A Comparison of Anisotropic PML to Berenger’s PML and Its Application to the Finite-Element Method for EM Scattering

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Abstract—The use of an anisotropic material for the boundary truncation of the finite-element method is considered. The anisotropic material properties can be chosen such that a plane-wave incident from free space into the anisotropic halfspace has no reflection. Because there is no reflection, the material is referred to as a perfectly matched layer (PML). The relationship between the anisotropic PML and the original PML proposed by Berenger [2] is considered. The anisotropic PML is applied to the finite-element solution of electromagnetic (EM) scattering from three-dimensional (3-D) objects. Numerical results are presented to demonstrate the accuracy of the PML.

Index Terms—Finite-element methods, electromagnetic scattering.

I. INTRODUCTION

The concept of using a lossy material to absorb an outgoing wave to simulate an infinite region of free space for finite methods is not a new one [1]. However, this method of truncation has not gained widespread use because of the reflections which occur at the free space/material interface. One idea to minimize the reflections at this interface is to choose a low loss for the material. Unfortunately, the lossy region must be sufficiently large to attenuate the wave which can significantly reduce the computational efficiency.

Berenger [2] has introduced a modification to Maxwell’s equations to allow for the specification of material properties which result in a reflectionless lossy material. The material is reflectionless in the sense that a plane wave propagating through an infinite free space/lossy material interface has no reflection for any angles of incidence. Berenger refers to this material as a perfectly matched layer (PML). Although Berenger demonstrates the validity of his approach with numerical experiments, the physical meaning of his modifications to Maxwell’s equations is not very clear. Chew and Weedon [3] provide a systematic analysis of the PML in terms of the concept of “coordinate stretching.” They demonstrate that Berenger’s modifications to Maxwell’s equations can be derived from a more generalized form of Maxwell’s equations. There has also been additional work done on the PML, including that of Mittra and Pekel [6] and Rappaport [7].

Recently, it has also been discovered by Sacks et al. [4] that the reflectionless properties of a material can be achieved if the material is assumed to be anisotropic. Unlike Berenger’s approach, this one does not require a modification of Maxwell’s equations, making it easier to analyze in the general framework of electromagnetics. One of the important issues to resolve is the differences between the PML proposed by Berenger and the anisotropic PML proposed by Sacks et al. In this paper, we review the derivation of the two PML methods to demonstrate that they produce different field results even though the associated differential equation and tangential field boundary conditions are the same.

The anisotropic PML has been implemented into an edge-based finite-element code. Special care is taken to treat the edges and corners of the computation domain to minimize the reflections. Results are generated for several canonical geometries to validate the theory of the anisotropic PML.

II. REVIEW OF THE TWO PML METHODS AND THEIR RELATIONSHIP

A. Review of Berenger’s PML Based on Coordinate Stretching

The derivation of Berenger’s PML is taken from Chew and Weedon [3] with a change of time convention. Only the equations which are relevant to our analysis are presented here. One should consult [3] for a more complete derivation.

For a general source-free homogenous medium we define the modified Maxwell’s equations, assuming an \( e^{j\omega t} \) time variation, to be

\[
\begin{align*}
\nabla_h \cdot \mathbf{E} &= 0 \\
\nabla_e \cdot \mathbf{H} &= 0 \\
\n\nabla_e \times \mathbf{E} &= -j\omega \mu \mathbf{H} \\
\n\nabla_h \times \mathbf{H} &= j\omega \varepsilon \mathbf{E}
\end{align*}
\]

(1)
In the above equation, $c_i, h_i, i = x, y, z$ are coordinate stretching variables which are, in general, complex. The permittivity $\varepsilon$ and permeability $\mu$ are assumed to be real and invariant to position.

For the medium defined by the modified Maxwell’s equations to satisfy the proper matching conditions, we must choose $\nabla_{\varepsilon} = \nabla_{h}$. Thus, the modified Maxwell’s equations can be written as

\[
\begin{align*}
\nabla_{\varepsilon} \cdot \varepsilon \vec{E} &= 0 \\
\nabla_{\varepsilon} \cdot \mu \vec{H} &= 0 \\
\nabla_{\varepsilon} \times \vec{E} &= -j\omega \mu \vec{H} \\
\nabla_{\varepsilon} \times \vec{H} &= j\omega \varepsilon \vec{E}.
\end{align*}
\]  

(3)

If the two curl equations in (3) are combined, we obtain the following:

\[
\nabla_{\varepsilon} \times \nabla_{\varepsilon} \times \vec{E} - k^2 \vec{E} = 0
\]

