Quality Control of CNT Forests via EM Probing

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***Abstract*—In this work, a study of CNT length distribution estimation is presented. The method involves the use of RF signals to excite the CNT forest at frequencies. The observations are then processed with an integer program solver to estimate the distribution of CNT lengths of an equivalent system. The numerical studies confirm the accuracy of this approach to estimate the distribution of CNT lengths in a forest.**

I. Introduction

Carbon Nanotubes (CNTs) are considered as ideal field emission devices, especially for flat-panel displays. During the process of growing the CNT forests, it is highly probable that CNTs of various lengths would be deposited on the surface of a substrate rather than CNTs with identical lengths. Furthermore, the density of CNTs may vary on this 2D space. This may lead to varying field emission intensity on (potentially) large surfaces. While it is beyond the scope of this article to comment on how uniformity in CNT density and length can be achieved, in this article, we provide a means to characterize the CNT length distribution in such CNT forests. This knowledge can be used to identify whether the manufactured field emission device conforms to quality standards.

Our approach to determine the length distribution in a CNT forest is based on the unique oscillatory response of CNTs in electric fields to EM waves. Rather than relying on the oscillation of electrons inside the antenna in response to EM waves, CNTs oscillate themselves when they are charged. Oscillations lead to variation of the distance of the tip of the CNT from a cathode plate. The distance variations are then detected as fluctuations of the emission current. The ground-breaking property of CNT-based communication systems is that it is possible to establish communication in the 100s of MHz range with systems that are 100s of nm in size. This response was first proposed in [1] to use CNTs as non-traditional antennas to receive RF signals, further analyzed in [2], and later presented in [3] as digital receivers.

As presented in [3], the response of CNTs to RF signals is maximized when the carrier frequency of the signal matches the resonance frequency of the CNT, which is inversely proportional to the length of the CNT. Moreover, the response dies off rather quickly when the carrier signal veers off the resonance frequency. This observation is the main motivation to use this phenomenon to determine the distribution of CNT lengths in a forest, which we briefly introduced in [4]. In this work, we provide details of the process to determine the CNT distribution and associated numerical results.

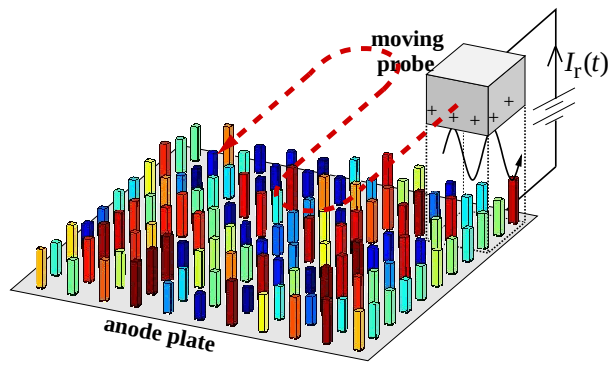

Fig. 1. Spatial CNT Length Distribution Measurement System [4]

II. System Model

Using a constant voltage between the anode plate on which CNT forest resides and a small probe serving as the cathode that moves close to the anode plate, it is possible to measure the combined field emission effect of CNTs in a given area. Unfortunately, this does not yield the desired information about the CNT density. To this end, we combine this basic setup with EM waves at various carrier frequencies. Each of the probed carrier frequencies generates a different response, which allows us to estimate the number of CNTs belonging to a group of comparable lengths. The details of the frequency dependent response of CNTs to EM radiation can be found in [3]. The operation can be further extended to a larger area using a moving probe that scans the surface area of the CNT forest by applying the aforementioned sampling procedure. The proposed system is shown in Figure 1.

The system is assumed to be comprised of N single-walled CNTs, which is assumed to be either known or estimated. Their actual lengths are $L_1, \cdots, L_N$ with associated resonance frequencies $f_1, \cdots, f_N$. In the following, we discuss the distribution of CNT lengths directly under the probe when it is stationary. The CNT forest patch is probed with M, $M \ll N$, equally spaced RF signals between $f_{\min}$ and $f_{\max}$, i.e., $F_1, F_2, \cdots, F_M$, where $F_1 = f_{\min}$, $F_M = f_{\max}$, and $F_{i+1} - F_i = \frac{f_{\max} - f_{\min}}{M-1} \triangleq \Delta f$, $\forall i = 1, \cdots, M-1$. The minimum and maximum frequencies correspond to a range we estimate all CNTs to reside in. We note that Δf should not be much larger than the 3-dB bandwidth of the system so as to ensure that we obtain non-trivial responses from all CNTs. When excited with the RF signals with the above mentioned center frequencies, the system responds with currents Y_i,

