Analysis of Electromagnetic Scattering from a Cavity with a Complex Termination by Means of a Hybrid Ray–FDTD Method

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Abstract—The electromagnetic modeling of engine cavities is a very difficult task because the electrical size of the cavity is very large, while the engine termination is geometrically complex. High-frequency techniques can adequately model the cavity, but perform poorly when applied to the termination. Low-frequency techniques are currently infeasible for such large geometries because of the large memory and computation time requirements. In this paper, we present a hybrid method which combines the most attractive features of the low- and high-frequency techniques. The finite-difference time-domain (FDTD) method is applied to the small region surrounding the termination. The remainder of the cavity is modeled with ray methods. To validate this method, we consider two-dimensional cavities with complex terminations. Our results are compared against those found from a hybrid combination of the modal method and the method of moments.

I. INTRODUCTION

THE PROBLEM of electromagnetic scattering from an open-ended cavity structure is an important problem in the area of radar scattering. Among other things, it models the radar scattering from a jet engine inlet. Traditionally, high-frequency methods have been applied to solve such problems. In recent years, two ray-based methods have been widely used to model the scattering properties of electrically large cavities of arbitrary shape. These are the “shooting and bouncing ray” (SBR) method [1] and the “generalized ray expansion” (GRE) method [2]. However, these two methods have only been shown to work for cavities with planar or very simple terminations. The terminations of practical interest are often geometrically complex (such as the fan blades in a jet engine), and the ray methods are not expected to be able to properly account for the diffraction effects of the termination. In addition, the termination may contain materials which are not perfectly conducting. Thus, a solution based purely on ray methods may be drastically different from the correct solution. Low-frequency techniques such as the moment method (MM) [3], the finite-element method (FEM) [4], and the finite-difference time-domain method (FDTD) [5], [6] can accurately model the complex termination as well as the rest of the cavity, but unfortunately, the computation costs to model such a large geometry exceed even the most powerful supercomputer.

In this paper, we present a hybrid method which combines both high- and low-frequency methods to overcome the shortcomings of either methods. A ray method is used to track the fields from the mouth of the cavity up to an arbitrarily defined planar surface close to the termination. These fields act as the excitation for the termination region, which is modeled by a low-frequency method. In our case, we have chosen the FDTD method because of its computational efficiency and its capability to obtain multiple-frequency data from a time history solution. It is expected that this hybrid method can provide a solution that is almost as accurate as if the solution had been obtained from low-frequency techniques but at a very small fraction of the computation costs.

The paper is divided into three sections after this introduction. Section II describes the details of the hybrid method including a description of the coupling between the ray method and the FDTD method. Although the formulation of the hybrid method is for the general case of a three-dimensional (3D) cavity with an arbitrary termination, the validation of this method is performed in two dimensions (2D) in order for us to more easily analyze the accuracy and robustness of this method before implementation in 3D. In Section III, results are presented for the 2D geometry of a parallel plate waveguide containing perfectly conducting terminations. These results are compared to a hybrid modal-MM method to demonstrate the accuracy of the hybrid ray–FDTD technique. Finally, a brief summary of this paper is provided in the last section.

II. THE HYBRID RAY–FDTD METHOD

In this section, a brief discussion of the GRE and the FDTD methods is given, followed by a description of how the coupling between the frequency-domain GRE method and the FDTD method is accomplished. The use of absorbing boundary conditions and the computation of the cavity scattered field are then considered.

A. The GRE Method

SBR and GRE are the two ray methods that have been considered for use in combination with FDTD for the hybrid method. Many papers have already been published on the two ray methods and their applications in cavities,
corner reflectors, and antennas [7]–[12]. For the hybrid method, we have chosen GRE over SBR after due consideration of their accuracy and efficiency. As in any hybrid method, the accuracy of the solution at the interface between the methods is an important factor in the overall accuracy of the hybrid method. It has been shown in [13] that the ray fields predicted by GRE inside a semi infinite parallel-plate waveguide cavity is more accurate than those obtained by SBR. The difference in the accuracy of these ray methods is attributed to the difference in the coupling of the (externally applied) incident fields into the waveguide cavity. GRE intrinsically includes the incident fields diffracted into the cavity by the open end through the integration of the equivalent currents over the aperture of the cavity. SBR, on the other hand, does not include the edge-diffracted fields. Depending on the electrical size of the aperture and the length of the cavity, the edge-diffracted fields may eventually affect the geometrical optics prediction of the fields intercepted by the aperture. In terms of efficiency, SBR generally requires less rays, which also means less ray tracing to perform, than in GRE for a single incident angle. However, a new set of rays has to be traced in SBR, including computing their divergence factors and reflection coefficients, for every new incident angle, whereas the same set of rays is used in GRE regardless of the incident angle of the field (the latter will become evident later). So GRE is better suited for scattering problems where the scattered field is required over a range of angles. Therefore, for accuracy and efficiency, we have chosen GRE over SBR for the hybrid method.

