The Nested \( h \) Refinement in 2D FETD Via the Hanging Variable Technique
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**Formulation: Hanging Edges**

Buffer triangles

\[ (1.2, 1.4), (1.2, 1.5), (1.2, 1.3) \]

Hanging edges:

- \( \text{Edge}(06), \text{Edge}(62), \text{Edge}(27) \)
- \( \text{Node}(6), \text{Node}(7), \text{Node}(8) \)

- The basis functions of the buffer elements must provide continuity condition across the interface to all refined and non-refined adjacent elements.
- The FE space of each buffer triangle must contain the FE space of the parent triangle.
- For edge element, the FE space of each buffer element must include the gradient of all nodal basis functions associated with mid nodes.

**Irregular refinement**

Buffer triangles

- \( \text{Edge}(06), \text{Edge}(62), \text{Edge}(27) \)
- \( \text{Node}(6), \text{Node}(7), \text{Node}(8) \)

**Regular refinement**

Refine element stay the same

**Intermediate Level Fine Level**

\[ \delta = a + b \]

**Condition across the interface to all refined and non-refined adjacent element.**

**3-Level Refinement: Allows grids of 3 different levels of grid spacing to be joined together.**

**Adaptive Grid Refinement Algorithm**

Procedure: Our approach is to take advantage of research for refinement in the frequency domain:

1. A short time run is done to generate a solution for refinement. Simulation time is chosen such that energy reaches every point in the grid.
2. A DFT is employed at specified frequencies to generate frequency domain solutions.
3. After DFT, an a posteriori estimator is calculated in each element for all specified frequencies.
4. Nested mesh refinement is done in every element in which the error criteria is exceeded.
5. Another short time run is done and the refinement procedure is repeated.
6. This process continues until the error criteria is met for all elements or the maximum refinement level is reached.

**A Posteriori Error Estimation**

Both the numerical and exact solution (sourceless) satisfies the following equation:

\[ \sum \int \rho \left( \nabla \times \left( \frac{1}{\mu} \nabla \phi - \frac{1}{\varepsilon} \rho \phi \right) \right) \cdot \hat{n} dS = 0 \]

Subtracting one from the other:

\[ \sum \int \rho \left( \nabla \times \left( \frac{1}{\mu} \nabla \phi - \frac{1}{\varepsilon} \rho \phi \right) \right) \cdot \hat{n} dS = 0 \]

\[ \rho_x \cdot \hat{n} dS \]

But we know that

\[ \nabla \cdot \left( \frac{1}{\mu} \nabla \phi - \frac{1}{\varepsilon} \rho \phi \right) = 0 \]

\[ \rho_x \cdot \hat{n} = 0 \]

**TEz Scattering from the NACA Air Foil**

**Error in RCS Relative to Reference**

**Initial Mesh with Error**

**First Refinement with Error**

**Second Refinement with Error**

**Third Refinement with Error**

**Refined Mesh with Error**

**Fourth Refinement with Error**

**Final Mesh with Error**

<table>
<thead>
<tr>
<th>Refinement</th>
<th>Number of Elements</th>
<th>Number of Meshes</th>
<th>Limit 1 Tolerance Element</th>
<th>Limit 2 Intermediate Elements</th>
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