Near Optimal Power and Rate Control of Multi-hop Sensor Networks with Energy Replenishment: Basic Limitations with Finite Energy and Data Storage

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***Abstract*—Renewable energy sources can be attached to sensor nodes to provide energy replenishment for extending the battery life and prolonging the overall lifetime of sensor networks. For networks with replenishment, conservative energy expenditure may lead to missed recharging opportunities due to battery capacity limitations, while aggressive usage of energy may result in reduced coverage or connectivity for certain time periods. Thus, new power allocation schemes need to be designed to balance these seemingly contradictory goals, in order to maximize sensor network performance. In this paper, we study the problem of how to jointly control the data queue and battery buffer to maximize the long-term average sensing rate of a wireless sensor network with replenishment under certain QoS constraints for the data and battery queues. We propose a unified algorithm structure and analyze the performance of the algorithm for all combinations of finite and infinite data and battery buffer sizes. We also provide a distributed version of the algorithm with provably efficient performance.**[1]

I. INTRODUCTION

Wireless sensor networks have been widely used in monitoring [1], maintenance [2], and environmental sensing [3]. The lack of easy access to a continuous power source and the limited lifetime of batteries have hindered the wide-scale deployment of such networks. However, new and exciting developments in the area of renewable energy [4] [5] provide an alternative to a limited power source, and may help alleviate some of the deployment challenges. These renewable sources of energy could be attached to the nodes and would typically provide energy replenishment at a slow rate (compared to the rate at which energy is consumed by a continuous stream of packet transmissions) that could be variable and dependent on the surroundings.

Energy management in networks equipped with renewable sources is substantially and qualitatively different from energy management in traditional networks. For example, conservative energy expenditure could lead to (i) very long delays because the energy is not being fully used to transmit at high enough data rates, and (ii) missed recharging opportunities because the battery buffer is full. On the other hand, an over aggressive use of energy may lead to lack of coverage or connectivity for certain time periods. Further, if the battery of a node discharges completely, it could be temporarily incapable of transferring time-sensitive data. This may have undesirable consequences for many applications. Thus, new techniques and protocols must be developed for networks with replenishment to balance these seemingly contradictory goals.

In this paper, we first consider a single link communication, in which the transmitting node has a data buffer that holds the incoming variable-rate sensing data and a battery buffer, which is being replenished at a variable rate. In our model, we allow any combination of finite and infinite data and battery buffer sizes. If the data buffer size is infinite, the concern is the stability of the data queue, while for finite data buffer, excessive data losses is the undesirable event. Likewise for the battery buffer, frequent occurrences of battery discharge should be avoided. We investigate the problem of maximizing the long-term average sensing rate, subject to the constraints on the stability of the data queue (or the desired data loss ratio when the data buffer size is finite) and the desired rate of visits to zero battery state. We provide a simple and unified joint rate control and power management framework, and show that the performance of our scheme is close to optimum.

We then extend our algorithm to a multihop network setting with multiple source destination flows. We generalize our joint power and rate rate control algorithms along with a multihop routing scheme. We assign each link a weight that is a function of the data queue size, the battery state, and the chosen power level for that link. Then, we schedule a subset of links at a given point in time, depending on their weights using maximum weight matching or maximal matching, under the primary (or node exclusive) interference model, that has been shown to be a good model for Bluetooth of FH-CDMA systems with perfect orthogonal spreading codes and low power-spectrum density and used widely in wireless network control [6] [7]. We extend the performance analysis and show that our algorithm achieves provable performance in the multihop setting as well.

While the problem of energy management in sensor networks has seen considerable attention, there have been a number of works [8], [9], [10], [11], [12], [13], [14], [15], [16] that also address scenarios with energy replenishment. In [8], the authors consider the problem of dynamic node activation in rechargeable sensor networks. They provide a distributed threshold policy on the number of nodes that can be activated, which achieves a performance within a certain factor of the

[1]This work was in part supported by NSF grants CNS 0831919, CCF 0916664, and CNS 1054738.

optimal solution for a set of sensors whose coverage areas overlap completely. In [9], the authors study the problem of computing the lexicographically maximum data collection rate for each node in a sensor network such that no node will be out of energy. In [10], the authors consider the problem of energy-aware routing with distributed energy replenishment. They provide an algorithm that achieves a logarithmic competitive ratio and is asymptotically optimal with respect to the number of nodes in the network. In [14], the authors formulate a convex joint rate allocation and routing problem in an energy replenishing wireless sensor network. They provide an implementation of the optimal solution based on a primal-dual approach as well as an asymptotically optimal heuristic power allocation scheme for finite battery sizes. Power allocation for wireless networks without replenishment has been widely studied, e.g., [17], [18]. In [18], the authors assume that the data buffer is large enough so that packet loss does not occur and provide a dynamic programming based solution. In [17], the authors develop approximate algorithms to minimize the average allocated power, or maximize the throughout given average power constraint, and at the same time keep the data queue stable. The approximation improves at the cost of increasing the data queue length and queueing delay. Most of these works assume a constant energy supply, i.e., there is no battery issue. The idea of [15] and this paper are motivated by [17]. Both papers model energy replenishment as a time-varying process and consider jointly managing the data and battery buffers. This coupling between the data and battery buffers is what makes the problem notoriously difficult to solve using standard optimization based approaches. For example, unlike [17] that utilizes the fact that a static allocation policy is optimal, it is not even clear whether a static policy would even be optimal in our setting. Specifically, with a battery buffer, there is an additional energy constraint that the allocated energy should be within the battery state, and this constraint is even more difficult than the average power constraint. In [15], the authors consider infinite data buffer and finite battery buffer sizes. They assume that the replenishing process is i.i.d (for non i.i.d process, they claim that a K slot analysis can be applied, which is complex and depends on the network size), and show that the probability of battery state being less than the peak power or close to the full battery state vanishes as the battery size grows, under an index policy. In this paper, we construct a framework that accommodates all combinations of finite and infinite data and battery buffer sizes by defining minimum number of virtual queues in a general format. In addition to the constraint on the stability of the data queue (constraint on the data loss ratio when the data buffer size is finite), we also have a constraint on the frequency of battery discharge. Rather than assuming an i.i.d replenishing process as in [15], we allow for a general replenishment process without assuming ergodicity and carefully design the virtual queues and explore the relations between actual queues and virtual queues to show that our algorithm is efficient. Further, we design a distributed algorithm in detail.

The main intellectual contributions and challenges of the paper are as follows:

- We formulate a sensing rate maximization problem with QoS constraints on both the data and battery queues. Due to the coupling between the battery and data buffers, a stationary policy could be suboptimal, hence traditional resource allocation techniques do not directly apply. Nonetheless, we are able to develop a simple and unified framework for all combinations of finite and infinite data and battery buffer sizes, and our algorithm is provably efficient.
- We extend the algorithm from a single-link case to a multi-hop network under node-exclusive interference model with practical settings and develop an efficient distributed algorithm.

II. Single Link Model

We first consider a single link control model in this paper, as illustrated in Figure 1. We assume a time slotted system and in time slot t, the amount of data available for the sensor node to sense is denoted by $A(t)$, which is upper bounded by $A_{\max}$, ($0 < A_{\max} < \infty$). In the same time slot, the amount of data the node actually senses and places in the data buffer is $R(t)$. The amount of energy expended by the node at time t for data transmission is $P(t)$, which is upper bounded by P_{peak}, ($0 < P_{\text{peak}} < \infty$) and the achievable data rate at that power level is $\mu(P(t))$, where we assume $\mu(\cdot)$ to be monotonically increasing, reversible and differentiable on the half real line $\Re^+ \bigcup \{0\}$. The node has a battery of size B_b (either $B_b < \infty$ or $B_b = \infty$) with zero initial battery state. We let $r(t)$ denote the amount of replenishment energy arriving to the node at time t. The energy state of the battery and the data state of the data buffer at time t is given by $q_b(t)$ and $q_d(t)$, respectively. The data buffer has size B_d (again, either $B_d < \infty$ or $B_d = \infty$). Under this setting, we describe the objective in the following section.

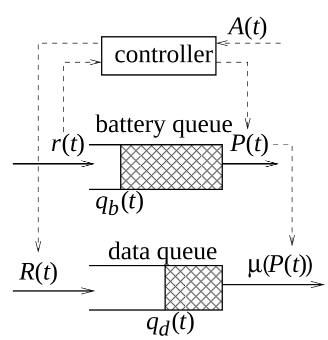

Fig. 1. Single Link Control Model

A. Problem Formulation

Our general objective is to maximize the long-term average sensing rate, subject to the QoS constraints on both data and battery queues:

$$(A) \qquad \max_{P,R} \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t)$$

$$\textit{s.t.}\ q_d(t+1) = \min\left[\Big(q_d(t) - \mu\big(P(t)\big)\Big)^+ + R(t), B_d\right], \tag{1}$$

$$q_b(t+1) = \min\Big[q_b(t) - P(t) + r(t), B_b\Big], \tag{2}$$

$$0 \le R(t) \le A(t), \tag{3}$$

$$0 \le P(t) \le \min\Big[q_b(t), P_{\text{peak}}\Big], \tag{4}$$

$$p_o \le \eta_o, \tag{5}$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_d(t) < \infty\ (B_d = \infty),\ \text{or}$$
$$p_d \le \eta_d\ (B_d < \infty), \tag{6}$$

where $(\cdot)^+ = \max[\cdot, 0]$, $R = \{R(0), R(1), \ldots, R(T-1), \ldots\}$ is the actual sensing data vector, $P = \{P(0), P(1), \ldots, P(T-1), \ldots\}$ is the allocated power vector, and

$$p_d = \begin{cases} 0, & \text{if } \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t) = 0 \\ \limsup_{T\to\infty} \frac{\frac{1}{T}\sum_{t=0}^{T-1} D(t)}{\frac{1}{T}\sum_{t=0}^{T-1} R(t)}, & \text{otherwise} \end{cases} \tag{7}$$

$$p_o = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathit{1}_o(t) \tag{8}$$

are the long-term data loss ratio with an upper bound η_d, and the frequency of visits to zero battery state with given threshold η_o respectively, where

$$D(t) = \left(\Big(q_d(t) - \mu\big(P(t)\big)\Big)^+ + R(t) - B_d\right)^+, \tag{9}$$

$$\mathit{1}_o(t) = \boldsymbol{\mathit{1}}_{\textit{battery hits zero state in slot } t} = \begin{cases} 0 & \text{if } P(t) < q_b(t) \\ 1 & \text{otherwise} \end{cases} \tag{10}$$

are the amount of data loss in slot and the indicator that the battery discharges completely in time slot t, respectively. Some applications may require the battery state to be always above certain positive level, then Equation (10) can be easily modified to represent how often the battery is below the desired level, and our solution structure works as well. Note that, we do not assume ergodicity of the system parameters, but if they are ergodic, then p_o represents the actual probability of a complete discharge event as $t \to \infty$.

In Problem (A), Constraints (1) and (2) describe how the data and battery queues evolve, respectively. Especially, if $B_d = \infty$, (1) can be simplified to

$$q_d(t+1) = \Big(q_d(t) - \mu\big(P(t)\big)\Big)^+ + R(t), \tag{1'}$$

and if $B_b = \infty$, (2) can be simplified to

$$q_b(t+1) = q_b(t) - P(t) + r(t). \tag{2'}$$

Constraint (3) bounds the actual amount of sensed data $R(t)$ by the amount of available data $A(t)$ in slot t. Constraint (4) states that we cannot oversubscribe the energy that is unavailable in the battery nor can we exceed the peak power level. Constraint (5) is the battery QoS constraint η_o of the desired battery discharge rate. Constraint (6) is the QoS constraint for data queue: if $B_d = \infty$, we need to keep the data queue stable, and if $B_d < \infty$, the data loss ratio is required under a given threshold η_d.