Because the detailed formulation of the GRE method can be found in [2], only the working principles of the method will be described. In this method, the incident field at the aperture of the cavity is replaced by equivalent surface currents via Kirchhoff’s approximation. These equivalent currents radiate the desired fields into the cavity interior as shown in Fig. 1. For large cavities, the aperture is divided into a number of smaller subapertures. A cone of ray tubes is then launched into the cavity from the phase center of each subaperture. The ray tubes are tracked using the central rays of the respective ray tubes. The central rays are tracked as ordinary geometrical optics rays. The exiting ray tubes contribute to the scattered field of the cavity. The scattered field is computed by physical optics approximation, taking into account the wavefront curvature, the size, and the shape of the ray tube associated with each exiting ray [2], [14]. A more efficient method for obtaining the scattered field will be presented in a later section.

In GRE, each ray tube is weighted by the far-field approximation of the radiation pattern of the subaperture (from which that ray tube has originated) with the cavity walls absent. Specifically, the electric field along the pth ray of the lth subaperture prior to any reflection is given by

\[ E_{pl}(r) = C_l(\hat{r}_{pl}) \frac{\exp(-jk_0r_{pl})}{r_{pl}} \]  

(1)

where \( k_0 \) is the propagation constant of the medium in which the cavity is embedded, \( r_{pl} \) is the position vector of a point along the pth ray with respect to the phase center \( O_l \) of the lth subaperture. \( C_l(\hat{r}_{pl}) \) is the far-field vector radiation pattern of the electric field evaluated in the direction \( \hat{r}_{pl} \) in the absence of the cavity walls. It is given by

\[ C_l(\hat{r}_{pl}) = \frac{jk_0 Z_0}{4\pi} \int \int \int \left[ \hat{r}_{pl} \times \hat{r}_{pl} \times \hat{r}_{pl} \times J_{eq}(r'_l) \right] + Y_0 \hat{r}_{pl} \times M_{eq}(r'_l) \exp \left( -jk_0 \hat{r}_{pl} \cdot r'_l \right) ds'_l, \]  

(2)

where \( r'_l \) is the vector from \( O_l \) to the equivalent sources on the subaperture \( S_l \). Equation (2) can be evaluated numerically or in closed form depending on the field excitation and the shape of the subaperture [9].

As each ray tube undergoes multiple reflections within the cavity, its field amplitude is modified by divergence factors and reflection coefficients. In particular, the electric field \( E(r_l) \) of the ray immediately after reflection from the point \( r_i \) is found iteratively from the field \( E(r_{i-1}) \) immediately after the previous reflection point \( r_{i-1} \) through the relation

\[ E(r_i) = \Gamma \cdot E(r_{i-1}) (DF)_{i-1} e^{-jk_0s}, \]  

(3)

where \( s = |r_i - r_{i-1}| \), \( \Gamma \) is the planar dyadic reflection coefficient evaluated at \( r_i \) and \( (DF)_{i-1} \) is the divergence factor governing the spreading of the ray tube associated with the ray after reflection from the point at \( r_{i-1} \). The divergence factor is usually evaluated via the Q-matrix formulation of Deschamps [15].

From (1)–(3), it is clear that only one set of rays needs to be traced for all the incident angles because the ray divergence factors and reflection coefficients are functions of the ray paths and not the incident angle; only the initial amplitude of the ray field is a function of the incident angle.

The most serious limitation in ray methods (like the GRE and SBR methods) is their inability to account for fields arising from discontinuities like edges and tips inside the cavities. As such, ray methods are not expected to produce reasonable results for cavities with complex terminations.

