Laboratory 10

State Feedback Controller for Position Control of a DC Servo

10.1 Objective

The objective of this laboratory is to position the gears of a DC servo as quickly as possible with no overshoot using a state-space feedback controller. For this, we will use the state space model of the DC servo introduced in the laboratory 3 (refer to [1]) and then tune the feedback gain matrix K to find the best position tracking while minimizing steady state error.

10.2 Model

From Section 3.2 of the course textbook [1], the block diagram for the position Servo SRV-02 is shown in Figure 1. For this laboratory, the servo is used in the high gear ratio configuration (refer to Figure 3.5 in the textbook [1]).

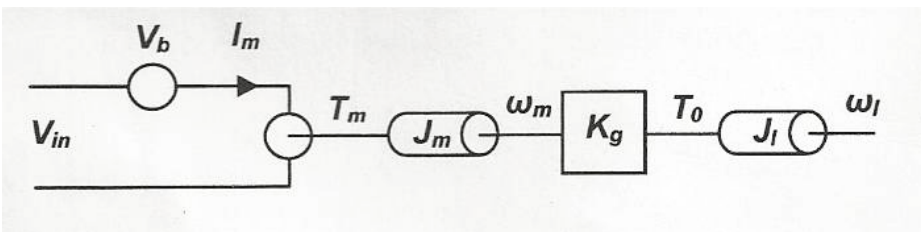

Figure 1: Block Diagram of Position Servo.

The following variables are used to describe the differential equations of the DC servomotor:

- θ: angle of the output shaft.
- ω_m: angular velocity of the motor shaft
- ω_l: velocity of the output shaft
- K_g: gear ratio
- T_m: motor torque
- T_0: output torque after the gearbox
- J_m: motor inertia
- J_l: load inertia

The parameter values needed for modeling this servomotor are: $R_m = 2.6\,\Omega$, $K_m = 0.00772\,\text{V/rad-}s^{-1}$, $K_g = 14 : 1$, $J_m = 3.87 \times 10^{-7}$ kg-m^2, and $J_l = 3 \times 10^{-5}$ kg-m^2.

The governing electrical equation is given by

$$V_{in} = I_m R_m + K_m \omega_m \tag{1}$$

The governing mechanical equations are

$$\omega_m = K_g \omega_l$$

$$T_o = K_g T_m$$

$$T_o = K_g T_m = K_g \left(J_m \dot{\omega}_m + \frac{J_l}{K_g} \dot{\omega}_l \right) = J_m K_g^2 \dot{\omega}_l + J_l \dot{\omega}_l = \left(J_m K_g^2 + J_l \right) \dot{\omega}_l$$

Let us define $J_{eq} = \left(J_m K_g^2 + J_l \right)$ be the equivalent inertia seen by the motor. This implies that

$$T_o = J_{eq} \dot{\omega}_l \tag{2}$$

From Equation (2), the torque-current relationship is defined by

$$T_m = K_m I_m \quad \Rightarrow \quad I_m = \frac{T_m}{K_m} = \frac{T_o}{K_m K_g} = \frac{J_{eq} \dot{\omega}_l}{K_m K_g}$$

Then, Equation (1) becomes

$$\frac{R_m J_e q}{K_m K_g} \dot{\omega}_l + K_m K_g \omega_l = V_{in} \tag{3}$$

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$ be the states of the system, $u = V_{in}$ the input and $y = \theta$ the output. The state-space model for the DC servo is the following

$$\begin{cases} \dot{x} & = Ax + Bu \\ y & = Cx \end{cases}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \ A = \begin{bmatrix} 0 & 1 \\ 0 & -41.5769 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 384.6154 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

10.3 Laboratory Preparation

1. Write a MATLAB script to design a pole placement based full state feedback controller for the model of the DC servomotor. Use the (A, B, C) state space matrices given in this laboratory. Check for controllability of the system. Choose appropriate locations for the closed loop poles such that the settling time (t_s) is less than 0.6 seconds and with no overshoot based in the simulation described in question 2.