In this paper, our purpose is to develop a simple algorithm, which performs arbitrarily close to the optimal performance. To achieve that purpose, we do the following:

- define *virtual queues* for both the data and battery buffer to avoid the difficulties involved in dealing with the data loss and battery discharge probability directly. We show that keeping the virtual queues stable ensures that the constraints on p_d and p_o are met.
- design a power allocation scheme based on simple index policies and show that our scheme keeps both virtual queues stable and at the same time performs arbitrarily close to the optimal performance.
- generalize our algorithm to the multihop scenario and develop distributed algorithms.

B. Virtual Queues

We define $\tilde{q}_d$ and $\tilde{q}_b$ as the virtual data and battery queues. The virtual queues evolve according to the following Lindley's queue evolution equations:

$$\tilde{q}_d(t+1) = \left(\Big(\tilde{q}_d(t) - \eta_d R(t)\Big)^+ - \mu\big(P(t)\big) + R(t) + I(t)\right)^+, \tag{11}$$

$$\tilde{q}_b(t+1) = \left(\Big(\tilde{q}_b(t) - \eta_o\Big)^+ + P(t) - r(t) + M(t) + \mathit{1}_o(t)\right)^+, \tag{12}$$

where $I(t) = (\mu(P(t)) - q_d(t))^+$ is the amount of transmitted idle packets when there is no enough data to transmit using the allocated energy, $M(t) = (q_b(t) - P(t) + r(t) - B_b)^+$ is the amount of missed replenishing energy due to full battery when $B_b < \infty$, and $\mathit{1}_o(t)$ is defined in Equation (10). Note that if $B_b = \infty$, then $M(t) = 0$ and Equation (12) reduces to

$$\tilde{q}_b(t+1) = \left(\Big(\tilde{q}_b(t) - \eta_o\Big)^+ + P(t) - r(t) + \mathit{1}_o(t)\right)^+. \tag{13}$$

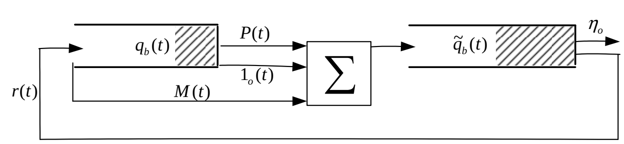

Fig. 2. Battery Queue and Virtual Battery Queue

Figure 2 shows the relationship between the actual and virtual battery queues when the battery size is finite. The amount of change in $q_b(t)$ from time t to $t+1$ is $P(t) - r(t) + M(t)$. Since the battery has a finite size, this term vanishes when averaged over an infinitely long period of time. Then, $p_o = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathit{1}_o(t)$ and η_o can be viewed as the long-term input and output rate of $\tilde{q}_b(t)$, respectively. Thus, it is reasonable to expect that $\tilde{q}_b(t)$ being stable implies $p_o \le \eta_o$.

Without loss of generality, the initial state $\tilde{q}_b(0)$ and $\tilde{q}_d(0)$ can be set to be zero. The following proposition shows that if the virtual queues $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are all strongly stable, p_d and p_o are guaranteed to meet their constraints.

Proposition 1: If the virtual queues $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are both strongly stable, i.e.,

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \Big(\tilde{q}_d(t) + \tilde{q}_b(t) + q_b(t)\Big) < \infty,$$

then $p_d \le \eta_d$ and $p_o \le \eta_o$.
The proof of this result can be found in Appendix A. Next, we present our scheme.

III. Joint Rate Control and Power Allocation Algorithm

A. Algorithm

The algorithm consists of two components: a *rate control* component and a *power allocation* component. Both components are *index policies*, i.e., the solutions are memoryless and they depend on the instantaneous values of the system variables.

Rate Control (RC):
We define $0 < V < \infty$ to be the control parameter of our algorithm. Let $Q_d(t) = q_d(t)$ when $B_d = \infty$, and let $Q_d(t) = (1-\eta_d)\tilde{q}_d(t)$ when $B_d < \infty$. If $Q_d(t) \le \frac{V}{2}$, the transmitting node chooses to sense all the available data, i.e., $R(t) = A(t)$. Otherwise, $R(t) = 0$.

Power Allocation (PA):
Solve the following optimization problem

$$\max_{P(t)\in\Pi(t)} Q_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t), \tag{14}$$

where $\Pi(t) = \{P(t) : 0 \le P(t) \le \min[q_b(t), P_{\text{peak}}]\}$ is a compact and nonempty set. Allocate $P(t)$.

The set, $\Pi(t)$ of possible power allocations guarantees Constraint (d) on $P(t)$ in Problem (A). If $\mu(\cdot)$ is concave[2], the objective function is a concave function of $P(t)$. Consequently, *PA* solves a simple convex optimization problem in each time slot. The positive term $Q_d(t)\mu(P(t))$ can be viewed as the utility of the allocated power $P(t)$ and the term $\tilde{q}_b(t)P(t)$ can be viewed as its associated cost. When the control parameter V is chosen to be large, $Q_d(t)$ tends to be large according to *RC*, and *PA* tries to allocate higher power $P(t)$ to increase the utility, whereas, when the virtual battery queue length $\tilde{q}_b(t)$ is large, *PA* avoids allocating a high amount of power to reduce the cost. Thus, this index policy of *PA* can be viewed as a *greedy profit maximization* scheme.
Observation 1: From the objective function in Equation (14), $Q_d(t)\mu\big(P(t)\big)$ can be viewed as the utility. When $Q_d(t)$ is large, i.e., the data queue or the virtual data queue is large, *PA* tends to allocate a larger power $P(t)$, so that $Q_d(t)$ will decrease.
Observation 2: Once the battery is discharged, i.e., $q_b(t) = 0$, $\tilde{q}_b(t)$ tends to increase as can be seen in Equation (12) or (13). Consequently, *PA* allocates a smaller power $P(t)$, so that the discharge event happens infrequently.
Observation 3: As $q_b(t)$ increases, $\tilde{q}_b(t)$ decreases, hence, *PA* tends to allocate a larger power $P(t)$ to avoid battery overflow in advance.
From *Observation 2* and *Observation 3*, *PA* expends energy neither too conservatively nor too aggressively.

[2] For instance, consider the additive white Gaussian noise channel capacity, $\mu(P) = \log(1 + P/N_0)$, where N_0 is the two sided noise power spectral density.

B. Performance Analysis

Recall that $A(t)$ is the available amount of sensing data and $R(t)$ is the actual amount of data the transmitting node chooses to sense. Clearly, using the rate controller, we make sure that the data queue remains within a certain bound. This has a positive effect on the battery as well, since a certain portion of the data packets are not allowed into the transmitting node. The natural question one would ask here is, whether our rate controller rejects too many packets in the first place to *synthetically* meet the constraints. In the following theorem, we show that this is not the case. Let $\lambda^* = \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R^*(t)$ be the optimal objective value of Problem (A) for any given exogenous process $\{A(t),\ t \ge 0\}$ and the replenishment process $\{r(t),\ t \ge 0\}$. In the following theorem, we show that the sensing rate achieved by *RC* and *PA* is asymptotically close to the optimal sensing rate λ^* as B_d and B_b tends to infinity. The optimal value can be achieved arbitrarily closely by increasing V. We use the notation $y = O(x)$ to represent y going to 0 as x goes to 0.

Theorem 1: If the following conditions hold:
(1) $\mu(\cdot)$ is concave on $\Re^+ \bigcup \{0\}$, and its slope at 0 satisfies[2] $0 \le \beta = \mu'(0) < \infty$,
(2) $0 < r(t) \le r_{\max}$, for all $t \ge 0$,
then the joint power allocation and admission control algorithm (with *RC* and *PA*) achieves:

$$Q_d(t) \le \frac{V}{2} + A_{\max}, \quad \forall\ t \ge 0 \tag{15}$$

$$\tilde{q}_b(t) \le \beta\left(\frac{V}{2} + A_{\max}\right), \quad \forall\ t \ge 0 \tag{16}$$

$$p_o \le \eta_o, \tag{17}$$

$$p_d \le \eta_d, \tag{18}$$

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t) \ge \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R^*(t) - O(\frac{1}{V}) - \eta_o(\mu_{\max} + \beta) - g(V, B_d, B_b), \tag{19}$$

where $\mu_{\max} = \mu(P_{\text{peak}})$ is the upper bound for the transmission rate, and

$$g(V,B_d,B_b) = \begin{cases} 0, & \text{if } B_d = \infty,\ B_b = \infty \\ O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}), & \text{if } B_d = \infty,\ B_b < \infty \\ O(\frac{(\frac{V}{2} - B_d)^+}{V}), & \text{if } B_d < \infty,\ B_b = \infty \\ O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}) + O(\frac{(\frac{V}{2} - B_d)^+}{V}), & \\ & \text{if } B_d < \infty,\ B_b < \infty. \end{cases}$$

The proof of Theorem 1 can be found in Appendix B. In Theorem 1, V is a finite tunable approximation parameter that controls the efficiency of the algorithm. Observe Equation (19), which compares the performance of our algorithm with that of the optimal solution of Problem (A), the term $\eta_o(\mu_{\text{max}} + \beta)$ captures the influence of battery outage, and it is small since the battery outage threshold η_o is usually set to be very small to avoid network disconnection. Function $g(V, B_d, B_b)$ represents the asymptotical property of the gap. If both battery and the data buffers are of infinite size, then the performance gap is identical to 0. Otherwise the performance gap is dictated by the finite (or the smaller) one of the buffers. One can observe that, by appropriately choosing the V-parameter, we can make the performance gap decay inversely proportional to the buffer sizes.

