ECE 551 HW # 1 Solution

Problem 1 Assume that the floor is a fractionless surface. For M_1, we have

$$M\ddot{x}_1 + K(2x_1 - x_2) = F(t)$$

For M_2, we have

$$M\ddot{x}_2 + b\dot{x}_2 - K(x_1 - x_2) = 0$$

Problem 2 Use the differential equation in class, and by using Laplace transform, we obtain

$$\begin{aligned} Js^2\Theta(s) + Ds\Theta(s) &= k_m I_a(s) \\ V_b(s) &= k_b s\Theta(s) \\ V_b(s) + R_a I_a(s) + L_a s I_a(s) &= V_a(s) \end{aligned}$$

Then, the transfer fuction (from the input V_a to the output Θ) is

$$G(s) = \frac{\Theta(s)}{V_a(s)} = \frac{k_m}{JL_a s^3 + (JR_a + DL_a)s^2 + (DR_a + k_b k_m)s}$$

Problem 3 The differential equation of the quarter-car system is

$$u + mg - kx - b\dot{x} - \delta = m\ddot{x}$$

At rest, $\ddot{x} = \dot{x} = x = u = 0$, such that, $\delta = mg$. By using Laplace transform, we obtain

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + bs + k}$$

In MATLAB, let m to be 1, 1.25, and 0.75, find the step response. Then we obtain the settling time: (a) 4.22, (b) 4.74, (c) 2.38; and the overshoot: (a)4.3%, (b)7.7%, (c) 1.2%. You should explain clearly the influence of the mass on the vertical position of the car.

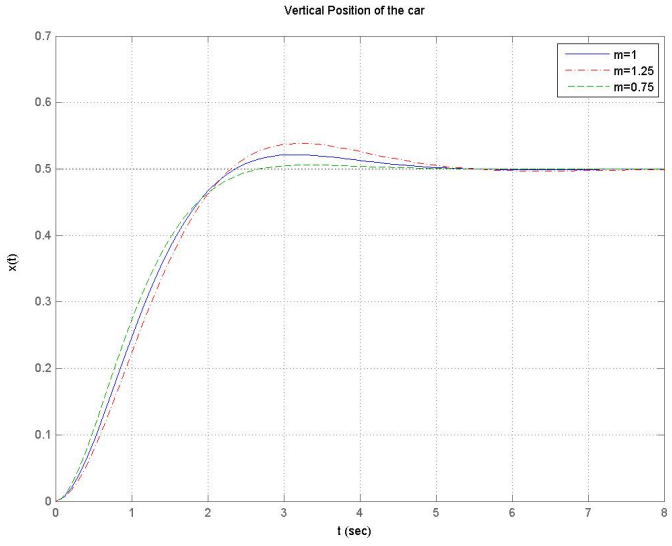

Figure 1: Vertical Position (step response)