(4)

where $k^2 = \omega^2 \mu \varepsilon$. However, there is one more condition that needs to be specified for perfect matching. Let us consider a space consisting of two homogeneous half-spaces, as shown in Fig. 1, with the coordinate system defined in the figure. The fields in Region 1 satisfy Maxwell’s equations with material properties $(\mu, \varepsilon)$. The fields in Region 2 satisfy (3). If $c_z = c_y = 1$ with the only constraints on $c_z$ being that the $\text{Re}(c_z) = 1$ and the $\text{Im}(c_z) < 0$, then a plane wave passing from Region 1 to Region 2 has no reflection and decays exponentially as it propagates through Region 2.

It should be noted that the $\text{Re}(c_z)$ is set to one in Berenger’s original time-domain equations. However, an arbitrary choice also produces the perfectly matched condition. Chew and Weedon [3] point this out in their paper, but do not carry this through to the final time domain equations, so that they can obtain Berenger’s original equations. The choice of the $\text{Re}(c_z)$ is important for the proper absorption of evanescent waves [5] and therefore should not be fixed.

B. Review of the Anisotropic PML

The derivation here is based on [4], but there are some slight changes in notation to help in the comparison with Berenger’s PML. Referring to Fig. 1, the time-harmonic form of Maxwell’s equations in Region 2 can be written as

\[
\begin{align*}
\nabla \cdot [\varepsilon] \vec{E} &= 0 \\
\nabla \cdot [\mu] \vec{H} &= 0 \\
\nabla \times \vec{E} &= -j\omega [\mu] \vec{H} \\
\n\nabla \times \vec{H} &= j\omega [\varepsilon] \vec{E}.
\end{align*}
\]  

(5)

where $[\mu]$ and $[\varepsilon]$ are the effective permeability and permittivity of Region 2, respectively. In this paper, we concentrate on materials with $[\mu]$ and $[\varepsilon]$ diagonal in the same coordinate system.

\[
\begin{align*}
[\mu] &= \begin{pmatrix} 
\mu_x + \frac{\sigma_x}{j\omega} & 0 & 0 \\
0 & \mu_y + \frac{\sigma_y}{j\omega} & 0 \\
0 & 0 & \mu_z + \frac{\sigma_z}{j\omega}
\end{pmatrix} \\
[\varepsilon] &= \begin{pmatrix}
\varepsilon_x + \frac{\sigma_x}{j\omega} & 0 & 0 \\
0 & \varepsilon_y + \frac{\sigma_y}{j\omega} & 0 \\
0 & 0 & \varepsilon_z + \frac{\sigma_z}{j\omega}
\end{pmatrix}.
\end{align*}
\]  

(6)

Furthermore, we select $[\varepsilon]$ and $[\mu]$ such that

\[
\frac{[\varepsilon]}{[\mu]} = \frac{[\mu]}{[\varepsilon]}
\]

to match Region 2 to Region 1. Thus, we can write

\[
\begin{align*}
[\mu] &= \mu[\Lambda] = \begin{pmatrix} 
a & 0 & 0 \\
0 & b & 0 \\
0 & 0 & c
\end{pmatrix} \\
[\varepsilon] &= \varepsilon[\Lambda] = \begin{pmatrix} 
a & 0 & 0 \\
0 & b & 0 \\
0 & 0 & c
\end{pmatrix}
\end{align*}
\]  

(7)

where $a, b, c$ are in general complex. Consequently, (5) reduces to

\[
\begin{align*}
\nabla \cdot [\Lambda] \vec{E} &= 0 \\
\nabla \cdot [\mu] \vec{H} &= 0 \\
\n\nabla \times \vec{E} &= -j\omega [\Lambda] \vec{H} \\
\n\n\nabla \times \vec{H} &= j\omega [\varepsilon] \vec{E}.
\end{align*}
\]  

(8)

The two curl equations in (8) can be combined to form

\[
[\Lambda]^{-1} \nabla \times [\Lambda]^{-1} \nabla \times \vec{E} - k^2 \vec{E} = 0.
\]  

(9)
As in Berenger’s PML, we must specify a further condition to the PML material to make it reflectionless to a plane wave. For the coordinate system defined in Fig. 1, we must specify $a = b = 1/c$. For a propagating wave to decay exponentially, we must specify that $\text{Im}(a) < 0$. Also, the $\text{Re}(a)$ can be specified for the desired absorption of evanescent waves.