IV. Multihop Network Model

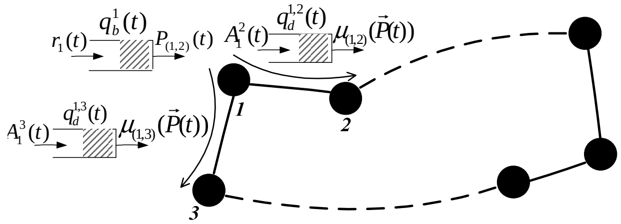

Fig. 3. Multihop Network Model

We consider a multihop wireless sensor network with N nodes and L links as illustrated in Figure 3. Each node $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ is attached to power sources for replenishment. Let $A_n^e(t)$ and $R_n^e(t)$ denote the amount of available data for sensing and the actual amount of sensed data, to node n that are destined to node e in slot t. We assume that each node n maintains an infinite data buffer with state $q_d^{n,e}(t)$ for flows destined to e, and also maintains a finite battery buffer with size B_b^n and state $q_b^n(t)$ (We focus on infinite data buffer and finite battery buffer in order to emphasize the difference from single link to multihop. For other combinations of data and battery buffer sizes, then extension can be made similarly without too much effort). Let $r_n(t)$ denote the replenishment at node n in time slot t. The transmit power is chosen to be $P_l(t)$ over link l. In the formulation, we assume that the power the receiving node consumes to receive and decode the packet is identical to $P_l(t)$ as well. The sole reason for this is simplicity and the generalization to the asymmetric case is straightforward (By defining a receiving transmitting power ratio, we can extend it to the general case with no technique difference). We use the node-exclusive interference model. Under this model, a node can only receive from or transmit to at most one node at any time slot. In each time slot t, with the assigned power $P_l(t)$, the achieved data rate at link l is $\mu_l(P_l(t))$ in that time slot, where the rate function $\mu_l(\cdot)$ is a non-decreasing, concave and differentiable function satisfying $\mu_l(0) = 0$. Let p_o^n be the frequency of visits to the zero battery state for node n. Let Ω_n and Θ_n denote the set of directed links originated from node n and terminate at node n, respectively. We say $\vec{P} = [P_1(t) \cdots P_L(t)]$ satisfies the node-exclusive model if $P_l(t) > 0$ for some $l \in \Omega_n \cup \Theta_n$, then $P_{l'} = 0$ for all $l' \in (\Omega_n \cup \Theta_n) \setminus \{l\}$. In a multihop network, we formulate the joint queue and energy management problem as follows:

$$(B) \qquad \max_{\vec{P}, \vec{R}} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{n,e\in\mathcal{N}} R_n^e(t)$$

$$\text{s.t. } \vec{P}(t) \text{ satisfies the node-exclusive model,} \tag{20}$$

$$q_d^{n,e}(t+1) \le \Big(q_d^{n,e}(t) - \sum_{l\in\Omega_n} \mu_l^e\big(P_l(t)\big)\Big)^+ + R_n^e(t) + \sum_{l\in\Theta_n} \mu_l^e(P_l(t)), \quad n \neq e, \tag{21}$$

$$q_b^n(t+1) = \min\Big[q_b^n(t) - \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) + r_n(t), B_b^n\Big], \tag{22}$$

$$0 \le \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) \le \min\Big[q_b^n(t), P_{\text{peak}}\Big], \tag{23}$$

$$\sum_{e=1}^{N} \mu_l^e\big(P_l(t)\big) = \mu_l\big(P_l(t)\big),\ R_n^e(t) \le A_n^e(t), \tag{24}$$

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} q_d^{n,e}(t) < \infty, \quad n \neq e, \tag{25}$$

$$p_o^n \le \eta_o^n, \tag{26}$$

where η_o^n is the desired upper bound for p_o^n, $\vec{P}(t)$ is the power assignment vector for all links in slot t, $\vec{P}$ is the power assignment for all links over all time slots, and $\vec{R}$ is the actual sensing data vector for all node-destination pairs over all time slots. In Problem (B), the objective is to maximize the long-term average total sensing rate for all nodes destined to all destinations.

In Problem (B), (20) is the interference constraint. Constraints (21) and (22) describe how the data and battery queues evolve, respectively. Note that the destination node of each flow does not need to maintain a data buffer for that flow, as indicated in (21). Constraints (23) are the energy conservation equations stating that we cannot oversubscribe the energy that is unavailable in the battery nor can we exceed the peak power level. Constraints (24) are the rate conservation equations that bound the actual amount of sensed data $R_n^e(t)$ by the available amount of data $A_n^e(t)$, and share the transmission rate of a link among all the destinations in slot t. Constraint (25) is the QoS constraint for data queue: we need to keep all the data queues stable. Constraint (26) is the battery QoS constraint of the desired battery discharge rate η_o^n.

Similarly, we define virtual queues for all $n \in \mathcal{N}$

$$\tilde{q}_b^n(t+1) = \Big(\Big(\tilde{q}_b^n(t) - \eta_o^n\Big)^+ + \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) - r_n(t) + M_n(t) + 1_o^n(t)\Big)^+, \tag{27}$$

where $M_n(t) = \left(q_b^n(t) - \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) + r_n(t) - B_b^n\right)^+$ is the amount of missed replenishment and

$$1_o^n(t) = \textit{indicator that battery hits zero state in slot } t \textit{ for node } n = \begin{cases} 0 & \text{if } \sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) < q_b^n(t) \\ 1 & \text{otherwise} \end{cases}$$

Next, we generalize the main results and algorithms *RC* and *PA* to the multihop scenario.

Corollary 1: If all the virtual battery queues $\tilde{q}_b^n(t),\ \forall n \in \mathcal{N}$ are strongly stable, we have $p_o^n \le \eta_o^n,\ \forall n \in \mathcal{N}$.
The proof is identical to the single hop scenario and can be found in Appendix C. We give the algorithm in the following section.

V. Joint Rate Control, Power Allocation and Routing Algorithm for Multihop Networks

The joint rate control, power allocation and routing algorithm for multihop networks can either be implemented in a centralized or distributed manner. For the centralized solution, we use the classical Maximal Weighted Matching (MWM) based algorithm and for the distributed algorithm, we can use the Maximal Matching (MM) based algorithms as in [19] [20].

Multihop Rate Control (MRC):
Depending on whether Maximum Weight Matching or Maximal Matching is employed by the scheduler, there is a slight difference in the implementation of MRC:
Maximum Weighted Matching (MWM): If $q_d^{n,e}(t) \le \frac{V}{2}$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.
Maximal Matching (MM): If $q_d^{n,e}(t) \le V$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.

Multihop Power Allocation (MPA):
Here the goal is to ensure that no node transfers data of a flow to a relay node that is not the destination of that flow, unless the differential backlog for that flow is greater than a fixed value $\gamma > 0$. We will choose the value of γ such that the resulting backlog of the receiving node is not larger than that of the transmitting node after the transmission. This pushes the data flow from the source to the destination with a positive back pressure. Let tran(l) and rec(l) denote the transmitting and receiving node of link l, respectively. We first define

$$\gamma_l^e = \begin{cases} \gamma & \text{if rec}(l) \ne e \\ 0 & \text{otherwise} \end{cases}$$

where $\gamma > 0$ is some constant. Let $e_l(t) = \arg\max_e \left\{ q_d^{\text{tran}(l),e}(t) - q_d^{\text{rec}(l),e}(t) - \gamma_l^e \right\}$ be the flow on link l that has the maximal modified differential backlog, and $w_l(t) = \max\left[q_d^{\text{tran}(l),e_l(t)}(t) - q_d^{\text{rec}(l),e_l(t)}(t) - \gamma_l^{e_l(t)}, 0\right]$ is the nonnegative differential backlog of l at time t.

For each link l, solve

$$\max_{P_l(t)\in\Pi_l(t)} w_l(t)\mu_l\big(P_l(t)\big) - \Big(\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t)\Big) P_l(t) \tag{28}$$

where $\Pi_l(t) = \Big\{ P_l(t) : 0 \le P_l(t) \le \min\left[q_b^{\text{tran}(l)}(t), q_b^{\text{rec}(l)}(t), P_{\text{peak}} \right] \Big\}$.

Let $P_l^s(t)$ be the solution for link l. With the calculated power $P_l^s(t)$, let $W_l(t) = w_l(t)\mu_l\big(P_l^s(t)\big) - \Big(\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t)\Big) P_l^s(t)$ be the weight on link l.

For the whole network, we either use the MWM or MM as described below. We will analyze the performance of our solution when using each algorithm.
Maximum Weighted Matching Algorithm: link l has weight $W_l(t)$, then the weight of a matching $\mathcal{M}$ is $W_{\mathcal{M}}(t) = \sum_{l\in\mathcal{M}} W_l(t)$. The network chooses a maximum weighted matching in a centralized manner, the links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Maximal Matching Algorithm: the network calculates a maximal matching that achieves at least half of the total weight of MWM in a fully distributed manner as in [19] [20]. The links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Multihop Routing:
When $w_l(t) > 0$, transmit for flow that is destined to $e_l(t)$ with rate $\mu_l\big(P_l(t)\big)$, i.e., $\mu_l^{e_l(t)}\big(P_l(t)\big) = \mu_l\big(P_l(t)\big)$ and $\mu_l^e\big(P_l(t)\big) = 0,\ \forall e \ne e_l(t)$.

Note that *MRC* and routing can be done by each node independently. We then give our main theorem for the multihop scenario:

Theorem 2: If
(1) $\mu_l(\cdot)$ is concave on $\Re^+ \bigcup\{0\}$, and its slope at 0 satisfies $0 \le \beta = \mu_l'(0) < \infty,\ \forall l \in \mathcal{L}$,

(2) $\forall n \in \mathcal{N}$: $0 < r_n(t) \le r_{\max},\ \forall t \ge 0$,

then the maximum weighted matching based joint rate control *MRC*, power allocation *MPA*, and routing algorithm achieves:

$$q_d^{n,e}(t) \le \frac{V}{2} + A_{\max}, \tag{29}$$

$$\tilde{q}_b^n(t) \le \beta\Big(\frac{V}{2} + A_{\max}\Big), \tag{30}$$

$$\begin{aligned} &\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R_n^e(t) \right\} \\ &\ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R_n^{e*}(t) - \eta_o^n(\mu_{\max} + \beta) - O(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}) \right\} - O(\frac{1}{V}), \end{aligned} \tag{31}$$

and the maximal matching based joint rate control *MRC*, power allocation *MPA*, and routing algorithm achieves:

$$q_d^{n,e}(t) \le V + A_{\max}, \tag{32}$$

$$\tilde{q}_b^n(t) \le \beta(V + A_{\max}), \tag{33}$$

$$\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R_n^e(t) \right\}$$
$$\ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e \frac{R_n^{e*}(t)}{2} - \eta_o^n(\mu_{\max} + \beta) \right.$$
$$\left. - O\Big(\frac{(\beta V - B_b^n)^+}{V}\Big) \right\} - O\Big(\frac{1}{V}\Big). \tag{34}$$

The proof of Theorem 2 can be found in Appendix D. The results can be interpreted similarly as Theorem 1.

VI. Numerical Example

We consider a network topology, shown in Figure 4 (a). There are 6 nodes, 7 links, and 2 flows with source-destination pair $(3, 1)$ and $(5, 2)$, respectively. In all simulations, the simulation time is $T = 10^6$ time slots. We use the rate power function $\mu_l(P_l) = 10\log_2(1 + \frac{g_l P_l}{N_l})$ packets/slot $\forall l \in \mathcal{L}$. Let the power of the background noise $N_l = 1.6 \times 10^{-14}W$, $\forall l \in \mathcal{L}$, and the channel gains $g_l = 1.6 \times 10^{-13}$, $\forall l \in \mathcal{L}$. Each node is equipped with an infinite data buffer for each flow through it. The number of arrivals $A_3^1(t),\ t \ge 0$ and $A_5^2(t),\ t \ge 0$, are modeled as independent Poisson random variables with mean $\lambda = 20$ packets/slot and $A_{\max} = 30$ packets/slot. We set η_o^n, the threshold of battery outage probability to 0.03 for all $n \in \mathcal{N}$ and the peak power $P_{\text{peak}} = 1.5W$. The backlog threshold $\gamma = 80 \ge \max_{n,e}\left(\sum_{\text{rec}(l)=n} \mu_l + A_n^e\right) = 2 \times 10\log_2(1 + 10P_{\text{peak}})$, so that the resulting backlog of the receiving node is not longer than that of the transmitting node.