B. The FDTD Method

FDTD is a direct solution of Maxwell’s time-dependent curl equations [5], [6]. It applies second-order accurate central-difference approximations for both the space and...
time derivatives of the electric and magnetic fields directly to the differential operators of the curl equations. We do not derive the difference equations here because they can be found in many papers, including those given above. Instead, we consider some of the attractive features of FDTD that are useful for our hybrid method.

FDTD models the actual real-time behavior of EM fields which makes it suitable for impulse and transient analysis. The frequency spectrum can also be easily obtained via Fourier transformation of the FDTD time solution. In contrast, the use of frequency-domain methods to get the frequency spectrum or time variation requires solving the problem repeatedly for multiple frequencies in the frequency range of interest.

In FDTD, the field computations involve only simple arithmetic operators with no matrix manipulation. The number of floating-point operations per time step and the memory storage are both proportional to the number of unknowns, \( n \), compared to \( n^3 \) and \( n^2 \), respectively, in frequency-domain methods such as the method of moments (assuming LU decomposition). The equations in FDTD can also be easily vectorized for the Cray computer. Despite these advantages, FDTD is computationally costly for solving electrically large problems. For example, a rectangular cavity, with a cross section of \( 30 \lambda \times 40 \lambda \) and a depth (or length) of \( 100 \lambda \), requires 5.76 billion unknowns for a mesh density of 20 cells/\( \lambda \). Moreover, the FDTD field equations have to be repeatedly evaluated for at least 8000 time steps (or iterations, assuming two time steps for the field to traverse from one cell to another along the length of the cavity) before the interior irradiated field can be observed at the aperture of the cavity. Finally, FDTD cannot easily model curve boundaries/surfaces although some solutions have been proposed to partly overcome this shortcoming [16], [17].

C. Coupling between Ray and FDTD

As was alluded to earlier, FDTD can be computationally inefficient when it is used to solve electrically large geometries. For the previous example of the rectangular cavity, most of the computational time and effort are wasted in the two-way propagation of the field along the length of the cavity. The proposed hybrid ray–FDTD method overcomes this inefficiency by using a ray method, specifically GRE, to propagate the fields from the aperture to the vicinity of the termination. However, since it is difficult to apply ray methods to complex terminations, FDTD is used to determine the interaction of the cavity fields with the termination.

As with most EM hybrid methods, the coupling between the individual methods has to be handled with care. Consider the cavity shown in Fig. 2. The cavity has been divided into three regions corresponding roughly to the air intake, the engine, and the exhaust sections of a jet engine inlet. The imaginary surface \( S_{T1} \) separates regions 1 and 2, while \( S_{T2} \) separates regions 2 and 3. In region 1, the cavity is assumed to be smoothly varying for high-frequency methods like ray methods to be valid. Ray tubes are traced from the aperture of the cavity to \( S_{T1} \), where they are summed to form a high-frequency solution of the cavity fields across \( S_{T1} \). This solution on \( S_{T1} \) is used as the excitation for the FDTD computation in region 2.

We will now describe two approaches to the coupling between the GRE and the FDTD methods. Depending on the size of a ray tube when it reaches \( S_{T1} \), it will either not intersect any of the FDTD grid points on \( S_{T1} \) or it will intersect one or more of those grid points. Therefore, different ray tubes will contribute differently to the total field at a grid point. Moreover, it is possible that some grid points will not have any ray tubes intersecting them. As a result, the incident fields on \( S_{T1} \), evaluated via rays will not be smoothly varying. In order to obtain a smoothly varying as well as an accurate field on \( S_{T1} \), the size of the ray tube can be restricted so that it will only intersect one FDTD grid point. Alternatively, some form of interpolation can be applied to the ray tubes that intersect more than one grid point. In both of these approaches, a ray tube is launched and tracked via its central ray to \( S_{T1} \). The projected ray tube area (for the 3D case; width for the 2D case) on \( S_{T1} \) is then determined. If this area is greater than some specified area, \( A_p \), the ray tube is subdivided and the process of tracking and determining the projected area of the smaller ray tubes is repeated. In the first approach, \( A_p \) is equal to the FDTD grid spacing (usually \( \lambda/20 \)) for the 2D case so that if the projected ray tube width is less than \( A_p \), that ray tube can intersect at most one FDTD grid point on \( S_{T1} \). In such an event, the field specified by the central ray will be added to the intersected grid point. Ray tubes that do not intersect any grid point are ignored.

