Transient Current on a Wire Penetrating a Cavity-Backed Circular Aperture in an Infinite Screen

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Abstract—The problem considered is that of a wire penetrating a circular aperture in an infinite conducting screen and entering a circular cylindrical cavity. Results for the transient current propagating along the wire both inside and outside the cavity are presented. The current in both regions is evaluated in the frequency domain by the method of moments (MOM). An approximate method for evaluating the exterior current at an observation point far from the aperture is also discussed. To obtain the transient response, a numerical inverse Fourier transform is used. The current response is examined as a function of cavity and aperture dimensions. Results obtained with the approximate method are compared with the MOM solution. It appears that information concerning the interior cavity dimensions is present in these exterior observations.

I. INTRODUCTION

THE DEVELOPMENT of means to shield sensitive electronic equipment from electromagnetic interference is facilitated by analytical models that can describe the coupling between exterior and interior regions by wire penetrations and apertures. We consider a problem in which a wire penetrates an aperture into a conducting cavity. Frequency domain results for the current on the wire inside the cavity were presented in an earlier paper [1]. In the present paper, we are concerned primarily with the transient current response. We present results for both the exterior and interior currents. An approximate and computationally efficient method for evaluating the exterior current far from the aperture is also discussed.

The specific geometry to be considered is shown in Fig. 1 with reference to a cylindrical coordinate system (ρ, Φ, z). An infinitesimally thin conducting screen of infinite extent is located at z = 0. In this screen, there is a circular aperture of radius c centered at ρ = 0. The screen divides the problem into two regions: region 1, consisting of an infinite half space, and region 2, consisting of a cylindrical cavity of radius b and length h centered about the z axis. The walls of the cavity and the screen are perfectly conducting. A wire of radius a is aligned with the z axis. Extending from z = −∞ in region 1, it passes through the center of the aperture into region 2, where it follows the axis of the cavity and is shorted to the end at z = h. Both regions 1 and 2 are free space with permeability μ₀ and permittivity ε₀.

II. CURRENT ON THE WIRE

In this section, we briefly review the procedure for obtaining the current in both regions in terms of the aperture electric field. Two methods for approximating the aperture field are discussed. In addition, we consider the exterior current (region 1) for the special case of an observer located far from the aperture. Frequency domain analysis follows.

The given geometry is φ-symmetric, and we select for our source a φ-symmetric magnetic current source $M_φ$. Therefore, Maxwell's equations decouple and only the TMₖ modes (with field components $H_φ$, $E_ρ$, and $E_z$) are excited. We obtain the following scalar equation for $H_φ$:

$$\left( \nabla^2_{\rho z} + k^2 - \frac{1}{\rho^2} \right) H_φ(\rho, z) = i \omega \varepsilon_0 M_φ$$

(1)

where $\nabla^2_{\rho z}$ is the Laplacian with respect to $\rho$ and $z$, and $k$ is the wavenumber given by $k = \omega \sqrt{\mu_0 \varepsilon_0}$. The $e^{\text{i}t}$ time dependence has been suppressed.

In order to produce a TEM wave incident on the aperture plane, we choose

$$M_φ(\rho, z) = \frac{M_0 \delta(z + d)}{\rho} \quad \rho \in (a, \infty).$$

(2)

Although this idealized source is not physically realizable (having infinite extent and infinite energy), its use simplifies the analysis of the results because we do not have to contend with higher order modes in the incident field. The idealized source has been shown to be a good approximation to a physically realizable source at lower frequencies and in regions...
where $\rho \ll d$ [2]. This physically realizable source has the same form as in (2) but has only finite support in $\rho$, that is, $\rho \in (a, r_e)$ where $r_e$ is some finite radius. We control the separation between source and aperture, and since we are primarily interested in the fields near the wire ($\rho \ll d$), we assert that the approximation is valid. The incident TEM wave that is produced induces a TEM current on the wire. The magnitude of this current is defined as $I_0$.