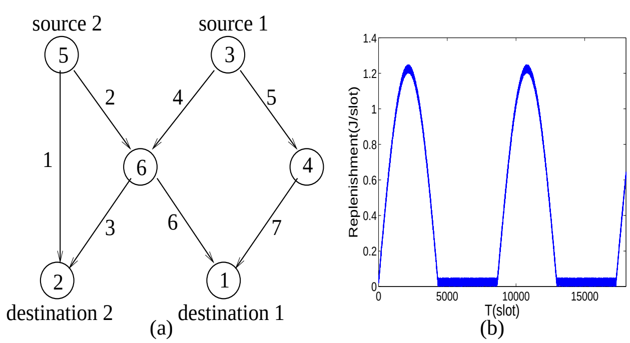

Fig. 4. (a) Network topology and (b) a sample replenishing process

Scenario 1: We first use a replenishment process which is formed by a periodic deterministic sine waveform ($r_{\max} = 1.2$ and period 8000) plus independent Gaussian noise with zero mean and variance 0.01, as shown in Figure 4 (b) (The cycles imitate the daily solar cycles for a solar battery and the average replenishing can be simply calculated $\bar{r} = 0.2$). All the battery buffer sizes are set to be $B_b = 100J$. We simulate both MWM based and MM based algorithms. We choose different values of the control coefficient V for the proposed algorithm and compare the results with the optimal value[3]. From Figure 5 (a), we see that as V increases, the average total sensing rates of the MWM and MM based algorithm keep increasing and get closer to the optimum and a value that is much larger than half optimum, respectively. This is consistent with Equation (31) and Equation (34). From Figure 5 (b), we see that as V increases, the average data queue length (we here only plot the data queue length of node 3 for flow 1 due to space limitation) keeps increasing but is upper bounded by the bound we get in Equation (29) and Equation (32). This means the queueing delay increases as we improve the sensing rate, which can be viewed as a tradeoff. From Figure 5 (c) we observe that the battery discharge probability (we only plot for node 5 here) increases to the threshold as V increases.

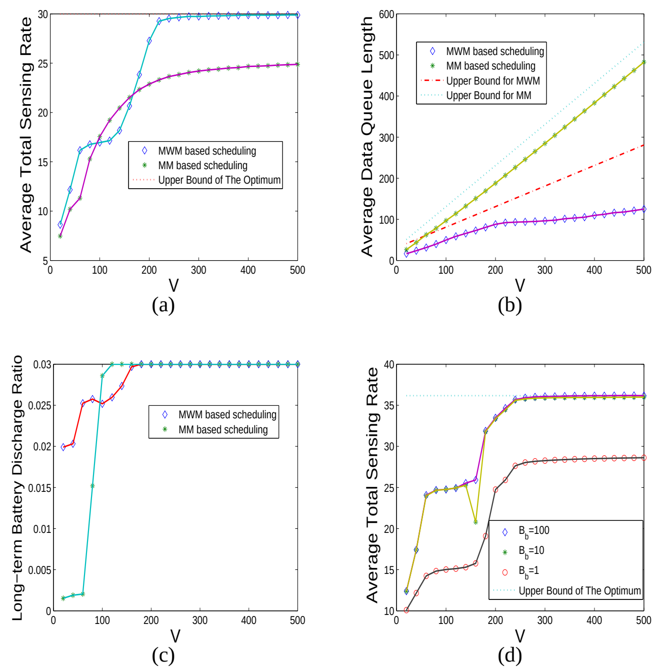

Fig. 5. Performance of the MWM and MM based algorithms. Impact of the control parameter V on (a) the average total sensing rate, (b) average data queue length, and (c) the battery discharge probability for *Scenario 1*. Impact of battery size on (d) the average total sensing rate for *Scenario 2*.

Scenario 2: We use different replenishment processes: $r_2(t)$ and $r_5(t)$ are i.i.d Bernoulli random variables *Bernoulli*(0.5) (i.e., $r_2(t) = 1$ w.p. 0.5 and $r_2(t) = 0$ w.p. 0.5); replenishing at all other nodes are independent Bernoulli random variables $0.2 \times$ *Bernoulli*(0.5) in even number slots, and $0.6 \times$ *Bernoulli*(0.5) in odd number slots ($\bar{r}_2 = \bar{r}_5 = 0.5$ and $\bar{r} = 0.2$ for other nodes), all plus Gaussian noise with zero mean and variance 0.01. This replenishing process is faster time-varying than the one in *Scenario 1*. We simulate[3] three

[3] The exact optimal objective value for Problem (B) is hard to obtain, so we here use an upper bound for the optimum. For this example, an upper bound for the optimum can be obtained by equal time sharing of schedules $\{1, 4, 7\}$ and $\{1, 5, 6\}$, and utilizing the link rate $\mu(\bar{r})$ under infinite battery size and no discharge constraint. In *Scenario 1*, the optimum is $2\mu(\bar{r}) = 30$. In *Scenario 2*, since $\mu(\bar{r}_2) = \mu(\bar{r}_5) > \lambda$, the optimum is $\lambda + \mu(\bar{r}_3) = 35$.

different battery sizes $B_b = 100J$, $B_b = 10J$ and $B_b = 1J$ (all nodes has the same battery sizes) for the MWM based algorithm. From Figure. 5 (d), we can see that the sensing rate increases as battery size increases. However, as long as the battery is large compared to the average replenishing rate, the improvement diminishes with increasing battery sizes.

Scenario 3: We simulate two tree topologies as shown in Figure 6 (a) and 6 (b). For both topologies, the replenishment processes for nodes $1, 2, 3$, i.e., $r_1(t), r_2(t)$ and $r_3(t)$ are i.i.d Bernoulli random variables *Bernoulli*(0.5); replenishment at all other nodes are as shown in Figure 4 (b). Further, Gaussian noise with zero mean and variance 0.01 are added to all replenishment process. The battery size for all nodes are $B_b = 200J$. Data arrives at leaf nodes $4, 5, 6$, destined to root node 1. The arrival processes $A_4^1(t),\ t \geq 0$ and $A_5^1(t),\ t \geq 0$, are independent truncated Poisson with mean $\lambda_4 = \lambda_5 = 5$ packets/slot and $A_{\text{max}} = 20$ packets/slot; and the arrival process $A_6^1(t),\ t \geq 0$ is an independent truncated Poisson process with mean $\lambda_6 = 10$ packets/slot and $A_{\text{max}} = 30$ packets/slot. From Figure 6 (c) and (d), we can see that the MWM based algorithm perform close to optimum and MM based algorithms perform much better than half the optimal performance[4]. Note that the MM algorithm for Figure 6 (a) achieves almost 100% of the optimum and MM algorithm for the topology in Figure 6 (b) achieves 80%, which is still higher than the performance (75%) for topology in Figure 4 (a) as shown in Figure 5 (a). The reasons are as follows: The number of schedules for the topology in Figure 6 (b) is more than that of Figure 6 (a); the tree topologies have fixed routing for each data flow, but for a general multihop topology such as the one in Figure 4 (a), there are multiple routing possibilities for each data flow.

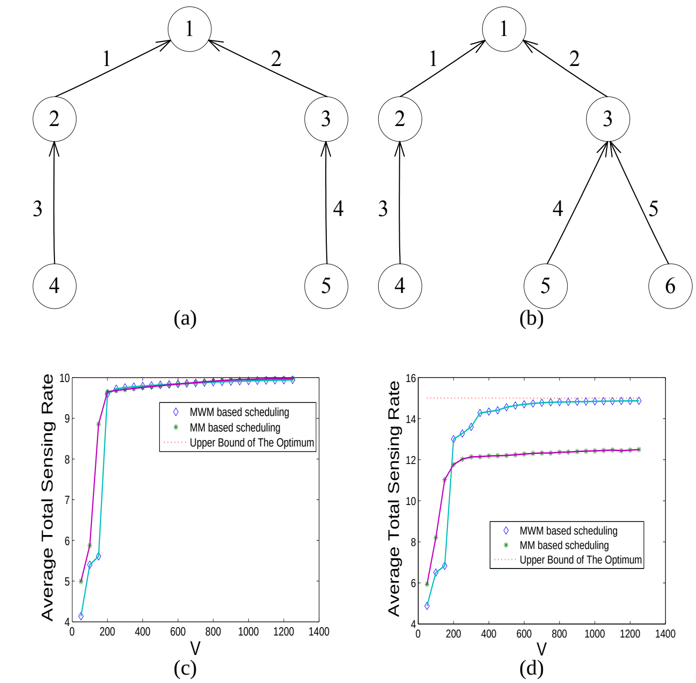

Fig. 6. Performance of the MWM and MM based algorithms for tree topology networks. Impact of the control parameter V on (c) and (d) the average total sensing rate for topology (a) and (b), respectively.

VII. Conclusion

In this paper, we studied the problem of energy management in rechargeable wireless sensor networks. Our objective was to maximize the average data sensing rate subject to QoS constraints on both data and battery queues. We provided a simple and unified framework of joint rate control and power allocation for all combinations of finite and infinite data and battery buffer sizes. We showed through both analysis and simulation that the performance of our strategy is close to that of the optimal solution. We extended our algorithm to the multihop scenario and showed that simple extensions of our index schemes for single hop generalize to the multihop scenario with a similar, close-to-optimal performance. We developed a distributed joint rate control, power allocation and routing algorithm for multihop networks under node-exclusive interference model. Although we do not consider channel variations in this paper, taking channel variations into consideration is technically straightforward. Our multihop network formulation does not consider the fairness among links, however, the opportunistic scheduling framework [21] used to resolve the fairness issue can be applied in this context. For future work, we are extending this framework to the general interference model where the rate power function is not concave.

[4]Note that the average replenishing rate of nodes $1, 2, 3$ are twice of the average replenishing rate of nodes $4, 5, 6$, so the replenishment processes of $4, 5, 6$ determines the upper bound of the optimum of total sensing rate. Furthermore, all leaf nodes are time sharing the transmission opportunity, then for topology in Figure 6 (a). Consequently, the upper bound of optimal total average sensing rate is 10, since $\lambda_4 + \lambda_5 = 10 < \mu(\bar{r}) = 15$, where $\bar{r}$ is the average replenishment rate in Figure 4 (b). Similarly, for the topology in Figure 6 (b), the upper bound of optimal total average sensing rate is 15 since $\lambda_4 + \lambda_5 + \lambda_6 = 20 > \mu(\bar{r}) = 15$.

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APPENDIX

A. Proof of Proposition 1

Using the idea similar to [17], we have the fact that if any queue represented with $Q(t)$ is strongly stable, then $\limsup_{T\to\infty}\frac{Q(T)}{T}=0$. Hence, if $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and $q_b(t)$ are strongly stable, then $\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T}=0$, $\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T}=0$ and $\limsup_{T\to\infty}\frac{q_b(T)}{T}=0$.

From Equation (11), we have $\tilde{q}_d(t+1)\geq\tilde{q}_d(t)-\eta_d R(t)+D(t)-D(t)-\mu(P(t))+R(t)+I(t)$. Note that $q_d(t+1)=q_d(t)-\mu(P(t))+I(t)+R(t)-D(t)$. By adding from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$ on both sides, we have

$$\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T}\geq\lim_{T\to\infty}\frac{\tilde{q}_d(0)}{T}+\lim_{T\to\infty}\frac{q_d(T)-q_d(0)}{T}+\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\Big[D(t)-\eta_d R(t)\Big].$$

Since $\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T}=0$, $\lim_{T\to\infty}\frac{\tilde{q}_d(0)}{T}=0$, and $\lim_{T\to\infty}\frac{q_d(T)-q_d(0)}{T}=0$, so we get $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}[D(t)-\eta_d R(t)]\leq 0$. Thus,

$$\begin{aligned}p_d-\eta_d&=\limsup_{T\to\infty}\left[\frac{\frac{1}{T}\sum_{t=0}^{T-1}D(t)}{\frac{1}{T}\sum_{t=0}^{T-1}R(t)}-\eta_d\right]\\&=\limsup_{T\to\infty}\left[\frac{\frac{1}{T}\sum_{t=0}^{T-1}[D(t)-\eta_d R(t)]}{\frac{1}{T}\sum_{t=0}^{T-1}R(t)}\right]\leq 0,\end{aligned}$$

i.e., $p_d\leq\eta_d$.