For the second approach, \( A_p \) is larger, but no greater than \( \lambda/2 \) and \( (\lambda/2)^2 \) for the 2D and 3D cases, respectively, for reasons given in [2]. In our 2D implementation, we have used \( A_p = \lambda/4 \) for greater accuracy in the evaluation of the fields on \( S_{T1} \). Since a larger ray tube may intersect more than one FDTD grid point on \( S_{T1} \), its contribution to the field on \( S_{T1} \) is determined by converting its ray field into modal fields. Specifically, the contribution of the ray field to the modal coefficients are determined by integrating the ray field over the projected area of the ray tube assuming a linear phase variation in the field over the projected area with respect to the field of the central ray. This approach assumes that the fields on \( S_{T1} \) are expressible in terms of parallel-plate waveguide modes for 2D problems. This assumption can usu-
ally be satisfied by a suitable choice of $S_T$. In realistic 3D problems, there is usually a narrow section in front of the termination which is cylindrical so that the fields in this narrow section can also be expressed in terms of modes. In any case, when the modal coefficients have been obtained by summing the contributions due to all the ray tubes, the desired field at each FDTD grid point on $S_T$ can be determined.

Comparing the two approaches, it is clear that the first approach requires more ray tracing (which means more computational time and storage) than the second approach because of the smaller number of the desired fields. However, in realistic 3D problems, the computational time and storage than the second approach because of the smaller number of terms which is cylindrical and is evaluated over a range of frequencies.

Another consideration in the coupling of the GRE and FDTD methods is the selection of a suitable time variation for the excitation since the former is a frequency-domain method while the latter is a time-domain method. There are two possible schemes for the time variation: the sinusoidal steady-state time variation [18] and the pulsed (usually Gaussian or raised-cosine) time variation. For the steady-state FDTD, the ray solution on $S_T$ is evaluated only at a single frequency of interest. Based on the complex ray field solution at $S_T$, the excitation can be made to vary sinusoidally with time. For the pulsed FDTD, there are two possible alternatives. For the first alternative, the ray solution on $S_T$ is computed over a range of frequencies corresponding to the frequency content of the pulse. The excitation can then be obtained by an inverse Fourier transform of the product of the ray solution and the Fourier transform of the pulse. This alternative is not attractive because the resultant inverse transform will have a wide time window with a number of significant pulses due to the different arrival times at $S_T$ of the reflected, diffracted, and reflected-diffraction fields. A better alternative is to use a basis (e.g., modes) as excitation for the pulsed FDTD to characterize the termination section in terms of a termination scattering matrix. This scattering matrix, together with the ray solution on $S_T$ (expressed also in terms of the basis set), can then be used to find the cavity scattered field. The pulsed time variation scheme is more efficient for problems which require multiple frequency solutions while the steady-state scheme is more efficient for problems which require only a single frequency solution.

Regardless of the time variation used, the excitation produces a wave which propagates toward the termination and interacts with it. For the geometry shown in Fig. 2, part of the wave may be transmitted to region 3 through $S_T$, while the remainder is reflected back toward $S_T$. If we assume that the waves leaving region 2 through the imaginary surfaces $S_{T1}$ and $S_{T2}$ do not return, then the absorbing boundary condition (ABC) such as the ones introduced by Higdon [19], [20] or Mur [21] can be applied in the FDTD computations at each of the two surfaces. The above assumption is reasonable because most jet engine inlets are shaped in such a way that there is very little energy that returns to region 2 upon its exit from there. Otherwise, we can convert the waves leaving region 2 back into rays (using GRE) and track those rays that return to region 2. These returning rays act as an additional excitation.

### D. Absorbing Boundary Conditions at $S_T$ and $S_{T2}$

As mentioned previously, absorbing boundary conditions (ABCs) are applied at the imaginary boundary surfaces $S_T$ and $S_{T2}$ for the proper transmission of waves through these surfaces. However, the two ABCs cannot be implemented in the same way because of different field conditions at their respective boundary surfaces. With reference to Fig. 2, the ABC at $S_T$ has to properly account for the waves which pass through $S_T$, in both directions; it has to account for the incident excitation at $S_T$ and the scattered field due to the termination. In contrast, the absorbing boundary at $S_{T2}$ only has waves transmitted through it from region 2 to region 3 assuming that there are no waves transmitted through $S_{T2}$ from region 3 to region 2 (recall discussion in previous section).