From (1) and (2), we may produce the following expressions for $H_\phi$ in the two regions [1]:

\[
H_\phi(\rho, z) = \frac{I_0}{\pi \rho} \cos k \rho - \frac{i k}{\eta} \int_0^c g_1(\rho, z|\rho', 0)E_\phi(\rho', 0)\rho' d\rho' \tag{3}
\]

\[
H_{\phi 2}(\rho, z) = \frac{i k}{\eta} \int_0^c g_2(\rho, z|\rho', 0)E_\phi(\rho', 0)\rho' d\rho' \tag{4}
\]

where $\eta$ is the free-space impedance given by $\eta = \sqrt{\mu_0/\epsilon_0}$, $E_\phi(\rho')$ is the electric field in the plane of the aperture, and $g_1$ and $g_2$ are the appropriate Green's functions for the two regions. Once the magnetic field is known, the surface current density on the wire is given by

\[
J_s = \hat{n} \times H_{\phi 2} = \hat{\rho} \times \phi H_\phi(a, z) = \hat{z} H_\phi(a, z) \tag{5}
\]

where $\hat{n}$ is the unit vector normal to the surface of the wire. To find the current $I(z)$ on the wire, we integrate $J_s$ around the circumference of the wire. This gives the following expressions for the current in the two regions [1], [3]:

\[
I(z) = \begin{cases} 
I_e e^{ikz} + I_e e^{-ikz} - \frac{4k}{\eta} \int_0^c \frac{B(\rho')e^{i\lambda_1z}}{\lambda_1 H_0^{(1)}(\lambda_1)H_0^{(2)}(\gamma a)} d\gamma E_\phi(\rho')\rho' d\rho', & z < 0 \\
-2\frac{2\pi}{\eta} \int_0^c \cos k(h - z) \frac{1}{\rho' \ln b/a} + \frac{\pi}{\rho' \ln b/a} \sum_{n=1}^{\infty} \cos \lambda_n(h - z) \sin \lambda_n h \left[ J_0(\gamma_n a) - J_0(\gamma_n b) \right] \frac{J_0(\gamma_n a) - J_0(\gamma_n b)}{J_0(\gamma_n b)} E_\phi(\rho')\rho' d\rho', & z > 0 
\end{cases} \tag{6}
\]

where

\[
B(\rho) = J_1(\gamma a)Y_0(\gamma b) - J_0(\gamma a)Y_1(\gamma b) \tag{7}
\]

\[
\lambda_1 = \sqrt{k^2 - \gamma^2}, \lambda_2 = \sqrt{k^2 - \gamma^2}, \text{ and } \gamma_0 \text{ are the roots of the transcendental equation}
\]

\[
J_0(\gamma_n a)Y_0(\gamma_n b) - J_0(\gamma_n a)Y_0(\gamma_n b) = 0. \tag{8}
\]

Note that the current in region 1 ($z < 0$) has three terms. The first term represents the incident current. The second term is the current resulting from a short circuit at $z = 0$. Finally, the third term is the perturbation caused by the aperture; we will refer to this later term as $I_A$.

To solve for the current on the wire, we must first evaluate the aperture field. Two approximate methods for doing so are presented in [1]. In the method-of-moments (MOM) formulation, the aperture field is approximated as

\[
E_\phi(\rho') = \sum_{m=1}^N \alpha_m P_m(\rho') \tag{9}
\]

where the $\alpha_m$'s are coefficients determined by the MOM solution. The $P_m$'s are nonuniform pulse functions defined by

\[
P_m(\rho') = \begin{cases} 
1 & Q_m < \rho' < R_m \\
0 & \text{otherwise}
\end{cases} \tag{10}
\]

where $R_m$ and $Q_m$ denote the outer and inner radial limits of the $m$th pulse. The integer $N$ specifies the number of pulse functions used to approximate the aperture field. For our weighting functions, we use delta functions located at the centers of the pulse functions. Then, from the MOM solution, we find that the aperture contribution to the exterior current (region 1) is [3]