Similarly, from Equation (12), we have $\tilde{q}_b(t+1)\geq\tilde{q}_b(t)-\eta_o+P(t)-r(t)+M(t)+1_o(t)$. Note that $q_b(t+1)=q_b(t)-P(t)+r(t)-M(t)$. By adding from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$ on both sides, we have

$$\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T}\geq\lim_{T\to\infty}\frac{\tilde{q}_b(0)}{T}-\eta_o+\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}1_o(t)+\lim_{T\to\infty}\frac{q_b(0)-q_b(T)}{T}.$$

Since $\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T}=0$, $\lim_{T\to\infty}\frac{\tilde{q}_b(0)}{T}=0$, and $\lim_{T\to\infty}\frac{q_b(0)-q_b(T)}{T}=0$, so we get $p_o=\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}1_o(t)\leq\eta_o$. ■

B. Proof of Theorem 1

Proof of Equation (15): Note that $I(t)\leq\mu(P(t))$ and $R(t)\leq A_{\max}$. The rate allocation unit *RC* is chosen to satisfy Equation (15).

Proof of Equation (16): Since $\mu(\cdot)$ is concave on $\Re^+\bigcup\{0\}$, we have $\mu(P(t))\leq\mu(0)+\beta P(t)$ for $P(t)\in\Pi(t),\ \forall t\geq 0$, where $0\leq\beta=\mu'(0)<\infty$. Then, $Q_d(t)\mu(P(t))-\tilde{q}_b(t)P(t)\leq Q_d(t)\mu(0)+\beta Q_d(t)P(t)-\tilde{q}_b(t)P(t)$ where $P(t)$ is the solution of *PA*.

If $\beta Q_d(t)P(t)-\tilde{q}_b(t)P(t)<0$, then we get $Q_d(t)\mu(P(t))-\tilde{q}_b(t)P(t)<Q_d(t)\mu(0)$. However, *PA* chooses $P(t)$ that maximizes $Q_d(t)\mu(P(t))-\tilde{q}_b(t)P(t)$ which means $Q_d(t)\mu(P(t))-\tilde{q}_b(t)P(t)\geq Q_d(t)\mu(0)$ since $0\in\Pi(t)$. Then we must have $\beta Q_d(t)P(t)-\tilde{q}_b(t)P(t)\geq 0$, i.e., $\beta Q_d(t)P(t)\geq\tilde{q}_b(t)P(t)$.

We now prove Equation (16) by induction. Without loss of generality, let $\tilde{q}_b(0)\leq\beta(\frac{V}{2}+A_{\max})$. Suppose for all $t\geq 1$, $\tilde{q}_b(t-1)\leq\beta(\frac{V}{2}+A_{\max})$ holds. In slot t, if $P(t)>0$, then $\tilde{q}_b(t)\leq\beta Q_d(t)\leq\beta(\frac{V}{2}+A_{\max})$. Otherwise, $P(t)=0$, from Equation (10) and the assumption $r(t)>0,\ \forall t\geq 0$, we have $1_o(t)=0$. We also note that $M(t)\leq r(t)$, then $\tilde{q}_b(t)\leq\tilde{q}_b(t-1)\leq\beta(\frac{V}{2}+A_{\max})$. Therefore, $\tilde{q}_b(t)\leq\beta(\frac{V}{2}+A_{\max})$ for all t, which is Equation (16).

Proof of Equation (17) and Equation (18): If $p_o=0$, we are done. In the following analysis, we assume $p_o>0$.

From Equation (16), we already have $\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T}=0$. In order to apply Proposition 1, we only need to show $\limsup_{T\to\infty}\frac{q_b(T)}{T}=0$.

Claim: For any $t\geq 0$, there exists $t'(t)>t$ such that $q_b(t'+1)=r(t')$ and $t'-t<\infty$. Suppose there exists $t\geq 0$ such that for any $t'>t$ with $q_b(t'+1)=r(t')$, $t'-t=\infty$. This means that $\exists N<\infty$ such that $q_b(t+1)>r(t),\ \forall t>N$, i.e., $1_o(t)=0,\ \forall t>N$. Thus, $p_o=\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}1_o(t)=\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{N}1_o(t)=0$, which leads to a contradiction.

Since $r(t) \leq r_{\max} < \infty,\ \forall t \geq 0$, and $q_b(t) - q_b(t+1) \leq P(t) \leq P_{\text{peak}}$. Combining with the above claim, we have $q_b(t) \leq r_{\max} + (t'(t) - t)P_{\text{peak}} < \infty,\ \forall t \geq 0$. Therefore, $\limsup_{T\to\infty} \frac{q_b(T)}{T} = 0$.

Equation (17) then follows from Proposition 1.

Equation (18) directly follows from Equation (15) Proposition 1.

Proof of Equation (19):

I) $B_d = \infty$ *and* $B_b = \infty$.

We define the Lyapunov function $L\big(q_d(t), \tilde{q}_b(t)\big) = q_d^2(t) + \tilde{q}_b^2(t)$, and $\Delta\big(q_d(t), \tilde{q}_b(t)\big) = L\big(q_d(t+1), \tilde{q}_b(t+1)\big) - L\big(q_d(t), \tilde{q}_b(t)\big)$, where $q_d(t)$ and $\tilde{q}_b(t)$ are the resulting queue dynamics of our algorithm given a sample path. Note that our problem is a sample path optimization problem, so $L\big(q_d(t), \tilde{q}_b(t)\big)$ is defined without conditional expectation.

From Equation (13), we have $\tilde{q}_b^2(t+1) \leq (\tilde{q}_b(t) - \eta_o)^2 + (1_o(t) + P(t) - r(t))^2 + 2(\tilde{q}_b(t) - \eta_o)^+(1_o(t) + P(t) - r(t))$. Also from the data queue dynamics, we have $q_d^2(t+1) \leq q_d^2(t) + \mu^2(P(t)) + R^2(t) + 2q_d(t)R(t) - 2q_d(t)\mu(P(t))$, then

$$\begin{aligned}\Delta =&\Delta\big(q_d(t), \tilde{q}_b(t)\big)\\ \leq&\mu^2(P(t)) + R^2(t) + (r_{\max} + \eta_o)^2 - 2q_d(t)\mu(P(t)) +\\ &(1 + P_{\text{peak}})^2 + 2q_d(t)R(t) + 2\tilde{q}_b(t)\big(1_o(t) + P(t) - r(t)\big)\\ \leq&(1 + P_{\text{peak}})^2 + (r_{\max} + \eta_o)^2 + \mu_{\max}^2 + A_{\max}^2 + VR(t) +\\ &2\tilde{q}_b(t)\big(1_o(t) - r(t)\big) + 2\big[q_d(t) - V/2\big]R(t) -\\ &2\big[q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)\big].\end{aligned}$$

It is apparent that *RC* is trying to minimize the term $\big[q_d(t) - V/2\big]R(t)$, and *PA* is trying to maximize the value of the term $\big[q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)\big]$. Since the optimal solution for Problem (A) may not be unique, we let $\mathcal{P}^*$ be the optimal solution set and $P^* \in \mathcal{P}^*$ be any optimal solution, for Problem (A) given any sample path. Since the constraint set $\Pi(t)$ is queue dynamic related, it is possible that $P^*(t) \notin \Pi(t)$.

Lemma 1: If by solving Equation (14), we get $\big[Q_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t)\big] < \big[Q_d(t)\mu\big(P^*(t)\big) - \tilde{q}_b(t)P^*(t)\big]$, then $P(t) = q_b(t)$ and $1_o(t) = 1$.

Proof: In time slot t, let $P^m(t)$ be the value that maximizes the unconstrained objective function $Q_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t)$.

Claim 1: $q_b(t) < P_{\text{peak}}$. Otherwise, $\Pi(t) = [0, P_{\text{peak}}]$ which is not queue dynamic related, then $\big[Q_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t)\big] \geq \big[Q_d(t)\mu\big(P^*(t)\big) - \tilde{q}_b(t)P^*(t)\big]$.

Claim 2: $P^m(t), P^*(t) > q_b(t)$. If $P^m(t), P^*(t) \notin \Pi(t)$, we must have $\big[Q_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t)\big] \geq \big[Q_d(t)\mu\big(P^*(t)\big) - \tilde{q}_b(t)P^*(t)\big]$, then $P^*(t) > q_b(t)$. If $P^m(t) < 0$, we will have $Q_d(t)\mu(0) = \big[Q_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t)\big] \geq \big[Q_d(t)\mu\big(P^*(t)\big) - \tilde{q}_b(t)P^*(t)\big]$, then $P^m(t) > q_b(t)$.

By the above claims, $P(t) < P^*(t)$ and $P(t) < P^m(t)$. Suppose $P(t) < q_b(t)$, since the objective function of *PA* is concave in $P(t)$ and $P^m(t) > q_b(t)$, we can increase $P(t)$ to increase the objective without violating the constraint. Thus, $P(t) = q_b(t)$ and $1_o(t) = 1$. ■

By Lemma 1, we have

$$\begin{aligned}\Delta \leq&(1 + P_{\text{peak}})^2 + (r_{\max} + \eta_o)^2 + \mu_{\max}^2 + A_{\max}^2 + VR(t) -\\ &VR^*(t) + (V + 2A_{\max})(\beta + \mu_{\max})1_o(t) +\\ &2q_d(t)\big[R^*(t) - \mu\big(P^*(t)\big)\big] + 2\tilde{q}_b(t)\big[P^*(t) - r(t)\big].\end{aligned} \tag{35}$$

The following lemma gives the sufficient condition for strong stability under sample mean argument.

Lemma 2: For any queue with evolution $q(t+1) = \big(q(t) - \mu(t)\big)^+ + \lambda(t)$, if $q(0) < \infty$, $\mu(t) \leq \mu_{\max},\ \forall t \geq 0$ and $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\big(\lambda(t) - \mu(t)\big) < 0$, then $q(t)$ is strongly stable, i.e., $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q(t) < \infty$.