The ABC at $S_T$ has to transmit (or absorb) the waves that are scattered by the termination toward $S_T$ without destroying or affecting the incident excitation at $S_T$. We will demonstrate how this function can be accomplished with the second-order ABC of Mur [21] in a 2D problem. Applying the ABC given by (17) of [21] to the scattered electric field $E_{s,i}$ (for the TM case) at $S_{T1}$, we have

$$E_{s,i}^{n+1} (0,j) = E_{s,i}^n (1,j) a_1 \left[ E_{s,i}^{n+1} (1,j) - E_{s,i}^n (0,j) \right]$$

$$- a_2 \left[ H_{s,i}^{n+1/2} (0,j + \frac{1}{2}) - H_{s,i}^{n+1/2} (0,j - \frac{1}{2}) \right] + H_{s,i}^{n+1/2} (1,j + \frac{1}{2}) - H_{s,i}^{n+1/2} (1,j - \frac{1}{2})$$

(following Mur’s FDTD indexing; see Fig. 3 also for indexing), where $E_{s,i}$ and $H_{s,i}$ are the fields scattered by (or reflected from) the termination. The constants $a_1$ and $a_2$ are given by

$$a_1 = \frac{c_0 \Delta t - \Delta x}{c_0 \Delta t + \Delta x}$$

$$a_2 = \frac{\mu_0 c_0^2 \Delta t \Delta x}{2 \Delta y (c_0 \Delta t + \Delta x)}$$

Replacing the scattered field components $(E_{s,i}, H_{s,i})$ with

$$E_{s,i} = (E_s, H_s) - (E_{s,i}, H_{s,i})$$

(5)
where the subscript $i$ in $E_x, (H_y)$ denotes the incident component of the total $E_x, (H_y)$ field, (4) becomes

$$E_{x,i}^{n+1}(0, j) = E_x^n(1, j) - E_x^n(1, j) + a_1$$

$$\left[ E_x^{n+1}(1, j) - E_x^{n+1}(1, j) - E_x^n(0, j) \right]$$

$$- a_2 \left[ H_x^{n+1/2}(0, j + \frac{1}{2}) - H_x^{n+1/2}(0, j - \frac{1}{2}) \right]$$

$$+ H_x^{n+1/2}(0, j - \frac{1}{2}) + H_x^{n+1/2}(1, j + \frac{1}{2})$$

$$- H_x^{n+1/2}(1, j - \frac{1}{2}) + H_x^{n+1/2}(1, j - \frac{1}{2}) \right].$$

(6)

Except for $E_{x,i}^{n+1}(1, j)$ and $E_{x,i}^{n+1}(1, j)$, all the other field components in (6) are computed values for the previous (one or half) time step. $E_{x,i}^{n+1}(1, j)$ can be calculated for the current time step from the regular (total-field) difference equation. $E_{x,i}^{n+1}(1, j)$ can be obtained via the ray method (as was done for the excitation at the absorbing boundary) or from the propagation of the incident excitation. After $E_{x,i}$ on $S_T$, has been found, the total field components of $H_x$ and $H_y$ in region 2 can be updated via the regular FDTD equations.

The procedure described above for obtaining the scattered fields at $S_T$ due to the reflection of the fields from the termination has to be used whenever the steady-state sinusoidal time variation is chosen for the FDTD computation in region 2. However, this procedure may or may not be necessary when the pulsed time variation is chosen, depending on the time window of the excitation and the closest distance between $S_T$ and the termination.

The ABC at $S_T$ does not require any special treatment like the one at $S_T$ if we assume that waves are transmitted through $S_T$ only from region 2 to region 3 and not vice versa. In this case, the transmitted field is also the total field. Therefore, any suitable ABC for the absorption of the total field at $S_T$ can be applied. Note that the ABC (4) can also be applied at $S_T$ with the scattered field variables replaced appropriately by the total field variables (with the proper spatial indices).

### E. Scattered Field Computation

To determine the cavity scattered fields, the appropriate field solutions in regions 1 and 2 have to be used. One possible way of finding the cavity scattered field is to launch GRE rays into region 1 using the FDTD solution at $S_T$. These rays are tracked to the front aperture of the cavity where aperture integration can be applied to find the scattered field. Unfortunately, this method requires that rays be traced both into the cavity for the incident excitation at $S_T$ and out of the cavity for the cavity scattered field computation.