\[
I_A(z) = -\frac{4}{\eta} \sum_{m=1}^N \alpha_m \int_0^c \frac{[A(kR_m z) - A(kQ_m z)]}{\xi(1 - \xi^2)^{1/2} H_0^{(1)}(ka \xi)H_0^{(2)}(ka \xi)} \cdot e^{i k(z - \xi^2)z} d\xi \tag{11}
\]

whereas the interior current (region 2) is given by [1]

\[
I(z) = -\frac{2\pi k}{\eta} \sum_{m=1}^N \left( \cos k(h - z) \frac{\ln b/a}{\ln b/a} + \frac{\ln b/a}{\ln b/a} \right) \right) \left[ J_0(\gamma_n a) - J_0(\gamma_n b) \right] \right) \frac{J_0(\gamma_n a) - J_0(\gamma_n b)}{J_0(\gamma_n b)} E_\phi(\rho')\rho' d\rho', \tag{12}
\]

where

\[
A(\gamma_n a) = J_0(\gamma_n a)Y_0(\gamma_n a) - J_0(\gamma_n a)Y_0(\gamma_n b). \tag{13}
\]

In the second method for approximating $E_\phi$, which is referred to as the zeroth-order approximation (ZO), the aperture field is approximated as

\[
E_\phi(\rho') = \frac{C}{\rho'} \tag{14}
\]

where $C$ is a constant to be determined. Although (14) does not correctly represent the field locally, it still yields good results at lower frequencies and allows us to evaluate $E_\phi$ more quickly than does the MOM. Using the ZO approximation, we obtain the following expression for the aperture contribution to the exterior current [3]:

\[
I_A(z) = -\frac{8\pi}{\eta Y_A \ln \frac{b/a}{a}} \int \frac{A(kb \xi)}{\xi(1 - \xi^2)^{1/2} H_0^{(1)}(ka \xi)H_0^{(2)}(ka \xi)} e^{i k(z - \xi^2)z} d\xi \tag{15}
\]
where \( Y_A \) is the admittance of the equivalent circuit model described in [1]. Note that \( Y_A \) is a function of the coefficient \( C \).

Let us now consider the exterior current at an observation point located far from the aperture. For \( kz \gg 1 \), Casey has evaluated the integral in (15) by assuming that the main contribution occurs in the neighborhood of the origin \( \xi = 0 \) [4]. The procedure is as follows. We first expand \( \sqrt{1 - \xi^2} \) about \( \xi = 0 \) as

\[
\sqrt{1 - \xi^2} = 1 - \frac{\xi^2}{2} - \cdots. \tag{16}
\]

Then, approximating the remainder of the integrand near \( \xi = 0 \), we obtain

\[
I_A(z) = \frac{16\pi I_0}{\eta Y_A} e^{ikz} \int_0^\infty e^{-i\xi^2/2} \frac{d\xi}{[\xi^2 + 4 \ln^2(k \xi)]}. \tag{17}
\]

We next apply the definitions \( \varphi = (k \xi)^2 \) and \( \xi = -z/(2ka^2) \) to produce

\[
I_A(z) = \frac{8\pi I_0}{\eta Y_A} e^{ikz} \int_0^\infty e^{-\varphi/2} \frac{d\varphi}{\varphi^{1/2 + \ln^2 \varphi}}. \tag{18}
\]

The integral in (18) is known [5]. Therefore, the current becomes

\[
I_A(z) = \frac{8\pi I_0}{\eta Y_A} e^{ikz} \left[ e^{i \Psi(1, \varphi)} - \Psi(1, \varphi) \right] \tag{19}
\]

where

\[
\Psi(\alpha, \varphi) = \int_\alpha^\infty \frac{\nu^{-1}}{\Gamma(\nu)} d\nu. \tag{20}
\]

and \( \Gamma(\cdots) \) is the gamma function. Finally, using an asymptotic expansion for \( \Psi(1, \varphi) \) as \( |\varphi| \to \infty \) [6], we obtain

\[
I_A(z) = \frac{8\pi I_0}{\eta Y_A} e^{ikz} \frac{\ln(\varphi)}{\varphi^{1/2}} \left( -\frac{z}{2ka^2} \right) \tag{21}
\]

The expression for the current is clearly much easier to compute than that given in (11). In the next section, we shall attempt to verify that at a wire position far from the aperture, (21) produces comparable results to (11).