Proof: Without loss of generality, assume $q(0) = 0$. Let $T_0 = 0$, $T_1 = \min\Big\{t > 0 :\ q(t) \leq \mu(t)\Big\}$, $T_2 = \min\Big\{t > T_1 :\ q(t) \leq \mu(t)\Big\}$,..., $T_k = \min\Big\{t > T_{k-1} :\ q(t) \leq \mu(t)\Big\}$,..., and so on. Let $N_k = T_k - T_{k-1},\ \forall k \geq 1$. We first claim $N_k < \infty,\ \forall k \geq 1$. Suppose there exits $1 \leq k < \infty$ such that $N_k = \infty$, then it means $\sum_{t=T_{k-1}}^{T}\big(\lambda(t) - \mu(t+1)\big) > 0,\ \forall T > T_{k-1}$, and as a result, $\liminf_{T\to\infty} \frac{1}{T}\sum_{t=T_{k-1}}^{T-1}\big(\lambda(t) - \mu(t+1)\big) \geq 0$. However,

$$\begin{aligned}&\sum_{t=T_{k-1}}^{T-1}\big(\lambda(t) - \mu(t+1)\big)\\ =&\sum_{t=0}^{T-1}\big(\lambda(t) - \mu(t)\big) - \sum_{t=0}^{T_{k-1}-1}\big(\lambda(t) - \mu(t)\big)\\ &+ \mu(T_{k-1}) - \mu(T)\\ &/*\ \text{In each of the } k \text{ segaments, the sum of the arrival}\\ &\text{departure difference becomes negative once, and the}\\ &\text{negative drift is no smaller than } -\mu_{\max} */\\ \leq&\sum_{t=0}^{T-1}\big(\lambda(t) - \mu(t)\big) + k\mu_{\max} + \mu(T_{k-1}) - \mu(T),\end{aligned}$$

then,

$$\begin{aligned}\sum_{t=T_{k-1}}^{T-1}\big(\lambda(t) - \mu(t+1)\big) \leq&\sum_{t=0}^{T-1}\big(\lambda(t) - \mu(t)\big)\\ &+ k\mu_{\max} + \mu(T_{k-1}) - \mu(T).\end{aligned}$$

By dividing by T and taking $\limsup_{T\to\infty}$ of both sides, we have

$$\begin{aligned}
&\liminf_{T\to\infty} \frac{1}{T} \sum_{t=T_{k-1}}^{T-1} \big(\lambda(t) - \mu(t+1)\big) \\
\le &\limsup_{T\to\infty} \frac{1}{T} \sum_{t=T_{k-1}}^{T-1} \big(\lambda(t) - \mu(t+1)\big) \\
\le &\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \big(\lambda(t) - \mu(t)\big) + \\
&\limsup_{T\to\infty} \frac{k\mu_{\max} + \mu(T_{k-1}) - \mu(T)}{T} \\
< &0,
\end{aligned}$$

which is a contradiction.

Let $N = \max_{k\ge1} N_k$, then $N < \infty$. This implies $q(t) \le N\mu_{\max}$, $\forall t \ge 0$. Now we suppose $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q(t) = \infty$. This means there exists a subsequence of times $\{t_n\}$ such that $t_n \to \infty$ and $\frac{1}{t_n}\sum_{t=0}^{t_n-1} q(t) \to \infty$. In other words, $\forall Q > 0$, $\exists\ T(Q) \in \{t_n\}$ such that $\forall t_k \in \{t_n\}$ and $t_k \ge T(Q)$, we have $\frac{1}{t_k}\sum_{t=0}^{t_k-1} q(t) > Q$. Let $Q = N\mu_{\max}$, and find the corresponding $T(N\mu_{\max})$ such that $\frac{1}{T(N\mu_{\max})}\sum_{t=0}^{T(N\mu_{\max})-1} q(t) > N\mu_{\max}$. Then there must exists $0 \le \tilde{T} \le T(N\mu_{\max})$ with $q(\tilde{T}) > N\mu_{\max}$ which leads to a contradiction. Therefore, $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q(t) < \infty$. ■

Lemma 3:

$$\frac{1}{V}\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \tilde{q}_b(t)\big[P^*(t) - r(t)\big] \le O(\frac{1}{V}).$$

Proof: Note that

$$\sum_{t=0}^{T-1} P^*(t) - \Big[q_b(0) + \sum_{t=0}^{T-1} r(t)\Big] \le 0,$$

then

$$\sum_{t=0}^{T-1}\Big(P^*(t) - r(t) - \epsilon\Big) < q_b(0),$$

where $\epsilon > 0$ can be arbitrarily small. By divided by T and taking $\limsup_{T\to\infty}$ of both sides, we have

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\Big(P^*(t) - r(t) - \epsilon\Big) < 0.$$

Construct an auxiliary queue with the following evolution:

$$\bar{q}_b^*(t+1) = \Big(\bar{q}_b^*(t) - r(t) - \epsilon\Big)^+ + P^*(t),$$

where $\epsilon > 0$ can be arbitrarily small. By Lemma 2, $\bar{q}_b^*(t)$ is strongly stable. By multiplying $\tilde{q}_b(t)$ for both sides of the inequality $\bar{q}_b^*(t+1) \ge \bar{q}_b^*(t) - r(t) - \epsilon + P^*(t)$ and rearranging terms, we obtain $\tilde{q}_b(t)\left[P^*(t) - r(t)\right] \le \tilde{q}_b(t)\Big[\bar{q}_b^*(t+1) - \bar{q}_b^*(t) + \epsilon\Big]$. By summing from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$, we have

$$\begin{aligned}
&\frac{1}{V}\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \tilde{q}_b(t)\big[P^*(t) - r(t)\big] \\
\le &\frac{1}{V}\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \bar{q}_b^*(t)\big(\tilde{q}_b(t-1) - \tilde{q}_b(t)\big) + \\
&\frac{1}{V}\limsup_{T\to\infty} \frac{\tilde{q}_b(T)\bar{q}_b^*(T) - \tilde{q}_b(0)\bar{q}_b^*(0)}{T} + \limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\frac{\tilde{q}_b(t)}{V}\epsilon \\
\le &\frac{1}{V}(\eta_o + r_{\max})\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \bar{q}_b^*(t) + \frac{\epsilon\beta}{V}(\frac{V}{2} + A_{\max}) \\
= &O(\frac{1}{V}) + (\frac{\beta}{2} + \frac{\beta A_{\max}}{V})\epsilon,
\end{aligned}$$

since the average queue length of the auxiliary queue remains finite no matter how large the system parameter B_b becomes and is not related to the algorithmic parameters V. By letting $\epsilon \to 0$, we finish the proof. ■

Note that $q_d^*(t+1) = (q_d^*(t) - \mu(P^*(t)))^+ + R^*(t) \ge q_d^*(t) - \mu(P^*(t)) + R^*(t)$. By multiplying both sides with $q_d(t)$ and rearranging terms, we obtain $q_d(t)(R^*(t) - \mu(P^*(t))) \le q_d(t)(q_d^*(t+1) - q_d^*(t))$. With the optimal policy, q_d^* is strongly stable. By summing from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$, we have

$$\begin{aligned}
&\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_d(t)\big[R^*(t) - \mu(P^*(t))\big] \\
\le &\limsup_{T\to\infty} \frac{q_d(T)q_d^*(T) - q_d(0)q_d^*(0)}{T} + \\
&\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} q_d^*(t)(q_d(t-1) - q_d(t)) \\
\le &\mu_{\max}\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} q_d^*(t),
\end{aligned} \tag{36}$$

which is a finite constant that is not related to V.

By summing from 0 to $T-1$, dividing by T and V, taking $\limsup_{T\to\infty}$ over Equation (35), combined with Lemma 3, and Equation (36), we get

$$\begin{aligned}
\liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t) \ge \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R^*(t) - \eta_o(\mu_{\max} + \beta) \\
- O(\frac{1}{V}).
\end{aligned}$$

II) *$B_d = \infty$ and $B_b < \infty$.*
From Equation (12), we have $\tilde{q}_b^2(t+1) \le (\tilde{q}_b(t) - \eta_o)^2 + (1_o(t) + P(t) - r(t) + M(t))^2 + 2(\tilde{q}_b(t) - \eta_o)^+(1_o(t) + P(t) -$

$r(t) + M(t))$, then

$$\begin{aligned}\Delta =&\Delta\big(q_d(t), \tilde{q}_b(t)\big)\\ \le&(1+P_{\text{peak}})^2 + (\eta_o + r_{\max})^2 + \mu^2(P(t)) + R^2(t) + VR(t)\\ &+ 2\big[q_d(t) - V/2\big]R(t) - 2\big[q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)\big]\\ &+ 2\tilde{q}_b(t)\big(1_o(t) - r(t) + M(t)\big)\\ \le&(1+P_{\text{peak}})^2 + (\eta_o + r_{\max})^2 + \mu_{\max}^2 + A_{\max}^2 + 2\tilde{q}_b(t)M(t)\\ &+ V\big(R(t) - R^*(t)\big) + 2q_d(t)\big[R^*(t) - \mu(P^*(t))\big] + 2\tilde{q}_b(t)\\ &\big[P^*(t) - r(t) + M^*(t)\big] + (V + 2A_{\max})(\beta + \mu_{\max})1_o(t)\end{aligned}$$

Lemma 4:

$$\frac{1}{V}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_b(t)M(t) \le O\Big(\frac{(\frac{\beta}{2}V - B_b)^+}{V}\Big).$$

Proof: We first provide a rough idea of the proof: by exploring the relations between $\tilde{q}_b(t)$ and $q_b(t)$, we notice that as $q_b(t)$ increases from 0 to B_b, $\tilde{q}_b(t)$ will decrease below $(\frac{\beta}{2}(V + A_{\max}) - B_b)^+$ at some slot. We now give the proof details.

Without loss of generality, let $q_b(0) = 0$. We have the following cases:
i) if $P(t) \ge r(t)$, $1_o(t) = 0$ and $\tilde{q}_b(t) > 0$, then $M(t) = 0$, $q_b(t+1) \le q_b(t)$ and $\tilde{q}_b(t+1) - \tilde{q}_b(t) \le q_b(t) - q_b(t+1)$, i.e., even if $\tilde{q}_b(t)$ increases, the increment is no larger than the decrement of $q_b(t)$;
ii) if $P(t) < r(t)$, $1_o(t) = 0$, then if $\tilde{q}_b(t+1) > 0$, $\tilde{q}_b(t) - \tilde{q}_b(t+1) = r(t) - P(t) - M(t) - \eta_o + (\eta_o - \tilde{q}_b(t))^+ \ge q_b(t+1) - q_b(t) = r(t) - P(t) - M(t)$, i.e., the decrement of $\tilde{q}_b(t)$ is no less than the increment of $q_b(t)$, else if $\tilde{q}_b(t+1) = 0$, it goes to case iv);
iii) if $1_o(t) = 1$, then $q_b(t+1) = \min[r(t), B_b]$ by definition of discharging event Equation (10). If $M(t) > 0$, then this means $r(t) > B_b$ and $-r(t) + M(t) = -B_b$;
iv) if $\tilde{q}_b(t) = 0$, then $\tilde{q}_b(t)M(t) = 0$.