A more suitable and efficient way for finding the cavity scattered field is based on the termination reciprocity integral developed by Pathak and Burkholder [22]. This integral is given by

$$E_s(P) \cdot P = \int_{S_F} (E_x \times H_y - E_y \times H_x) \cdot \hat{n} ds,$$  \hspace{1cm} (7)

where $E_s(P)$ is the desired cavity scattered field at the observation point $P$ and $P$ is the strength of an electric current point (test) source. $(E_x, H_y)$ are the fields scattered by the termination in the cavity while $(E_x, H_y)$ are the fields radiated by the test source in the presence of the cavity structure without the termination. $\hat{n}$ is the unit vector normal to the surface $S_F$ as shown in Fig. 2. The approximation in (7) assumes that the source and observer are in direct view of the open front end so that the contribution to $E_s(P)$ from the scattered field exiting through $S_F$ is negligible compared to the fields exiting through $S_F$.

To see the usefulness of (7), consider how the cavity scattered fields are obtained using a purely ray-based approach. For example, in the original implementation of SBR [1, 7], two-way (in and out of the cavity) ray tracing has to be performed before the scattered field can be determined via aperture integration. With the reciprocity integral, the rays are only traced from the open end of the cavity to the termination and back to $S_T$. Therefore, the amount of ray tracing is basically reduced by about half. This reduction is even more significant when the reciprocity integral is used in the hybrid method because the rays are only traced from the open end to $S_T$.

From (7), we see that the integral is independent of the method(s) used to find the two sets of tangential fields on $S_T$. It is particularly easy to apply the integral to find the backscattered field in our hybrid method since both sets of fields at $S_T$ are readily available. Specifically, for backscatter computation, $(E_x, H_y)$ is the initial excitation.
on $S_T$, evaluated by the GRE method for the FDTD computation in region 2; its evaluation has already been described earlier. $(E_x, H_y)$ is the termination scattered field that is transmitted through $S_T$ from region 2. It has to be evaluated on $S_T$ from the FDTD algorithm. However, the tangential $E$ and $H$ fields in a Yee cell lie on different planes. In particular, the tangential $E$ and $H$ planes are half a cell width apart in the direction normal to both of those planes. Consider for example, the 2D TE case shown in Fig. 3, where the tangential magnetic field $H_y(0, j)$ lies on $S_T$, and the tangential electric field $E_x(\frac{j}{A}, j)$ lies on a plane parallel to $S_T$, at $\Delta x/2$ away. To obtain $E_y(0, j)$ (which is symbolically, the termination scattered field $E_y$, on $S_T$), an extrapolation of the neighboring values of $E_y$, is used. Specifically, we assumed that the gradient of the scattered field $E_y$, in the normal direction of $S_T$, d$E_y$/dx, at $x = \Delta x/4$ is equal to that at $x = \Delta x$, so that

$$E_y, (0, j) = \frac{1}{2} E_y, (\frac{1}{A}, j) - \frac{1}{2} E_y, (\frac{1}{A}, j).$$

For the steady-state FDTD, the scattered field components $(E_x, y)$ on the right-hand side of (8) are replaced by the difference between their respective total $(E_x)$ and incident $(E_y)$ field components. This extrapologic scheme can be similarly applied to the tangential electric or magnetic field in the 3D case. When all the tangential fields within the integral of (7) have been appropriately manipulated into the frequency domain, the cavity scattered field $E_x, (P)$ can be computed.

In summary, the hybrid ray–FDTD method uses high-frequency ray solutions as input excitation for the evaluation of the termination scattered field via the low-frequency FDTD method; the scattering of the cavity is then obtained via the termination reciprocity integral. The hybrid ray–FDTD method combines the efficiency of the ray method (and the reciprocity integral) with the modeling flexibility of the FDTD method. At the same time, it overcomes some of the limitations of the individual methods in analyzing the scattering from a cavity.

### III. RESULTS

The hybrid GRE–FDTD method has been implemented in three different programs (see Table I) using various combinations of ray tracing and time variations to determine the radar cross section (RCS) of parallel-plate cavities with different PEC plug terminations. Program 1 implements the GRE method such that the projected areas of the ray tubes are less than $\lambda/20$ to find the incident field on $S_T$, while the other two programs implement the GRE method such that the ray fields are converted into modes to find the same fields. Programs 1 and 2 implement the FDTD algorithm using the steady-state sinusoidal time variation, while program 3 implements the FDTD algorithm using the basis-pulsed time variation approach.