2. Construct the following Simulink model in order to test the pole placement based full state feedback controller designed in question 1. You will plug-in the following information:

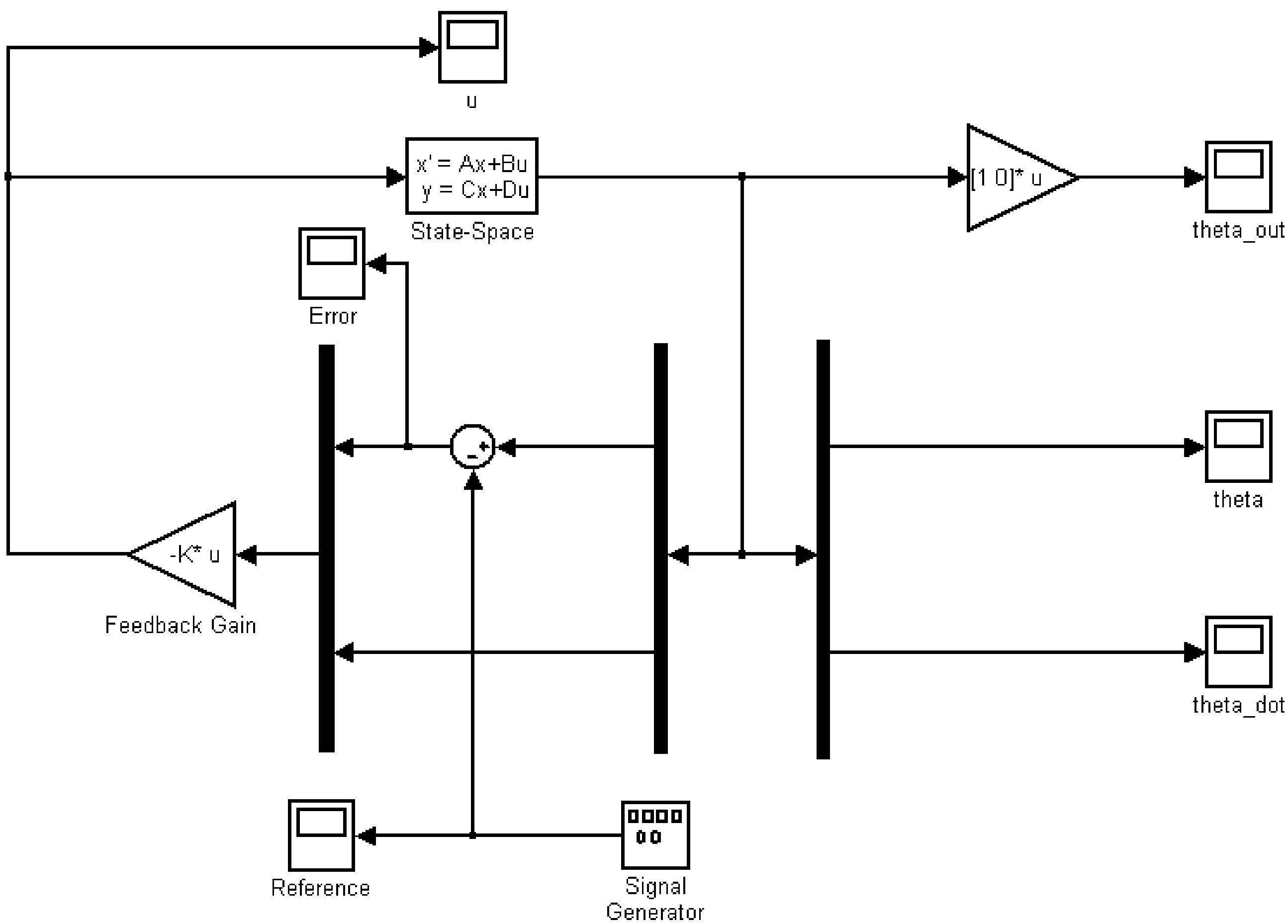

Figure 2: Simulink model for simulation of state feedback controller.

- Set the Signal Generator amplitude to $45 * pi/180$ and frequency to 0.05 Hz.
- Set the simulation time to 60 seconds.
- Enter the corresponding A, B, C, D matrices in the State-Space box. Remember that we are interested in all the states at the output so the matrix C is different from the one given by the model.

- Tune the feedback gain matrix K until you obtain the desired design performance. Remember to take into account the input voltage constraint of the motor, that is $\pm$5V.
- Save all the scopes' data as an array into the workspace. Do not limit the size of the output data.

3. Generate the following plots:

   - Comparison between the plant output (θ) and reference input (θ_r). Indicate in the same plot the settling time of the system (2% criterion).
   - Control signal applied to the plant. Make sure to that it is in the range $\pm$5V.
   - Error between the plant output and reference input (i.e., error = θ - θ_r).
   - Subplot including the states variables θ and $\dot{\theta}$.

10.4 Laboratory Procedure

10.4.1 Connections

Connect the motor in the position control configuration.