C. A Comparison of the Two PML Methods

The first thing to note is that one cannot match the equations in (3) to (8), term by term, no matter what one chooses for the variables $\alpha$, $\beta$, $\gamma$, or $\delta$. However, the important question is whether the combined equations in (3) are equivalent to the combined equations in (8). Let us consider the associated differential equations in (4) and (9). If one expands out the double curl terms, it is immediately clear that the two equations are not equivalent. However, if we incorporate the divergence conditions in (3) and (8) into the corresponding double curl terms, we get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} - \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E})$$

$$\nabla_e \times \nabla_e \times E - \nabla_e \cdot (\nabla_e \cdot E) = -\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{c^2} \frac{\partial^2}{\partial z^2} \right) E$$

(10)

$$[\Lambda]^{-1} \nabla \times [\Lambda]^{-1} \nabla \times E - \frac{1}{a} \nabla \cdot ([\Lambda] \nabla E) = -\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{a^2} \frac{\partial^2}{\partial z^2} \right) E.$$  (11)

From (10) and (11), we see that the two PML formulations lead to the same modified vector Helmholtz equation. In addition, at the material interface in Fig. 1, we know that both formulations satisfy the boundary conditions on the continuity of the tangential electric and magnetic fields. It is, therefore, natural to conclude that both formulations produce the same solution. However, this conclusion is incorrect since Berenger’s PML does not produce Maxwellian fields. Because the divergence condition is Maxwellian for the anisotropic PML and non-Maxwellian for Berenger’s PML, the continuity conditions on the normal fields are different. Thus, the two PML formulations produce the same solutions for the fields tangential to the interface but different solutions for the fields normal to the interface.

One advantage of the anisotropic PML is that it provides a better physical understanding of the PML because it can be explained within the framework of Maxwell’s equations. It is immediately obvious from a study of the material properties that the perfectly matched condition is achieved by allowing one component of the permittivity and permeability tensors to amplify the field while the other two components cause decay in the fields. Another advantage of the anisotropic PML is that the finite-element method (FEM) is well developed for Maxwell’s equations. It is unnecessary to derive or validate the variational expression for the PML region.

III. MODELING OF EDGE AND CORNER REGIONS OF THE PML

The theory for the anisotropic PML is based on the assumption that a plane wave is propagating through a planar media interface of infinite extent. However, the main interest for the PML in our case is to use it to absorb the scattered field from an object in free space. Thus, the PML material must totally surround the scatterer, and the outer surface of the PML must also be terminated in some way. For this work, we terminate the PML with a perfect electric conductor (PEC). The PML material is placed in the shape of a box (Fig. 2) to best approximate the reflectionless properties of the PML.

The choice for the material properties of the side regions of the box ($\Lambda_{side}$, $\mathbf{e} = x, y, z$) is straightforward and is shown in the figure, where $s_x, s_y, s_z$ are arbitrary complex numbers. However, the method for determining material properties at the edge and corner regions of the box is not so clear. From electromagnetic theory, one expects that any choice would cause some diffracted field from the edges and corners. One approximate approach for the edge region is to choose the edge properties such that they are perfectly matched to the adjacent side regions when the edge/side interface is of infinite extent. We can use a similar approach for the corner region by matching the corner properties to the adjacent edges. If we go through the corresponding analysis of a plane wave propagating through an infinite media interface for the edges and corners, we arrive at the interesting relationship that $\Lambda_{edge}$ is equal to the matrix multiplication of $\Lambda_{side}$ of the two adjacent sides and $\Lambda_{corner}$ is equal to the matrix multiplication of $\Lambda_{side}$ of the three adjacent sides. As an example, let us consider the edge in Fig. 2 with material properties $\Lambda_{side}^e$.

Fig. 2. Geometry of the PML region surrounding the scatterer, regular view, and cross-sectional view.
The matching condition is

$$\Lambda_{\text{match}} = \begin{pmatrix} \varsigma_x/\varsigma_z & 0 & 0 \\ 0 & \varsigma_z/\varsigma_y & 0 \\ 0 & 0 & \varsigma_y/\varsigma_z \end{pmatrix}.$$  \hspace{1cm} (12)

IV. NUMERICAL RESULTS

To test the capabilities of the anisotropic PML, we consider its use in the edge-based FEM with the edge elements (Whitney 1-forms) as the basis functions [8]. The implementation of the anisotropic media into the finite element formulation is simple and straightforward [9] and, therefore, is not discussed here. A biconjugate gradient solver (BiCG) with diagonal preconditioning is used to solve the resulting matrix equations.

