Laboratory 11

State Feedback Controller for Position Control of a Flexible Joint

11.1 Objective

The objective of this laboratory is to design a full state feedback controller for endpoint position control in the face of flexibility effects for a flexible joint mounted on the SRV-02DC servomotor. For this, we will use the state space model of the combined system (i.e., servomotor and flexible joint) introduced in the Laboratory 8 (refer to [1]) and then tune the feedback gain matrix K to find the best position tracking while minimizing endpoint oscillation.

11.2 Model

From Section 8.2 of the course textbook [1], the default connection that we will use in this laboratory is illustrated. For this laboratory, the servo is used in the high gear ratio configuration (refer to Figure 3.5 in the textbook [1]). The flexible joint is described with a fourth-order system and is shown in Figure 1.

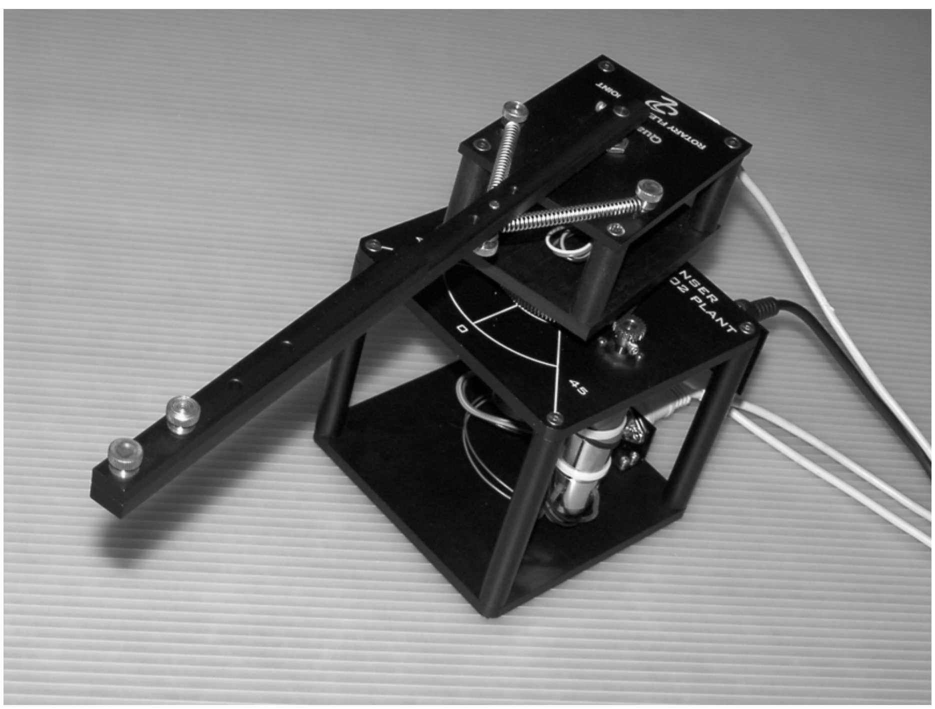

Figure 1: Flexible Joint.

The following variables are used to describe the state-space model of the flexible joint:

- θ: motor shaft position, measured using channel 1 encoder "ENC1"
- $\dot{\theta}$: angular velocity of the motor shaft, measured using channel 3 ADC
- α: angular deflection of the joint's arm, measured using channel 2 encoder "ENC2"
- $\dot{\alpha}$: angular rate of the joint's angle, computed with a derivative filter of the form $\frac{150s}{s+150}$
- K_{stiff}: linear approximation of the joint stiffness. $K_{stiff} = 1.6108$ Nm/rad
- J_{hub}: total inertia of the motor. $J_{hub} = 0.0021$ Kg.m^2
- J_{load}: inertia of the arm. $J_{load} = 0.0059$ Kg.m^2
- R_m: armature resistance of the motor. $R_m = 2.6\ \Omega$
- K_m: one of the motor torque constants. $K_m = 0.00767$
- K_g: gear ratio of the motor. In the high gear ratio configuration, $K_g = 70$

