Design Project #2 Solution

P10.1 a $\zeta = 0.6$. $T_s \leq 2.5$. $\Rightarrow \omega_n \geq 2.67$

Lead Compensator: $G_c(s) = \dfrac{\alpha\tau s+1}{\alpha(\tau s+1)}$

The loop gain $k = \omega_n^2 \geq 2.67^2 = 7.13$. choose $k = 10$.

$\phi_{pm} = \dfrac{\zeta}{0.01} = 60°$. here we choose $\phi_{pm} = 75°$.

$$\alpha = \frac{1+\sin\phi_{pm}}{1-\sin\phi_{pm}} = 57.7. \quad \text{we choose } \alpha = 60.$$

Then we can obtain the bode plot of the uncompensated system with loop gain k. i.e. $G(s) = \dfrac{k}{s^2}$, $k = 10$.

$10\log\alpha = 17.78$, in this bode plot. we can find the $\omega_m = 8.81$, where at this freqency. the magnitude is $-10\log\alpha = -17.78$, such that.

$$\tau = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{8.81\cdot\sqrt{60}} = 0.0147$$

Therefore. $G_c(s) = \dfrac{0.882s+1}{60(0.0147s+1)}$. to obtain a loop gain $k = 10$. k_2 should be chosen to equal $k\cdot\alpha = 600$ to balance $\frac{1}{\alpha}$ term in the compensator.

$$G_c(s) = \frac{0.882s+1}{60(0.0147s+1)}. \quad k_2 = 600, \quad k_1 = 1.$$

Then we can obtain the step response in Matlab.

overshoot $= 9.0415\%$. settling time $= 1.76$ s.

This is only a candidate solution.

P10.9. ②

$$G(s) = \frac{50}{s(0.1s+1)^2}$$

Lag compasentor $G_c(s) = \frac{k(\tau s+1)}{\alpha \tau s+1}$.

for a ramp input. $e_{ss} \le 2.5\%$. $\Rightarrow$ $k_v \ge 40$.

$\Rightarrow k \ge 0.8$ $(k_v = k \cdot 50)$. choose $k=2$

overshoot $\le 5\%$. $\zeta \ge 0.7$. $\phi_{pm} \ge 70°$. choose $\phi_{pm} = 75°$

From the uncompensated Bode diagram (with loop gain $k_v = 100$), allowing 5° for the phase-lag compensator. we locate the frequenly ω, where $\phi(\omega) = -100°$. to be the new crossover frequency $\omega_c' = 0.8$ (allows for a small margin of safety). the magnitude at the new crossover frequency is 41.9 dB $\Rightarrow$ 41.9 dB $= 20 \log \alpha$. $\alpha = 124.45$

zero is one decade below the crossover frequency.

$\omega_z = \omega_c'/10 = 0.08$. $\omega_p = \omega_z/\alpha = 6.42 \times 10^{-4}$

such that $G_c(s) = \frac{4(12.5s+1)}{1558s+1}$

Finally. choose $k=4$ to ensure overshoot

overshoot $= 4.53\%$

$e_{ss} = 0.5\%$ for ramp input.