III. NUMERICAL RESULTS FOR THE CURRENT

We shall examine the current on the wire both inside and outside the cavity. Since we are primarily interested in the transient current response, only a few representative frequency domain plots are included. We evaluate the current in the frequency domain and then use the inverse Fourier transform to obtain the time domain current. Let us begin with a description of the input pulse used to excite the system.

A. The Input Pulse

The input pulse that we will use is of double exponential form. The time domain expression for this pulse is given by

\[
f(t) = A_0(e^{-\alpha t} - e^{-\beta t})u(t) \tag{22}\]

where \( u(t) \) is the unit step function, and \( A_0 \) is a normalization constant, which ensures that the maximum of \( f(t) \) is unity. The value of \( A_0 \) is found to be

\[
A_0 = \frac{1}{e^{-\alpha t_0} - e^{-\beta t_0}} \tag{23}
\]

where \( t_0 = \ln(\beta/\alpha)/(\beta - \alpha) \). We take the Fourier transform of (22) to obtain the frequency spectrum of the pulse

\[
F(f) = A_0 \left[ \frac{1}{\alpha + i 2\pi f} - \frac{1}{\beta + i 2\pi f} \right]. \tag{24}
\]

The shape of the transient pulse and its frequency content are characterized by the values chosen for \( \alpha \) and \( \beta \). Let us consider two different cases. In the first case, we wish to model the pulse from the transient range at Lawrence Livermore National Laboratory. We set \( \alpha = 2 \times 10^9 \text{s}^{-1} \) and \( \beta = 3 \times 10^9 \text{s}^{-1} \) (Fig. 2). Note that the spectrum of this pulse drops by approximately 50 dB as we increase in frequency from dc to 7 GHz (Fig. 3). From our second case, we
choose \( \alpha = 1.5 \times 10^8 \, \text{s}^{-1} \) and \( \beta = 3 \times 10^8 \, \text{s}^{-1} \) (Fig. 2). The shape of this "trailing edge" pulse makes it relevant to EMP studies. We observe that its spectrum drops by more than 50 dB as we increase in frequency from dc to 3 GHz (Fig. 3). Since the high-frequency content of this pulse is much lower than that of the previous pulse, we refer to this pulse as the "low-frequency" pulse and the first as the "high-frequency" pulse.

It is necessary to truncate the frequency spectrum at some point in order to use the numerical inverse fast Fourier transform (IFFT) algorithm. Beyond the point of truncation, the magnitude of the frequency spectrum should be negligible to minimize error. For the high-frequency pulse, we truncate at 7 GHz, and for the low-frequency pulse, we truncate at 3 GHz. In each case, the magnitude of the spectrum at the truncation point is at least 50 dB down from its magnitude at zero frequency. In order to increase the time resolution of our data, we zero fill past the point of truncation. The number of frequency samples is 512 for both pulses. For the high-frequency pulse, we zero fill to 28 GHz so that \( \Delta f = 17.72 \times 10^{-12} \, \text{s} \); for the low-frequency pulse, we zero fill to 12 GHz so that \( \Delta f = 41.34 \times 10^{-12} \, \text{s} \).

B. Exterior Current

We shall excite our system using each of the input pulses described. Only the reflected portion of the transient current response outside the cavity will be considered, that is, the incident current (first term in (6)) will be subtracted, leaving the reflected current \( I_R \). Our aim is essentially twofold. First, we want to see how well the approximation given in (21) compares with the MOM solution in (11). Second, we wish to observe how the response is affected by changes in observer position, cavity length, and aperture size.