Combine the above discussion with Equation (16), we have that $\forall t \ge 0$, if $M(t) > 0$ and $\tilde{q}_b(t) > 0$, we must have $\tilde{q}_b(t) \le \beta(\frac{V}{2} + A_{\max}) + \max[r_{\max}, 1] - B_b$. Thus,

$$\begin{aligned}&\frac{1}{V}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_b(t)M(t)\\ \le&\frac{(\beta(\frac{V}{2} + A_{\max}) + \max[r_{\max}, 1] - B_b)^+ r_{\max}}{V}\\ =&O\Big(\frac{(\frac{\beta}{2}V - B_b)^+}{V}\Big). \quad \blacksquare\end{aligned}$$

The remaining argument is similar to case I), and we obtain

$$\begin{aligned}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t) \ge& \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t) - \eta_o(\mu_{\max} + \beta)\\ &- O\Big(\frac{1}{V}\Big) - O\Big(\frac{(\frac{\beta}{2}V - B_b)^+}{V}\Big).\end{aligned}$$

III) $B_d < \infty$ *and* $B_b = \infty$.
From Equation (13), we have $\tilde{q}_b^2(t+1) \le (\tilde{q}_b(t) - \eta_o)^2 + (1_o(t) + P(t) - r(t))^2 + 2(\tilde{q}_b(t) - \eta_o)^+(1_o(t) + P(t) - r(t))$. Also from Equation (11), we have $\tilde{q}_d^2(t+1) \le (\tilde{q}_d(t) - \eta_d R(t))^2 + (I(t) + R(t) - \mu(P(t)))^2 + 2(\tilde{q}_d(t) - \eta_d R(t))^+(I(t) + R(t) - \mu(P(t)))$, then

$$\begin{aligned}\Delta =&\Delta\big(\tilde{q}_d(t), \tilde{q}_b(t)\big)\\ \le&(1+P_{\text{peak}})^2 + (\eta_o + r_{\max})^2 + \mu^2(P(t)) + (1+\eta_d^2)R^2(t)\\ &+ VR(t) + 2\tilde{q}_b(t)\big(1_o(t) - r(t)\big) + 2\tilde{q}_d(t)I(t) + 2\\ &\Big[(1-\eta_d)\tilde{q}_d(t) - \frac{V}{2}\Big]R(t) - 2\Big[\tilde{q}_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t)\Big]\\ \le&(1+P_{\text{peak}})^2 + (\eta_o + r_{\max})^2 + \mu_{\max}^2 + (1+\eta_d^2)A_{\max}^2\\ &+ 2\tilde{q}_d(t)I(t) + (V + 2A_{\max})(\beta + \mu_{\max})1_o(t) + VR(t)\\ &- VR^*(t) + 2\tilde{q}_d(t)\big[R^*(t) - \mu(P^*(t)) + I^*(t) - D^*(t)\big]\\ &+ 2\tilde{q}_d(t)\big[D^*(t) - \eta_d R^*(t)\big] + 2\tilde{q}_b(t)\big[P^*(t) - r(t)\big]\end{aligned}$$

Without loss of generality, let $q_d(0) = B_d$. Similar to the proof of Lemma 4, we have the following cases:
i) if $D(t) > 0$, then from $q_d(t+1) = (q_d(t) - \mu(P(t)))^+ + R(t) - D(t)$ and $I(t) = (\mu(P(t)) - q_d(t))^+$, we know that $R(t) > B_d$ when $I(t)$ is strictly positive. Further, whenever $D(t) > 0$, $q_d(t+1) = B_d$;
ii) if $D(t) = 0$, from $q_d(t+1) = q_d(t) - \mu(P(t)) + I(t) + R(t) - D(t)$ and $\tilde{q}_d(t+1) = ((\tilde{q}_d(t) - \eta_d R(t))^+ + D(t) - \mu(P(t)) + I(t) + R(t) - D(t))^+$, $\tilde{q}_d(t)$ decreases no slower and increases no faster than $q_d(t)$ until $\tilde{q}_d(t)$ hits zero.

Note that only when $q_d(t) < \mu_{\max}$, $I(t)$ may be strictly positive. Then combine with the above discussion, we have

$$\begin{aligned}\frac{1}{V}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_d(t)I(t) \le&\frac{(\frac{V}{2} + A_{\max} + \mu_{\max} - B_d)^+\mu_{\max}}{V}\\ =&O\Big(\frac{(\frac{V}{2} - B_d)^+}{V}\Big).\end{aligned}$$

Similar to Lemma 3, we can obtain

$$\frac{1}{V}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_d(t)[D^*(t) - \eta_d R^*(t)] \le O\Big(\frac{1}{V}\Big).$$

The remaining argument is similar to case I), and we obtain

$$\begin{aligned}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t) \ge& \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t) - \eta_o(\mu_{\max} + \beta)\\ &- O\Big(\frac{1}{V}\Big) - O\Big(\frac{(\frac{V}{2} - B_d)^+}{V}\Big).\end{aligned}$$

IV) $B_d < \infty$ *and* $B_b < \infty$.
Simply by combing case II) and case III), we obtain

$$\begin{aligned}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t) \ge& \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t) - \eta_o(\mu_{\max} + \beta)\\ &- O\Big(\frac{1}{V}\Big) - O\Big(\frac{(\frac{\beta}{2}V - B_b)^+}{V}\Big) -\\ &O\Big(\frac{(\frac{V}{2} - B_d)^+}{V}\Big). \quad \blacksquare\end{aligned}$$

C. Proof of Corollary 1

Similar to the proof of Proposition 1, we have the fact if $\tilde{q}_b^n(t)$ is strongly stable, then $\limsup_{T\to\infty} \frac{\tilde{q}_b^n(T)}{T} = 0$. From Equation (27), we have $\tilde{q}_b^n(t+1) \geq \tilde{q}_b^n(t) - \eta_o^n + \sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) - r_n(t) + M_n(t) + 1_o^n(t)$. Note that $q_b^n(t+1) = q_b^n(t) - \sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) + r_n(t) - M_n(t)$. By summing from 0 to $T-1$, dividing by T and taking $\limsup$ of both sides, we have

$$\limsup_{T\to\infty} \frac{\tilde{q}_b^n(T)}{T} \geq \lim_{T\to\infty} \frac{\tilde{q}_b^n(0)}{T} - \eta_o^n + \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} 1_o^n(t) + \lim_{T\to\infty} \frac{q_b^n(0) - q_b^n(T)}{T}.$$

Since $\limsup_{T\to\infty} \frac{\tilde{q}_b^n(T)}{T} = 0$, $\lim_{T\to\infty} \frac{\tilde{q}_b^n(0)}{T} = 0$, and $\lim_{T\to\infty} \frac{\tilde{q}_b^n(0) - q_b^n(T)}{T} = 0$, so we get $p_o^n = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} 1_o^n(t) \leq \eta_o^n,\ \forall n \in \mathcal{N}$. ■

D. Proof of Theorem 2

Proof of Equation (29) and Equation (32): We prove Equation (29) by induction. Let $q_d^{\max}(t)$ be the maximum data queue length for all flows at slot t. Assume that $q_d^{\max}(t) \leq \frac{V}{2} + A_{\max}$ (holds for $t = 0$ by letting $q_d^{n,e}(0) = 0,\ \forall n, e \in \mathcal{N}$), need to show that it holds at slot $t+1$. Consider the data queue $q_d^{n,e}(t+1)$ maintained at any node n for flow destined to any node $e \neq n$ at slot $t+1$. If node n received data destined to e from other node m at slot t, then by the routing policy in Section V and definition of $w_{(m,n)}(t)$, $q_d^{m,e}(t) - q_d^{n,e}(t) > \gamma_{(m,n)}^e$, where (m,n) is the link from node m to node n. Choose γ such that the resulting backlog of the receiving node is not longer than that of the transmitting node (let $\mu_{\max}^{\text{in}}$ to be the maximum endogenous arrivals, then $\gamma = \mu_{\max}^{\text{in}} + A_{\max}$ satisfy this condition), we then have $q_d^{n,e}(t+1) \leq q_d^{n,e}(t) + \gamma$, then $q_d^{n,e}(t+1) \leq q_d^{m,e}(t) \leq q_d^{\max}(t) \leq \frac{V}{2} + A_{\max}$. If node n did not receive any data destined to e from other nodes, then it can only have exogenous arrivals. Clearly $q_d^{n,e}(t+1) \leq q_d^{n,e}(t) \leq \frac{V}{2} + A_{\max}$ if there were no exogenous arrivals. If there were exogenous arrivals, by *MRC* of Section V, we must have $q_d^{n,e}(t) \leq \frac{V}{2}$, then $q_d^{n,e}(t+1) \leq \frac{V}{2} + A_{\max}$. Thus, Equation (29) holds. Equation (32) can be shown using the same argument.

Proof of Equation (30) and Equation (33): Consider any link l^*. Since $\mu_{l^*}(P_{l^*}(t)) \leq \mu_{l^*}(0) + \beta P_{l^*}(t)$ for $P_{l^*}(t) \in \Pi_{l^*}(t),\ \forall t \geq 0$, we have

$$\begin{aligned} & w_{l^*}(t)\mu_{l^*}(P_{l^*}(t)) - (\tilde{q}_b^{\text{tran}(l^*)}(t) + \tilde{q}_b^{\text{rec}(l^*)})P_{l^*}(t) \\ \leq & w_{l^*}(t)\mu_{l^*}(0) + \beta w_{l^*}(t)P_{l^*}(t) - (\tilde{q}_b^{\text{tran}(l*)}(t) + \tilde{q}_b^{\text{rec}(l^*)})P_{l^*}(t), \end{aligned}$$

where $P_{l^*}(t)$ is the solution of Equation (28). If $\beta w_{l^*}(t)P_{l^*}(t) - (\tilde{q}_b^{\text{tran}(l^*)}(t) + \tilde{q}_b^{\text{rec}(l^*)})P_{l^*}(t) < 0$, then we get $w_{l^*}(t)\mu_{l^*}(P_{l^*}(t)) - (\tilde{q}_b^{\text{tran}(l^*)}(t) + \tilde{q}_b^{\text{rec}(l^*)})P_{l^*}(t) < w_{l^*}(t)\mu_{l^*}(0)$. However, solution of Equation (28) chooses $P_{l^*}(t)$ that maximizes $w_{l^*}(t)\mu_{l^*}(P_{l^*}(t)) - (\tilde{q}_b^{\text{tran}(l^*)}(t) + \tilde{q}_b^{\text{rec}(l^*)})P_{l^*}(t)$ which means $w_{l^*}(t)\mu_{l^*}(P_{l^*}(t)) - (\tilde{q}_b^{\text{tran}(l^*)}(t) + \tilde{q}_b^{\text{rec}(l^*)})P_{l^*}(t) \geq w_{l^*}(t)\mu_{l^*}(0)$ since $0 \in \Pi_{l^*}(t)$. Then we must have

$$(\tilde{q}_b^{\text{tran}(l*)}(t) + \tilde{q}_b^{\text{rec}(l^*)})P_{l^*}(t) \leq \beta w_{l^*}(t)P_{l^*}(t). \tag{37}$$

We now prove Equation (30) by induction. Without loss of generality, let $\tilde{q}_b^n(0) \leq \beta(\frac{V}{2} + A_{\max}),\ \forall n \in \mathcal{N}$. Suppose for all $t \geq 1$, $\tilde{q}_b^n(t-1) \leq \beta(\frac{V}{2} + A_{\max}),\ \forall n \in \mathcal{N}$ holds. In slot t, if all the incident links of any node n is assigned zero power, then $\tilde{q}_b^n(t) \leq \tilde{q}_b^n(t-1) \leq \beta(\frac{V}{2} + A_{\max})$. Otherwise, let l^* be an incident link assigned non-zero power, and by Equation (37), we have $\tilde{q}_b^n(t) \leq \beta w_{l^*}(t) \leq \beta q_{\max}^d(t) \leq \beta(\frac{V}{2} + A_{\max}),\ \forall n \in \mathcal{N}$. Equation (33) can be shown using the same argument.