The initial validation of the anisotropic PML is done for a TEM wave propagating in a parallel plate waveguide. The PML is then applied to solve the problems of plane wave scattering from two PEC rectangular plates, a PEC sphere, and a PEC triangle-semicircle plate. The total field formulation is used for the waveguide problem, whereas the scattered field formulation is used for all the free-space scattering problems.

As noted in Section III, for a free-space scattering problem, the scatterer is enclosed by the PML material, which is in turn covered by PEC, as shown in Fig. 2. The material properties for all the sides, edges, and corners of the PML can be expressed in terms of $s_x, s_y, s_z$. For simplicity, we use $s_x = s_y = s_z = s' - j s''$ for all computations, where $s'$ and $s''$ are some real numbers to be used for identifying different results.

The FEM far-field radar cross section (RCS) patterns are compared to either the moment method (MoM) solutions for the plate problems or the series solutions for the sphere problem. It is important to note that the FEM patterns are sensitive to the selection of the surface on which the integration is performed. It is found in many cases that using a surface some distance away from the PEC boundary gives dramatically better results than on the PEC boundary itself. In this paper, all the FEM patterns are calculated from surfaces half a cell away from the PEC boundaries.

A. TEM Waveguide

Propagation within a simple parallel plate waveguide is studied. At one end, the boundary condition $E^0 = 1 \Omega \hat{x}$ is applied. At the opposite end, a metal backed PML layer is used. The geometry of this example is shown in Fig. 3, in which $\lambda$ stands for the free-space wavelength. Note that the material property of the PML can be characterized by $\Lambda_{\text{side}}$.

Although a two-dimensional (2-D) problem in nature, this problem is solved in three-dimensional (3-D) in order to understand the convergence behavior of the BiCG solver for 3-D problems involving PML. The computational domain in the $y$ direction is truncated to a thickness of $0.2 \lambda$, resulting in a domain shaped like a rectangular box. The perfect magnetic conducting (PMC) boundary condition is then applied to the two new boundaries. The resulting problem has the same field inside the computational domain as the original problem.

The FEM solution of this problem is compared to the analytic solution for different values of $\varsigma$ and $\varsigma'$, and good agreement is observed, which demonstrates that the edge-based FEM is modeling the anisotropic media properly. In the solution process, however, a drawback of using the anisotropic absorber is found. Specifically, the matrix becomes ill-conditioned as the parameter $\varsigma'$ is increased. This problem leads to slow convergence in the iterative matrix solver. Further investigation reveals that the condition number is sensitive to the value of $\varsigma'$ as well as $\varsigma''$. In Fig. 4, the number of iterations required for convergence to a relative residual of $10^{-6}$ is plotted for different values of $\varsigma'$ and $\varsigma''$. Theoretically, $\varsigma'$ has no effect on the absorbing capability of the PML material for plane waves; it only affects the wavelength in the PML [4]. Therefore, to reduce the discretization errors, small values of $\varsigma'$ are preferred. However, from Fig. 4, we observe that the convergence of the iterative solver is slower when $\varsigma'$ is smaller than $\varsigma''$. As a result, $\varsigma' = \varsigma''$ provides a good compromise and is used for all the following cases.

Note also from Fig. 4 that for $\varsigma' = \varsigma''$, the numbers of iterations needed are approximately in direct proportion to the values of $\varsigma'$. This result has also been observed for most of the cases we tried. So the larger the values of $\varsigma'$ and $\varsigma''$, the higher the computational cost, when the BiCG matrix solver is used. This is an important consideration that keeps one from using very large values of $\varsigma'$ and $\varsigma''$.

B. Scattering by Rectangular PEC Plates

The scattering from two rectangular PEC plates is considered in this section. For each case, the plate is placed in the $x$-$y$ plane, and the plane wave is incident in the $-\hat{z}$ direction of the spherical coordinate, as shown in Fig. 5. A sideview of the plate and PML configuration is shown in Fig. 6. Note that the PML is put $d_1$ distance away from the plate, and its thickness is $d_2$. This is by no means the only possible configuration, but is the one used in this example for ease of comparison.

It should be mentioned that uniform meshes are used for these two plate examples to control the quality of the mesh. The mesh is uniform in the sense that the computational domain is first divided into small cubes of the same size (whose edge length will be denoted as the mesh size); each cube is then subdivided into five tetrahedra, as shown in Fig. 7. As a result, the edges of all the tetrahedra have very similar lengths.