In all three programs, the aperture of the cavity is divided into three subapertures from which rays are launched within an angle of $+75^\circ$ of the cavity axis to compute the excitation at $S_T$, $S_T$ is fixed at $4\lambda$ from the closed end of the cavity. For the FDTD algorithm, a $\lambda/20 \times \lambda/20$ spatial grid is used (unless it is specified otherwise) with a time step of $\Delta x/2c_0$; Mur’s ABC [21] is applied at $S_T$ to absorb the fields scattered toward $S_T$ by the termination. No ABC is implemented at $S_T$ since region 3 (see Fig. 2) is nonexistent for our terminated cavity. The scattered field is obtained via the reciprocity integral (7) which is now exact since $S_E$ is perfectly conducting for the terminated (PEC) parallel-plate waveguide.

In the following examples, we will only present results obtained via program 2. These results are compared with reference solutions obtained via the hybrid asymptotic modal-method of moments (modal-MM) [23]. The results from the other two programs are not presented here because there is little difference between their results and that of program 2. In fact, the purpose of programs 1 and 3 is to check the accuracy of the approach employed in program 2. Moreover, program 3 validates the time-domain (pulsed-basis) approach which has great potential in (2D and 3D) scattering problems where the cavity RCS is required at multiple frequencies. For the purpose of this paper, the results from program 2 are sufficient to illustrate the utility and accuracy of the hybrid method.

Figures 4 and 5 show the RCS patterns of a 9.6$\lambda$-wide and 30$\lambda$-long parallel-plate waveguide cavity with a $2\times 4\lambda$ rectangular PEC plug termination for the TM and TE cases, respectively. The GRE–FDTD patterns in both figures show excellent agreement with the corresponding modal-MM patterns. The slight differences between the GRE–FDTD and the modal-MM solutions can be attributed to two causes. One of the causes is the inability of the ABC to absorb the higher-order modes excited by the rectangular plug termination. These higher-order modes result from the reflection and edge diffraction of the fields from the termination. They are particularly significant at larger incident angles since the termination reflected fields (which are the incident fields after undergoing multiple reflections in the termination section) are incident at large angles from the normal on the absorbing boundary $S_T$. These higher-order modes are not transmitted (or absorbed) as well as the lower-order modes by the second-order Mur’s ABC (see Table II [24]). In addition, their multiple reflections between the absorbing boundary and the termination result in appreciable errors in the scattered field. Another source of...
error is the ray solution on $S_T$. These solutions are less accurate at large incident angle $\theta$ than at small $\theta$. This difference in accuracy is a result of limiting the ray solution to those rays whose launch angles are within $\pm 75^\circ$ of the cavity axis. At large $\theta$, this (artificial) angular limit may not be adequate. For example, for an incident angle of $60^\circ$, the magnitude of the radiation power pattern of each subaperture at $75^\circ$ is $-8$ dB (40%) with respect to the beam maximum, while the magnitude of the first and second side lobes are $-9$ dB (35%) and $-13$ dB (22%), respectively. Therefore, significant contribution by rays whose launch angles are greater than $75^\circ$ are omitted from the final ray solution on $S_T$.

Also shown in Figs. 4 and 5 are the respective patterns obtained using the GRE method alone. In this purely GRE approach, the rays are launched from the aperture and traced to $S_T$, where they form the incident field. The rays are then traced beyond $S_T$ to the termination where they are scattered back to $S_T$ to form the termination scattered field. Both the incident and scattered fields are converted into modal fields before they are used in the reciprocity integral to obtain the cavity scattered fields. It is clear from Figs. 4 and 5 that the GRE solutions agree very well with the reference and hybrid solutions for $\theta$ up to about $15^\circ$. This agreement suggests that the diffracted fields from the termination are negligible for small $\theta$. Beyond that, the GRE solutions show significant deviations from the reference solutions at certain angles. From these results, it is envisaged that for a complex termination, the hybrid GRE-FDTD method will produce far more accurate results than the purely GRE approach which does not account for the termination diffracted fields.

Figure 6 shows the resultant RCS patterns for the TM case when the rectangular plug termination is replaced by a wedge-shaped plug termination. The height and base width of the wedge-shaped plug are $2\lambda$ and $4\lambda$, respectively. The wedge is modeled in the FDTD code with the electric field $E$, tangent to the wedge surface. The GRE-FDTD pattern is again in excellent agreement with the modal-MM pattern.