We begin by considering the variation of observer position. We select a cavity having dimensions \( b = 5a \) and \( h = 15a \), with an aperture size of \( c = 2a \). Throughout this section, we assume the wire radius \( a \) to be 1 cm. First, we excite the system with the high-frequency pulse. The MOM solution for the resulting transient current at \( z = -1a, -10a, -100a, \) and \( -1000a \) is shown in Fig. 4. The time scale has been adjusted so that \( t = 0 \) corresponds to the arrival of the first reflected pulse at the observation point. This first reflection is from the screen at \( z = 0 \). It takes 1 ns for the pulse to travel the length of the cavity and back. Therefore, the first reflection from the back of the cavity \( (z = h) \) arrives at \( t = 1 \) ns, the second reflection arrives at \( t = 2 \) ns, and so on. We are able to identify in Fig. 4 the first four reflections from the back of the cavity.

In addition, we observe that the magnitudes of these cavity reflections decrease as we travel away from the aperture. This is a result of radiation loss. Dispersion causes the pulse to broaden as it travels back down the wire. This result is expected since a single wire cannot support a pure propagating mode. In contrast, the magnitude of the screen reflection increases as the distance from the aperture increases. In order to understand this, we note from (21) that for \( z \to -\infty \), the aperture contribution to the current \( I_A \to 0 \). In effect, we see just the reflection from a short circuit. When the observation point is closer to the aperture, though, the aperture perturbation produces some cancellation with the short circuit current, and the magnitude of the total exterior current decreases. One final observation regarding Fig. 4 concerns the "ripple" visible at times greater than approximately 5 ns. We believe that the ripple is associated with the radial modes in the coaxial cavity. This idea will be discussed further in the next section.

We also excite the system with the low-frequency pulse. The time domain response at each of the four observation points is shown in Fig. 5. We observe essentially the same behavior as with the high-frequency pulse, except that now it is slightly more difficult to identify the multiple cavity reflections. We notice as well that the late-time ripple has disappeared. As we will soon show, for the case of the low-frequency pulse, we are truncating at a frequency lower than the onset of the first radial mode. Since we believe the ripple is related to the radial modes, its disappearance in this case is not surprising.

The results presented above were obtained using the MOM expression in (11). We now compare them with the transient current evaluated using the approximation in (21). The specific
case of $z = -100a$ for the high-frequency pulse is shown in Fig. 6. We note good agreement between the two solutions. However, for $z = -10a$ (Fig. 7) the agreement is rather poor because in deriving (21), we assume that $|kz| \gg 1$. If we evaluate the current at a position too close to the aperture, the approximation is no longer valid.

The same two observer positions are shown in Figs. 8 and 9 for the low-frequency pulse. Once again, the MOM solution and the approximation agree favorably for $z = -100a$. There is also improved agreement for $z = -10a$. In order to explain this, we examine the results in the frequency domain. The frequency spectra for the two observer locations are given in Figs. 10 and 11. There is excellent agreement for $z = -100a$. For $z = -10a$, we note that there is good agreement near dc. It may be shown that for frequencies near dc, the dominant contribution to the integral of (15) again occurs for $\xi \rightarrow 0$; therefore, the approximation in (21) holds. Since the low-frequency pulse has greater frequency content near dc, the approximation agrees more closely in this case. Figs. 10
and 11 also show that the onset of the first radial mode for $b = 5a$ at 3.64 GHz is above the point of truncation for the low-frequency pulse (3 GHz). Therefore, as discussed above, the ripple believed to be associated with the radial modes does not appear in this case.

Let us now consider the effects of varying the cavity length $h$. Again, for the cavity radius, we select $b = 5a$, and for the aperture size, we select $c = 2a$. Fig. 12 shows the transient current response to the high-frequency pulse at $z = -1000a$ for two cases: $h = 15a$ and $h = 25a$. The round-trip travel time for the pulse inside each cavity is 1 and 1.67 ns, respectively. We see good agreement between the MOM and approximate solutions for both cases. In addition, the time delays in the reflections from the back of the cavity caused by the longer length are apparent. The same observations can be made for the current response due to the low-frequency pulse as shown in Fig. 13.