The plate for the first example is of size $1\lambda \times 0.5\lambda$. Various values of $d_1, d_2, \varsigma'$, and $\varsigma''$ have been tried to find their effects
on the FEM solutions. In particular, four different meshes (denoted as M1, M2, M3, and M4) are selected for illustration. The parameters for these meshes are listed in Table I. Note that all four meshes have the same value of $d_2$ but decreasing values of $d_1$. The mesh size is chosen to be $0.05\lambda$, which should be able to keep the discretization error reasonably small.

First, we show the effects of the material properties $(s', s'')$ on the bistatic RCS patterns. The patterns are taken in the $x$-$z$ plane and for the $\phi$ polarization; the incident plane wave is $\hat{y}$ polarized with $\theta^{\text{inc}} = 30^\circ, \phi^{\text{inc}} = 0^\circ$. Fig. 8 shows the patterns for mesh M2, with increasing values of $s'$ and $s''$. The values of 1.0, 1.5, 2.0, and 3.0 are used for $s'$ and $s''$. The pattern calculated by the moment method (MoM) is also shown for
Fig. 8. Bistatic patterns for the $1 \lambda \times 0.5\lambda$ rectangular plate by using mesh M2 with different material properties $(s', s'')$. Patterns are in the $x$-$z$ plane for the $\phi$ polarization. The incident plane wave has $y$-polarized electric field with $\theta^{\text{inc}} = 30^\circ, \phi^{\text{inc}} = 0^\circ$.

TABLE I

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<tr>
<th>Parameters for the Four Meshes Used in the $1 \lambda \times 0.5\lambda$ Rectangular Plate Example</th>
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comparison. It is seen that the patterns keep changing as the values of $s'$ and $s''$ are increased from one to three. However, the rate of change slows down dramatically for $s', s'' \geq 1.5$. This indicates that the absorption of the PML for this mesh and incident wave is sufficient for $s' = s'' = 1.5$ (this value may change for other mesh or incident wave); any larger values of $s'$ and $s''$ do not improve the patterns very much. In fact, larger values of $s'$ and $s''$ result in larger discretization errors due to the fact that the fields inside the PML region change more and more drastically and the mesh becomes less and less able to model them correctly. It is observed that for a fine mesh such as M2, there exists a range of values for $s'$ and $s''$ in which the patterns do not change very much. So any value in this range can be used for $s'$ and $s''$. For a coarse mesh, however, the patterns will be much more susceptible to the changes of $s'$ and $s''$. Therefore, the choice of $s'$ and $s''$ must be made in conjunction with the mesh size.

Next we test the effect of $d_1$ on the patterns. Fig. 9 shows the patterns for these four meshes with the same material property $s' = s'' = 1.5$. It is seen from Fig. 9 that the patterns for M1, M2, and M3 are very similar. So for this example $d_1$ has very little effect on the patterns once $d_1 \geq 0.1\lambda$. However, for mesh M4, whose $d_1 = 0.05\lambda$, the pattern is seen to be somewhat worse than the other three. When the value of $s'$ and $s''$ is increased to 2.0, the pattern for M4 becomes even worse. This result suggests that the method becomes less effective when the PML is placed very close to the scatterer. However, we do not yet know whether this worsening in accuracy is due to the PML or due to the discretization error. To find out which one is the true cause of the problem, we use two meshes that have the same values of $d_1$ and $d_2$ as M3 and M4, but with the same smaller mesh size of 0.025. Fig. 10 shows their patterns for $s' = s'' = 2.0$. It is clearly seen that the patterns for both meshes are in good agreement with each other and with the MoM pattern. This shows that the accuracy problem is mainly due to the discretization error and not the PML. So we believe that in theory, $d_1$ can be very small without losing the effectiveness of the PML, but in practice there exists a lower bound for $d_1$ which is determined by the mesh size.

We comment that for a sufficiently fine mesh that does not introduce too much discretization error, increasing $d_2$ has the same effect as increasing $s'$. In summary, we find from this example that the parameters $d_1$, $d_2$, and $s'$ and $s''$ all depend on the mesh size. For a small mesh size, one can choose large values of $s'$ and $s''$ and small values of $d_1$ and $d_2$; however, for a large mesh size, one can only use small values of $s'$ and $s''$ and large values of $d_1$ and $d_2$. The product of $s''$ and $d_2$ determines the absorbing capability of the PML, so they have to be considered together. What is a better way to choose these parameters depends on the problem geometry and the accuracy one is looking for.
Fig. 9. Bistatic patterns for the 1 \( \times 0.5 \lambda \) rectangular plate by using different meshes (differ in \( d_1 \) only) but with the same material property \((s', s'') = (1.5, 1.5)\). Patterns are in the \( x-z \) plane for the \( \phi \) polarization. The incident plane wave has \( y \)-polarized electric field with \( \theta_{\text{inc}} = 30^\circ, \phi_{\text{inc}} = 0^\circ \).

