_d(s)} = k_aG(s)$

$$S_{k_a}^{T_{OL}} = \frac{k_a}{k_aG(s)}\cdot\frac{\partial T_{OL}}{\partial k_a} = 1, \qquad S_{k_1}^{T_{OL}} = 0$$

Closed-loop: $T_d(s)=0$ $\quad T_{CL}(s) = \dfrac{k_aG(s)}{1+k_ak_1G(s)}$

$$S_{k_a}^T = \frac{k_a}{T}\cdot\frac{\partial T}{\partial k_a} = \frac{1}{1+k_ak_2G(s)}$$

$$S_{k_1}^T = \frac{k_1}{T}\cdot\frac{\partial T}{\partial k_1} = -\frac{k_1k_aG(s)}{1+k_ak_1G(s)}$$

(b) Open-loop: $\theta_d(s)=0$.

$$\theta(s) = T_d(s)\cdot G(s) = T_d(s)\cdot\frac{9}{s^2+1.2s+9}$$

Wave effect: step input: $T_d(t) = k$ rads

$\theta(\infty) = \lim\limits_{s\to 0} s\theta(s) = k$. $\Rightarrow$ open-loop system has a poor ability to reduce disturbance.

Closed-loop: $\theta_d(s)=0$.

$$\theta(s) = T_d(s)\cdot\frac{G(s)}{1+k_ak_1G(s)} = \underbrace{\frac{k}{s}}_{T_d(s)}\cdot\frac{9}{s^2+1.2s+9+9k_ak_1}$$

$\theta(\infty) = \lim_{s\to 0} s\theta(s) = K \cdot \frac{1}{1+K_a K_1}$

If we choose large K_a, K_1, the steady-state error will be very small. $\Rightarrow$ closed-loop system has a strong ability to redue disturbance.

(3). (a) Proportional Controller.

(20 pts). Open-loop TF: $G(s) = \frac{K}{s+1}$. Closed-loop TF: $T(s) = \frac{K}{s+K+1}$

$E(s) = \frac{1}{1+G(s)} R(s)$, $e(\infty) = \lim_{s\to 0} s E(s) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{s+1}{s+K+1} = \frac{1}{1+K}$

This is first order system $\Rightarrow$ no overshoot!

$y(t) = \frac{K}{K+1}(1 - e^{-(1+K)t})$ using inverse Laplace.

Let $y(t) = 0.9$. rise time $= -\frac{1}{1+K} \ln(1 - 0.9\frac{K+1}{K})$

Zero steady state error $\Rightarrow$ K large!

for large K $\Rightarrow$ rise time can be less than 1 sec.

For example: pick $K = 1000$

(b). PI Controller $T(s) = \frac{T_I K s + 1}{T_I s^2 + T_I (1+K) s + 1}$

$E(s) = \frac{1}{1+G(s)} R(s)$. $G(s) = \frac{T_I K s + 1}{T_I s(s+1)}$

$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{T_I s(s+1)}{T_I s(s+1) + T_I K s + 1} = 0$!

If the closed-loop system has real poles, it can be represented as the sum of several first order system $\Rightarrow$ no overshoot! $\Rightarrow [T_I(1+K)]^2 > 4T_I$

And according to part (a), we know if we pick large K, for the first order system, we can ensure that rise time is less than 1 sec. For example, here we pick $T_I = 1$, $K = 1000$! or $K = 10$, ect.