Finally, we examine the current for three different aperture sizes: $c = 1.01a$, $1.1a$, and $2a$. We set $b = 5a$, $h = 15a$, and $z = -1000a$. The current responses to the high- and low-frequency pulses are shown in Figs. 14 and 15. As the aperture size decreases, the screen looks more like a short circuit, and therefore, the magnitude of the screen reflection increases. Less current penetrates into the cavity, and that which does is more likely to be trapped inside the cavity longer since reducing the aperture size essentially closes off the cavity, reduces radiation damping, and increases the cavity $Q$. This explains why the magnitude of the first cavity reflection is reduced, whereas later cavity reflections become more significant.

C. Interior Current

For the transient current response inside the cavity, we use the high-frequency pulse as our excitation. We first consider a cavity with dimensions $b = 5a$ and $h = 15a$. The aperture has radius $c = 1.01a$, and the observer is located at the center of the cavity ($z = h/2$). The interior current for this case is shown in Fig. 16. We observe a ripple having a period of 0.5 ns in the transient response. The round-trip time between the center of the cavity and either end is 0.5 ns; therefore, this suggests that the ripple is associated with reflections in the $z$ direction. There is an additional ripple superimposed on the first, and it has a shorter period. It is most easily observed at times greater than 8 ns. We propose that this effect is related to radial reflections, that is, to the presence of the radial modes. To test this theory, we consider a second case in which the radius of the cavity is increased to $b = 7a$. Since the observer position remains constant, we again expect the 0.5-ns ripple. It is indeed present, as is shown in Fig. 17. However, we expect the period of the ripple caused by the radial modes to increase since the radial separation between the wire and the cavity has increased. This appears to be the case. If we count the number of oscillations in a given time period, say 9 to 12 ns, we see that this total number decreases. As pointed out earlier, this ripple associated with the radial modes is also present in plots of the exterior current response.

IV. CONCLUSIONS

In our study of the exterior current response, we had two objectives. First, we wanted to compare results obtained us-
Fig. 14. Transient current response to the high-frequency pulse evaluated using MOM for different aperture sizes ($b = 5a, h = 15a, z = -1000a$).

Fig. 15. Transient current response to the low-frequency pulse evaluated using MOM for different aperture sizes ($b = 5a, h = 15a, z = -1000a$).

Fig. 16. Transient current response to the high-frequency pulse at the center of the cavity ($z = h/2$) evaluated using MOM ($b = 5a, h = 15a, c = 1.01a$).

Fig. 17. Transient current response to the high-frequency pulse at the center of the cavity ($z = h/2$) evaluated using MOM ($b = 7a, h = 15a, c = 1.01a$).
ing the method-of-moments solution to those obtained by an approximate method. Second, we were interested in how the transient response was affected by changes in various dimensional parameters. Using two different double exponential input pulses to excite the system, a “low-frequency” pulse and a “high-frequency” pulse, we made the following observations.

It was possible in the transient response to identify the reflection from the conducting screen as well as the first few reflections from the back of the cavity. The cavity reflections were easier to identify when the high-frequency pulse was used as the input. A broadening of the reflected pulse due to dispersion was also evident. Results obtained by the approximate method agreed favorably with those obtained using the method of moments for observation points far from the aperture. As we increased the length of the cavity, we observed a corresponding delay in the cavity reflections. This demonstrated an important idea, namely, that information concerning one of the cavity’s dimensions might be obtained from exterior observations. Finally, as we would expect, reducing the aperture radius caused an increase in the magnitude of the screen reflection and a decrease in the magnitude of the first reflection from the back of the cavity.

To examine the transient current response inside the cavity, we employed only the high-frequency input pulse. The results contained two interesting features taking the form of “ripples” in the transient signal. From the observations based on the periods of oscillation of these ripples, one was shown to be a function of the cavity length, and the other was shown to be a function of the cavity radius. It is not surprising that information regarding the cavity dimensions was present in the interior current response; however, it is significant that the features associated with the two dimensions were distinct. Even more importantly, the ripple related to the cavity’s radial dimension also appeared in the exterior current response. This suggests that information concerning both interior cavity dimensions might be obtained from exterior observations.

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