Fig. 10. Bistatic patterns for the 1 \( \times 0.5 \lambda \) rectangular plate. The FEM patterns use \( d_2 = 0.2\lambda \), mesh size \( 0.025\lambda \), \( s' = s'' = 2.0 \), and \( d_1 = 0.1\lambda \) and 0.05\( \lambda \). Patterns are in the \( x-z \) plane for the \( \phi \) polarization. The incident plane wave has \( y \)-polarized electric field with \( \theta_{\text{inc}} = 30^\circ, \phi_{\text{inc}} = 0^\circ \).
C. Scattering by a PEC Sphere

The scattering from a PEC sphere of diameter $0.4\lambda$ is considered. The PML material is $0.2\lambda$ thick and is placed $0.3\lambda$ away from the sphere, as shown in Fig. 12. The incident plane wave is traveling in the $-z$ direction.

For this problem, a nonuniform mesh with element edge lengths no longer than $0.1\lambda$ is created. The total number of unknowns is 139,710. Fig. 13 shows the bistatic RCS patterns of this sphere calculated using $s' = s'' = 1.5$ for the PML. The series solutions are shown for comparison. It is seen that the FEM patterns are in good agreement with the series solutions. This result demonstrates that the PML is effective on absorbing spherical waves.

D. Scattering by a Triangle-Semicircle PEC Plate

For the last example, we consider the scattering of a plane wave by a triangle-.semicircle plate. The problem geometry is shown in Fig. 14. The plate is composed of a half circle of radius $1\lambda$ and an equilateral triangle of side-length $2\lambda$. The PML material is placed as close as $0.1\lambda$ to the plate.
in the $x$ and $y$ directions, and $0.2\lambda$ in the $z$ direction. A nonuniform mesh with element edge lengths no longer than $0.15\lambda$ is created. The total number of unknowns is 104,218. Since this mesh is coarser than previous ones, its accuracy is expected to be worse.

Figs. 15 and 16 show the bistatic RCS patterns ($x$-$z$ plane; $\phi$ polarization) for two incident waves $\theta_{\text{inc}} = \phi_{\text{inc}} = 0^\circ$ and $\theta_{\text{inc}} = 30^\circ, \phi_{\text{inc}} = 0^\circ$, respectively, both having $\phi$-polarized electric field. The material properties $\epsilon' = \epsilon'' = 1.5$ and 2.0 are used for both cases. It is seen that although the accuracies of the FEM patterns are not as good as previous cases, especially for the wide angles, the general agreements with the MoM patterns are still very good considering the large mesh size used.

V. CONCLUSION

In this paper, we study the relationship between Berenger’s PML and the anisotropic PML. We show that the two methods are different. Although both methods produce the same tangential fields, the normal fields are different because the two PML formulations satisfy different divergence conditions. FEM can be implemented in a straightforward manner for the anisotropic PML region since the conversion of an FEM code for isotropic media to one with anisotropic media requires only minor changes.

From the numerical results, we see that the anisotropic PML can be very effective for the scattering problems, provided the appropriate material properties and geometry of the PML are used. We also show that the computational efficiency can be improved by carefully considering the relations of the mesh densities, the PML material properties, and the size of the computational domain. Based on the results in this paper, we believe that the anisotropic PML shows great promise as a material absorber for FEM. However, these results also indicate that more work is necessary to develop this concept to its full potential.
Fig. 15. Bistatic patterns for the triangle-semicircle plate. Patterns are in the $x$-$z$ plane for the $\phi$ polarization. The incident plane wave has $y$-polarized electric field with $\theta_{\text{inc}}^{\phi} = \phi_{\text{inc}}^{\phi} = 0^\circ$.

Fig. 16. Bistatic patterns for the triangle-semicircle plate. Patterns are in the $x$-$z$ plane for the $\phi$ polarization. The incident plane wave has $y$-polarized electric field with $\phi_{\text{inc}}^{\phi} = 0^\circ$ and $\theta_{\text{inc}}^{\phi} = 30^\circ$. 
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REFERENCES


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