The fourth-order linearized differential equation representing the dynamics of the plant can be written in a first-order matrix differential equation, representing the state variable description of the system. The state variable representation is a compact notation combining four coupled first-order differential equations each describing the dynamic of one state of the plant. We define the state vector, x, for the flexible joint as follows

$$x = \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} \tag{1}$$

Let y be the output of the system and u the voltage input to the motor (V_{in}). The state variable model is of the form

$$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases} \tag{2}$$

The derivation of the model equations from physics involves mathematical modeling and linearization techniques that are beyond the scope of this lab. The following matrices will comprise the design model for the system

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{stiff}}{J_{hub}} & -\frac{K_m^2 K_g^2}{R_m J_{hub}} & 0 \\ 0 & -\frac{K_{stiff}(J_{load}+J_{hub})}{J_{hub}J_{load}} & \frac{K_m^2 K_g^2}{R_m J_{hub}} & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ \frac{K_m K_g}{R_m J_{hub}} \\ -\frac{K_m K_g}{R_m J_{hub}} \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

11.3 Laboratory Preparation

1. Write a MATLAB script to design a pole placement based full state feedback controller for the model of the flexible joint. Use the (A, B, C) state space matrices given in this laboratory. Check for controllability of the system. Choose appropriate locations for the closed loop poles such that the settling time (t_s) is less than 0.9 seconds and with no overshoot based in the simulation described in question 2.

2. Construct the following Simulink model in order to test the pole placement based full state feedback controller designed in question 1. You will plug-in the following information:

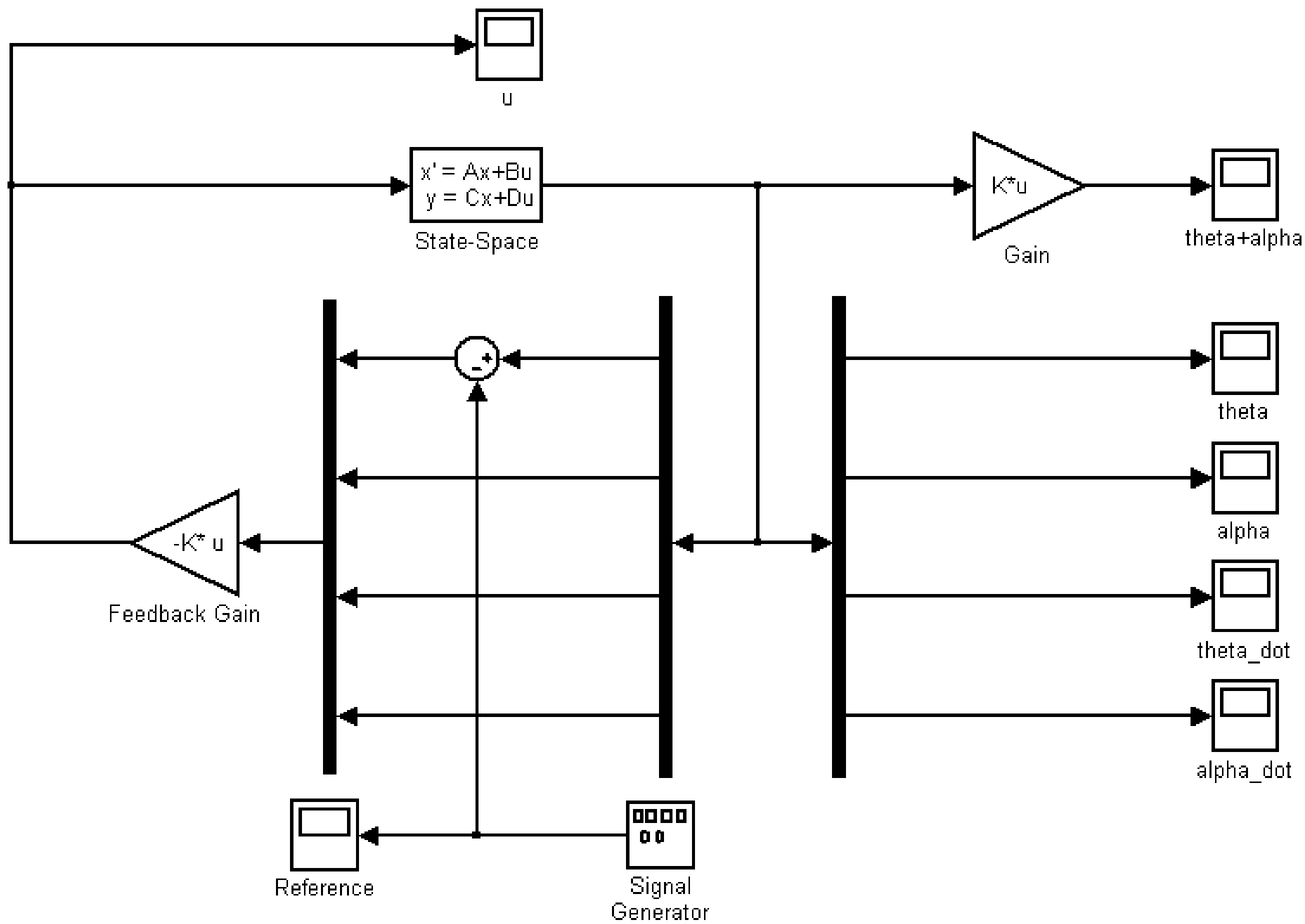

Figure 2: Simulink model for simulation of state feedback controller.

- Set the Signal Generator amplitude to $45 * pi/180$ and frequency to 0.05 Hz.
- Set the simulation time to 60 seconds.
- Enter the corresponding A, B, C, D matrices in the State-Space box. Remember that we are interested in all the states at the output so the matrix C is different from the one given by the model.
- Define the gain K of the box "Gain" (this is not the feedback gain) such that $y = \theta + \alpha$.

- Tune the feedback gain matrix K until you obtain the desired design performance. Remember to take into account the input voltage constraint of the motor, that is ±5V.
- Save all the scopes' data as an array into the workspace. Do not limit the size of the output data.

3. Generate the following plots:

- Comparison between the plant output ($\gamma = \theta + \alpha$) and reference input (θ_d). Indicate in the same plot the settling time of the system (2% criterion).
- Control signal applied to the plant. Make sure to that it is in the range ±5V.
- Error between the plant output and reference input (i.e., error = θ - θ_d).
- Subplot including all the states variables θ, $\dot{\theta}$, α, and $\dot{\alpha}$.

11.4 Laboratory Procedure

11.4.1 Connections

Connect the motor in the position control configuration then connect the encoder port located on the back of the flexible joint to the channel 2 encoder "ENC2" on the DS1104 interface board.

11.4.2 Simulink Diagram and ControlDesk

- Open MATLAB and Simulink and create a new model called Lab11_Group_#.mdl, where # corresponds to the lab group number. Set the simulation parameters to a fixed step size of 1 ms and a simulation time of 60 seconds. Turn off block reduction and set the simulation initial state to STOP.
- The model should look as follows:

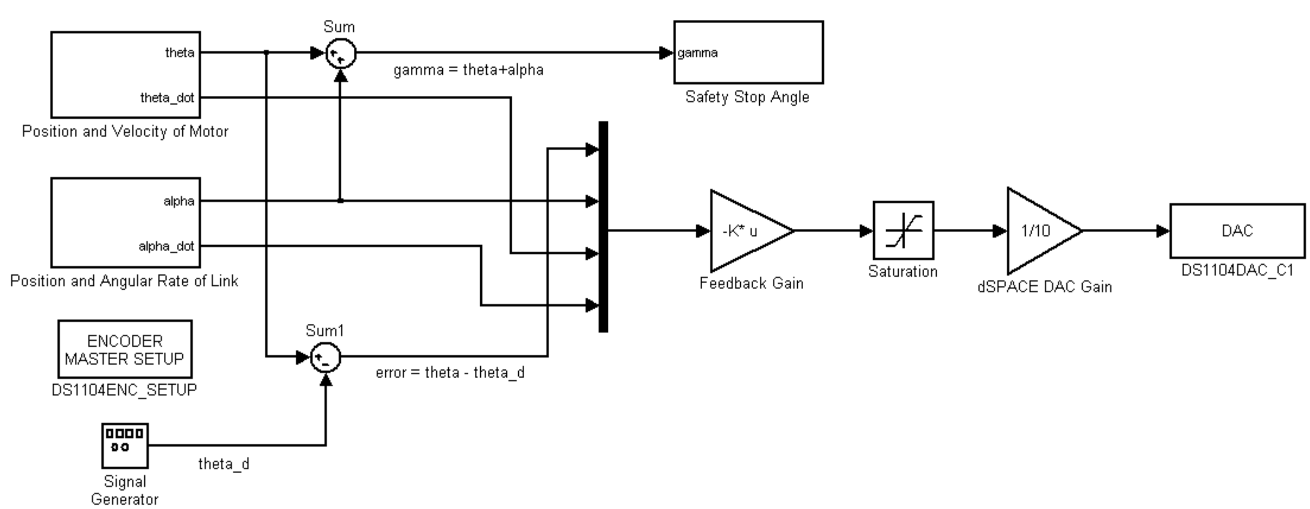

Figure 3: Simulink model for implementation of state feedback controller in dSPACE.

- The Signal Generator amplitude is $45 * pi/180$ and frequency is 0.05 Hz.

(a) Configure the safety stop condition of the Simulink Diagram for a save operation between +45 and −45 degrees (remember all the angles should be in radians). *Hint*: use the blocks "relational operator", "stop simulation", and "constant".

(b) Configure the Position and Velocity of Motor box such that you can measure the angular position (θ) using channel 1 encoder and the angular velocity ($\dot{\theta}$) via channel 3 ADC (Analog input 2 in the dSPACE board).

(c) Configure the Position and Angular Rate of Joint box such that you can measure the angular position of the joint (α) using channel 2 encoder and compute the angular rate of the joint using a derivative filter of the form $\frac{150s}{s+150}$. Remember that the encoder for angle α of the joint needs a $\times - 1$ gain as well.

(d) Develop a graphical interface in dSPACE control desk where you can observe the simulation time, control the simulation states (i.e., STOP, PAUSE, and RUN), change the feedback gain matrix K, and show plots of the reference input, angular position of the motor shaft, angular velocity of the motor shaft, angular position of the joint, angular rate of the joint angle, error, and control signal.

(e) Start with the feedback gain matrix K found in the pre-laboratory. Can you obtain a good performance? If not, try to re-tune your feedback gain matrix K.

(f) Perform several iterations until you reach a steady-state error of less than 0.5% and a settling time less than 1.1 seconds.

11.5 Post-Laboratory Exercises

- Plot the acquired data in MATLAB plots as you did for the pre-laboratory. Indicate the settling time and steady-state error on the plot whenever necessary. Also, include your feedback gain matrix K in at least one plot.
- Compare your simulation and implementation results. Do they perfectly match? Comment about the differences.
- Explain why the simulation and implementation results differ or perfectly match.

11.6 Pre-Lab Report Summary Sheet

After having completed the Laboratory Preparation problems, remove this sheet and use it to summarize and organize your results. Attach this as a cover sheet (required) when you turn in the Laboratory Preparation assignment. You may wish to keep a copy of this sheet for use during the Laboratory Procedure.

- **Final iteration tuned feedback gain matrix**:

$K = [\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad]$

- **Transient Characteristics**:

Setting time (t_s) =

Steady-state error (e_{ss})=

NAME:

Suggested MATLAB Commands

1. **Controllability**

```
% Controllability Matrix and its Rank
CO = ctrb(A,B);
rank_CO = rank(CO);
```

2. **Feedback Gain Matrix** K

```
% Controller Gain Matrix K
p = [first_pole second_pole];
K = acker(A,B,p); % A_cl = A - B*K;
```

References

[1] Yurkovich, S. and Abiakel, E. "Control Systems Technology Lab". Pearson Publishing. 2004.