Proof of Equation (31): Define $L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) = \sum_{n,e} \left(q_d^{n,e}(t)\right)^2 + \sum_n \left(\tilde{q}_b^n(t)\right)^2$, then

$$\begin{aligned} \Delta(t) =& L(\vec{q}_d(t+1), \vec{\tilde{q}}_b(t+1)) - L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) \\ \leq & \sum_{n,e} \left[(A_{\max} + \mu_{\max})^2 + \mu_{\max}^2\right] + \sum_n \left[(1 + P_{\text{peak}})^2 + \right. \\ & \left.(\eta_o^n + r_{\max})^2\right] + 2\sum_{n,e} q_d^{n,e}(t)R_n^e(t) + 2\sum_n \tilde{q}_b^n(t) \\ & \left[1_o^n(t) - r_n(t) + M_n(t)\right] - 2\sum_{l\in\mathcal{L}} \left[(q_d^{\text{tran}(l),e_l(t)}(t) - \right. \\ & \left. q_d^{\text{rec}(l),e_l(t)}(t))\mu_l(P_l(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t))P_l(t)\right] \end{aligned}$$

Note that if $q_d^{\text{tran}(l),e_l(t)}(t) - q_d^{\text{rec}(l),e_l(t)}(t) < \gamma_l^{e_l(t)}$, then $w_l(t) = 0$ and $\mu_l(P_l(t)) = 0$, then $(q_d^{\text{tran}(l),e_l(t)}(t) - q_d^{\text{rec}(l),e_l(t)}(t))\mu_l(P_l(t)) = w_l(t)\mu_l(P_l(t))$; otherwise, $q_d^{\text{tran}(l),e_l(t)}(t) - q_d^{\text{rec}(l),e_l(t)}(t) \geq w_l(t)$, then $(q_d^{\text{tran}(l),e_l(t)}(t) - q_d^{\text{rec}(l),e_l(t)}(t))\mu_l(P_l(t)) \geq w_l(t)\mu_l(P_l(t))$. We then have

$$\begin{aligned} \Delta(t) \leq & \sum_{n,e} \left[(A_{\max} + \mu_{\max})^2 + \mu_{\max}^2\right] + \sum_n \left[(1 + P_{\text{peak}})^2 + \right. \\ & \left.(\eta_o^n + r_{\max})^2\right] + 2\sum_n \tilde{q}_b^n(t)\left[1_o^n(t) - r_n(t) + M_n(t)\right] - \\ & 2\sum_{l\in\mathcal{L}} \left[w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t))P_l(t)\right] \\ & + 2\sum_{n,e} q_d^{n,e}(t)R_n^e(t). \end{aligned}$$

Note that $\sum_{l\in\mathcal{L}}[w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t))P_l(t)] = \sum_{l\in\mathcal{M}(t)}[w_l(t)\mu_l(P_l^s(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t))P_l^s(t)] \geq \sum_{l\in\mathcal{M}^*(t)}[w_l(t)\mu_l(P_l^s(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t))P_l^s(t)]$, where $\mathcal{M}(t)$ is the matching chosen by MWM algorithm and $\mathcal{M}^*(t)$ is the matching picked by optimal policy in slot t, and $P_l^s(t),\ \forall l \in \mathcal{L}$ are the suppositionally calculated power in *MPA*. Since the objective function of power allocation component is separable over links and for each link it is concave, using similar argument as in Lemma 1, we have

$$\Delta(t) \leq \sum_{n,e} \left[(A_{\text{max}}+\mu_{\text{max}})^2+\mu_{\text{max}}^2\right] + \sum_{n} \Big[(1+P_{\text{peak}})^2 + (\eta_o^n + r_{\text{max}})^2 + (V+2A_{\text{max}})(\beta+\mu_{\text{max}})1_o^n(t)\Big] + V\sum_{n,e} R_n^e(t) + 2\sum_{n,e}\left[q_d^{n,e}(t)-\frac{V}{2}\right]R_n^{e*}(t) - 2\sum_{n,e}\sum_{l\in\Omega_n}(q_d^{\text{tran}(l),e}(t) - q_d^{\text{rec}(l),e}(t)-\gamma)\mu_l^e(P_l^*(t)) + 2\sum_n \tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n} P_l^*(t) - r_n(t) + M_n^*(t) + M_n(t)\right]$$
$$\leq \sum_{n,e}\left[A_{\text{max}}^2 + 2(A_{\text{max}}+\gamma+\mu_{\text{max}})\mu_{\text{max}}\right] + \sum_n \Big[(V+2A_{\text{max}})(\beta+\mu_{\text{max}})1_o^n(t) + (1+P_{\text{peak}})^2 + (\eta_o^n+r_{\text{max}})^2\Big] + V\sum_{n,e}\big(R_n^e(t)-R_n^{e*}(t)\big) + 2\sum_n \tilde{q}_b^n(t)M_n(t) + 2\sum_{n,e} q_d^{n,e}(t)\left[R_n^{e*}(t) + \left(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\right)\mu_l^e(P_l^*(t))\right] + 2\sum_n \tilde{q}_b{}^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n} P_l^*(t) - r_n(t) + M_n^*(t)\right]. \tag{38}$$

Similar to Lemma 3, we have

$$\frac{1}{V}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n \tilde{q}_b^n(t)\Big[\sum_{l\in\Omega_n\bigcup\Theta_n} P_l^*(t) - r_n(t) + M_n^*(t)\Big] \leq O(\frac{1}{V}).$$

Similar to Lemma 4, we have

$$\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n \tilde{q}_b^n(t)M_n(t) \leq \sum_n O(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}).$$

Note that the data queue dynamics can be written as

$$q_d^{n,e}(t+1) = q_d^{n,e}(t) + R_n^e(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l(t)) - D_{\text{in}}^{n,e}(t) + D_{\text{out}}^{n,e}(t),$$

where $D_{\text{in}}^{n,e}(t)$ is the amount of overestimated endogenous arrivals to the queue $q_d^{n,e}$ since $\sum_{l\in\Theta_n}\mu_l^e(P_l(t))$ may be larger than the actual endogenous arrivals; $D_{\text{out}}^{n,e}(t)$ is the amount of overestimated departures since $\sum_{l\in\Omega_n}\mu_l^e(P_l(t))$ may be larger than the actual departures. Further, at slot t, if node n does not have endogenous arrivals for flow e, then $D_{\text{in}}^{n,e}(t)=0$; if flow e is transferred from node n to node m, then $D_{\text{out}}^{n,e}(t) = D_{\text{in}}^{m,e}(t)$. By choosing $\gamma = A_{\text{max}}+\mu_{\text{max}}$, the resulting backlog of the receiving node is no longer than that of the transmitting node. We then have

$$\sum_{n,e} q_d^{n,e}(t)(q_d^{n,e*}(t+1) - q_d^{n,e*}(t))$$
$$= \sum_{n,e} q_d^{n,e}(t)\left[R_n^{e*}(t) - D_{\text{in}}^{n,e*}(t) + D_{\text{out}}^{n,e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))\right]$$
$$\geq \sum_{n,e} q_d^{n,e}(t)\left[R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))\right].$$

Since $q_d^{n,e}(t) - q_d^{n,e}(t+1) \leq \mu_{\text{max}},\ \forall t\geq 0,\ \forall n,e\in\mathcal{N}$ and $q_d^{n,e*}$ are strongly stable $\forall n,e\in\mathcal{N}$, then

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e} q_d^{n,e}(t)\Big(q_d^{n,e*}(t+1) - q_d^{n,e*}(t)\Big)$$
$$\leq \sum_{n,e}\Big[\lim_{T\to\infty}\frac{q_d^{n,e}(T)q_d^{n,e*}(T) - q_d^{n,e}(0)q_d^{n,e*}(0)}{T} + \mu_{\text{max}}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} q_d^{n,e*}(t)\Big]$$
$$\leq \sum_{n,e}\mu_{\text{max}}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} q_d^{n,e*}(t), \tag{39}$$

which is a finite constant that is not related to V. Thus, we have $\frac{1}{V}\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e} q_d^{n,e}(t)[R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))] \leq O(\frac{1}{V})$. By summing from 0 to $T-1$, dividing by T and V, taking $\liminf_{T\to\infty}$ for Equation (38), we get

$$\sum_n\left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^e(t)\right\}$$
$$\geq \sum_n\left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^{e*}(t) - \eta_o^n(\mu_{\text{max}}+\beta) + O(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V})\right\} - O(\frac{1}{V}).$$

which is Equation (31).

Proof of Equation (34): Define $L(\vec{q}_d(t),\vec{\tilde{q}}_b(t)) = \frac{1}{2}\sum_{n,e}\left(q_d^{n,e}(t)\right)^2 + \frac{1}{2}\sum_n\left(\tilde{q}_b^n(t)\right)^2$.

By *MRC*, we have for any $n,e\in\mathcal{N}$, $n\neq e$ that $\left[q_d^{n,e}(t)-V\right]R_n^e(t) = \left[q_d^{n,e}(t)-V\right]A_n^e(t) \leq \left[q_d^{n,e}(t) - V\right]R_n^{e*}(t) \leq \frac{1}{2}\left[q_d^{n,e}(t)-V\right]R_n^{e*}(t)$ if $q_d^{n,e}(t)\leq V$; and $\left[q_d^{n,e}(t)-V\right]R_n^e(t) = 0 \leq \frac{1}{2}\left[q_d^{n,e}(t)-V\right]R_n^{e*}(t)$ if $q_d^{n,e}(t) > V$. Thus,

$$\left[q_d^{n,e}(t)-V\right]R_n^e(t) \leq \frac{1}{2}\left[q_d^{n,e}(t)-V\right]R_n^{e*}(t).$$

From the efficiency of the distributed matching algorithm [19] [20], we have $\sum_{l\in\mathcal{L}}[w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t))P_l(t)] \geq \frac{1}{2}\sum_{l\in\mathcal{M}(t)}[w_l(t)\mu_l(P_l^s(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t))P_l^s(t)] \geq \frac{1}{2}\sum_{l\in\mathcal{M}^*(t)}[w_l(t)\mu_l(P_l^s(t)) - (\tilde{q}_b^{\text{tran}(l)}(t) +$

$\tilde{q}_b^{\text{rec}(l)}(t))P_l^s(t)]$, where $\mathcal{M}(t)$ is the matching chosen by MWM algorithm and $\mathcal{M}^*(t)$ is the matching picked by optimal policy in slot t, and $P_l^s(t),\ \forall l \in \mathcal{L}$ are the suppositionally calculated power in *MPA*.

We then have

$$\begin{aligned}\Delta(t) =& L\big(\vec{q}_d(t+1), \vec{\tilde{q}}_b(t+1)\big) - L\big(\vec{q}_d(t), \vec{\tilde{q}}_b(t)\big)\\ \leq& \sum_{n,e}\big[\frac{1}{2}A_{\text{max}}^2 + (A_{\text{max}}+\gamma+\mu_{\text{max}})\mu_{\text{max}}\big] + \frac{1}{2}\sum_{n}\big[(V+\\ & 2A_{\text{max}})(\beta+\mu_{\text{max}})1_o^n(t) + (1+P_{\text{peak}})^2 + (\eta_o^n + r_{\text{max}})^2\big]\\ & + V\sum_{n,e} R_n^e(t) + \frac{1}{2}\sum_{n,e}\big[q_d^{n,e}(t) - V\big]R_n^{e*}(t) + \\ & \frac{1}{2}\sum_n \tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n} P_l^*(t) - r_n(t) + M_n^*(t)\right] + \\ & \frac{1}{2}\sum_{n,e} q_d^{n,e}(t)\left[\left(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\right)\mu_l^e(P_l^*(t))\right] + \\ & \sum_n \tilde{q}_b^n(t)M_n(t).\end{aligned}$$

The remaining argument is similar to the proof of Equation (31). ■

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