State Feedback for a DC Servo

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Abstract
Using full state feedback you will design a state-space feedback controller to position the gears of a DC servo as quickly as possible with no overshoot. “Full state feedback” is achieved by using a tachometer and encoder to measure the angular velocity and position of the DC servo output shaft.

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1 State Feedback Control for a DC Servo

1.1 Model Development and Analysis

You will develop a state space model for DC servo that will be used in the full state feedback controller design. For this you should refer to the model development section on DC servo that is in the design challenges document that is on the web. Suppose that the state space model

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

has an \(n \times 1\) state \(x\) and a \(1 \times 1\) input \(u\). Recall that for the DC servo

\[
\begin{align*}
x &= \begin{bmatrix} \theta, \dot{\theta} \end{bmatrix}^\top \\
y &= \theta
\end{align*}
\]

1. Find the \(A\), \(B\), and \(C\) matrices for a state space description of the plant.

2. Is the plant “controllable”? Use Matlab to provide your answer.

1.2 Full State Feedback Controller Design

Since we have enough sensors to measure the entire state, we use a controller

\[
u = -Kx
\]

that seeks to drive the states to zero. Controller design entails finding the \(n \times 1\) gain vector \(K\). You could use pole placement via Ackerman’s formula. Next, you will test your design via simulation:

1. Develop a simulation for the closed-loop system so that you can test your various designs. For your simulation you may want to use the Matlab command “initial”.

2. Develop a simulation in Simulink for the closed-loop system in which the output shaft position tracks a square waveform between \(\pm 30\) degree with a frequency of 0.05 Hz. Find a value of the feedback gain \(K\) that results in as fast of a response as possible with no overshoot when the initial condition has zero for all components. Also, check that the control input to the plant does not exceed \(\pm 5V\) (simply check this via simulation) for your choice of \(K\). Why?

3. Show the closed-loop response for the feedback gain \(K\) you choose.