Figure 7 shows the RCS patterns of the wedge-terminated cavity for the TE case. The wedge is now modeled in the FDTD code with the magnetic field $H$, tangent to the wedge surface. In contrast to the TM case where the tangential electric field $E$ vanishes on the wedge surface, the tangential magnetic field $H$ for the TE case does not vanish. Instead, it has to be specially treated. Specifically, we apply Stoke's theorem to one of Maxwell's curl equations to obtain a difference equation for updating the tangential $H$ correctly. Applying Stoke's theorem to Maxwell's curl-E equation results in

$$\oint_{\Delta ABC} E \cdot ds = -\mu_0 \sum_{\Delta ABC} \frac{\partial H}{\partial t} \cdot \hat{z} ds,$$  \hspace{1cm} (9)
becomes

\[ H_{x}^{n+1}(i_{m}, j_{n}) = H_{x}^{n}(i_{m}, j_{n}) + \frac{\Delta t}{\mu_{0} \Delta y} \left[ E_{x}^{n+1/2}(i_{m}, j_{n} + \frac{1}{2}) - E_{x}^{n+1/2}(i_{m}, j_{n} - \frac{1}{2}) \right] \]

\[ - \Delta t \left[ E_{y}^{n+1/2}(i_{m} + \frac{1}{2}, j_{n}) - E_{y}^{n+1/2}(i_{m} - \frac{1}{2}, j_{n}) \right] + \frac{\Delta t}{\mu_{0} \Delta x} \left[ E_{y}^{n+1/2}(i_{m} + \frac{1}{2}, j_{n}) - E_{y}^{n+1/2}(i_{m} - \frac{1}{2}, j_{n}) \right] \]

where the correction factor CF is given by

\[ CF = \frac{\Delta t}{\mu_{0} \Delta y} \left[ E_{x}^{n+1/2}(i_{m}, j_{n} + \frac{1}{2}) - E_{x}^{n+1/2}(i_{m}, j_{n} - \frac{1}{2}) \right] - \frac{\Delta t}{\mu_{0} \Delta x} \left[ E_{y}^{n+1/2}(i_{m} + \frac{1}{2}, j_{n}) - E_{y}^{n+1/2}(i_{m} - \frac{1}{2}, j_{n}) \right]. \]

Notice that without CF, (10) is just the regular FDTD equation for updating \( H_{x} \). Therefore, CF adjusts for the
H, 's that are tangent to the wedge surface. Once again, the GRE–FDTD pattern in Fig. 7 shows excellent agreement with the modal-MM pattern except for large θ.

For the final example, a PEC semicircular plug of radius 2h is substituted for the wedge-shaped plug. The curved surface of the semicircular plug is modeled using the stair-stepped approximation as shown in Fig. 8 for the TM₂ case. A similar approximation of the semicircular cross section is used for the TE₂ case. The RCS patterns of the new cavity are shown in Figs. 9 and 10 for the TM₂ and TE cases, respectively. In these figures, we have also plotted the patterns obtained with the smaller L/40 FDTD grid. The patterns obtained with the L/20 grid show some resemblance to the reference patterns, while those obtained with the finer grid show better agreement with the reference patterns, particularly for the TE₂ case. The significant improvement in the latter case is attributed to the fact that in 2D geometries, TE₂ modes suffer dispersion due to the stair-stepped approximation, while TM₁ modes do not [25]. This dispersion (or numerical error) reduces when a finer FDTD grid is used. Therefore, the pattern for the TE₂ case shows marked improvement, while that of the TM₂ case is basically unchanged.

IV. SUMMARY

In this paper, we have introduced a hybrid method which combines ray methods with FDTD. The use of the hybrid method allows us to overcome the deficiencies of a single method. The FDTD calculation, which is computationally intensive, is limited to a small region around the termination. The ray solution is used to evaluate the fields in the remainder of the cavity where it is assumed that the geometry is simple enough for high-frequency approximations to be valid. Results have been presented and compared to a hybrid modal-MM solution to demonstrate its accuracy. From these results, it is evident that the hybrid ray–FDTD method shows great potential for eventually providing a more accurate solution for the electromagnetic scattering from a realistic three-dimensional jet engine.

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