10.4.2 Simulink Diagram and ControlDesk

- Open MATLAB and Simulink and create a new model called Lab10_Group_#.mdl, where # corresponds to the lab group number. Set the simulation parameters to a fixed step size of 1 ms and a simulation time of 60 seconds. Turn off block reduction and set the simulation initial state to STOP.

- The model should look as follows:

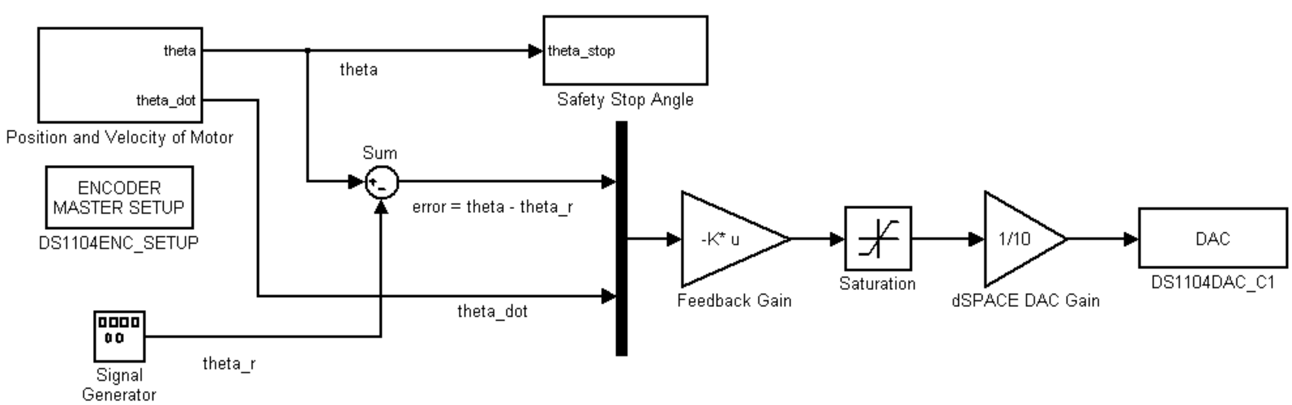

Figure 3: Simulink model for implementation of state feedback controller in dSPACE.

- The Signal Generator amplitude is $45 * pi/180$ and frequency is 0.05 Hz.

(a) Configure the safety stop condition of the Simulink Diagram for a save operation between +45 and −45 degrees (remember all the angles should be in radians). *Hint*: use the blocks "relational operator", "stop simulation", and "constant".

(b) Configure the Position and Velocity of Motor box such that you can measure the angular position (θ) using channel 1 encoder and the angular velocity ($\dot{\theta}$) via channel 5 ADC (Analog input 4 in the dSPACE board).

(c) Develop a graphical interface in dSPACE control desk where you can observe the simulation time, control the simulation states (i.e., STOP, PAUSE, and RUN), change the feedback gain matrix K, and show plots of the reference input, angular position, angular velocity, error, and control signal.

(d) Start with the feedback gain matrix K found in the pre-laboratory. Can you obtain a good performance? If not, try to re-tune your feedback gain matrix K.

(e) Perform several iterations until you reach a steady-state error of less than 1.5% and a settling time less than 0.8 seconds.

10.5 Post-Laboratory Exercises

- Plot the acquired data in MATLAB plots as you did for the pre-laboratory. Indicate the settling time and steady-state error on the plot whenever necessary. Also, include your feedback gain matrix K in at least one plot.
- Compare your simulation and implementation results. Do they perfectly match? Comment about the differences.
- Explain why the simulation and implementation results differ or perfectly match.

10.6 Pre-Lab Report Summary Sheet

After having completed the laboratory Preparation problems, remove this sheet and use it to summarize and organize your results. Attach this as a cover sheet (required) when you turn in the Laboratory Preparation assignment. You may wish to keep a copy of this sheet for use during the Laboratory Procedure.

- **Final iteration tuned feedback gain matrix**:

$K = [\qquad\qquad\qquad\qquad\qquad]$

- **Transient Characteristics**:

Setting time (t_s) =

Steady-state error =

NAME:

Suggested MATLAB Commands

1. **Controllability**

```
% Controllability Matrix and its Rank
CO = ctrb(A,B);
rank_CO = rank(CO);
```

2. **Feedback Gain Matrix** K

```
% Controller Gain Matrix K
p = [first_pole second_pole];
K = acker(A,B,p); % A_cl = A - B*K;
```

References

[1] Yurkovich, S. and Abiakel, E. “Control Systems Technology Lab”. Pearson Publishing. 2004.