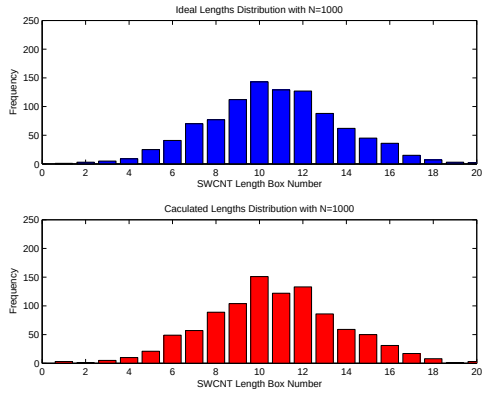

Fig. 2. Normal Length Distribution, $N = 1000$, $M = 20$

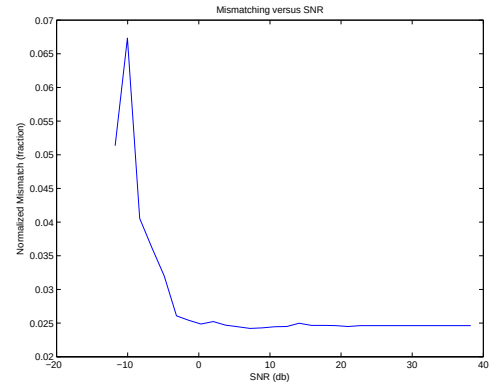

Fig. 3. Fraction of Misclassification vs. SNR

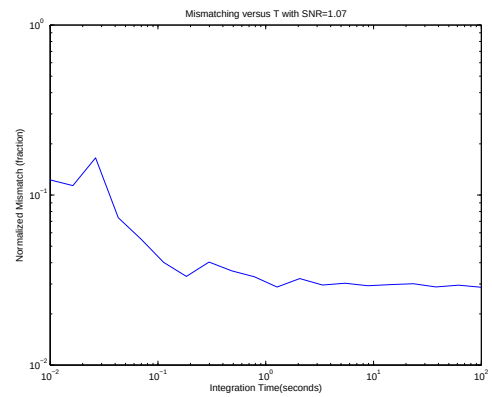

Fig. 4. Fraction of Misclassification vs. T

$i = 1, \cdots, M$. The Y_i values are used to estimate the PMF of an equivalent system that has N CNTs of M distinct length classes. As M grows, the system approximates the actual distribution of CNT lengths of the forest under investigation.

Assume that the mapped system is comprised of M groups of C_i CNTs with identical lenghts L_i. Under this assumption, the total response $\mathbf{Y}$ of the tested area is modeled as $\mathbf{Y} = \mathbf{HC}$, where $\mathbf{H}$ is a transfer matrix with elements $H_{ij} = \frac{1}{2}Ta^2H^2(F_i, f_j) + TN_aB_jH^2(F_i, f_j)$, where T is the duration of the EM exposure, a is the amplitude of the oscillation, $N_a/2$ spectral density of the acoustic noise, B_j is the 3-dB bandwidth for f_j, and $H(F_i, f_j)$ is the transfer function for the response of antennas with resonance frequency f_j for a signal with carrier frequency F_i. The detailed model is available in [3], [4]. For a given $\mathbf{H}$ and observation $\mathbf{Y}$, we compute the vector of number of CNTs $\mathbf{C}$ in each group by solving the following integer program using YALMIP [5]:

$$\min_{\mathbf{C}} \| \mathbf{HC} - \mathbf{Y} \|_\infty \tag{1}$$

$$\text{subject to} \sum_{i=1}^{M} C_i = N, \quad C_i \in \mathcal{Z}_0^+, \quad i = 1, \cdots, M$$

III. Numerical Study for Gaussian Distribution

In the following, we generated CNT forests of size N following a Gaussian length distribution and estimated the distribution solving the Problem (1). The $\mathbf{Y}$ vector is generated by computing the response of the system over T and using the actual length information. The results of Figure 2 show the grouping of $N = 1000$ CNTs into $M = 20$ bins (upper plot) and its estimation obtained by solving (1) (lower plot). These results confirm the effectiveness of our approach to estimate the CNT distribution.

We further analyzed the effect of SNR on the classification performance. Here, the classification performance is measured as the ratio of the sum of bin size differences to the total number of CNTs. The results are presented in Figure 3 As expected, the classification performance significantly improves with SNR values, achieving an almost asymptotic value of 2.5% mismatch decreasing from 6.5% at -10dB.

Finally, we have also analyzed the effect of the radiation time T on the classification performance, which are depicted in Figure 4. This parameter determines the execution time of the sampling sequence, which is $T \cdot M$. With larger T, the classification performance increases, but the execution time also increases linearly. The results of Figure 4 indicates that reasonable levels of performance can be achieved for $T \simeq 200ms$.

IV. Conclusion

In this work, we introduced an integer programming solution to estimate the distribution of CNT lengths of a CNT forest. Such distributions can readily be used to assess the quality of large CNT forest arrays. Our numerical studies confirm that such studies can be performed reliably without the use of electron microscopes.

References

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