Numerical Studies of the Nonlinear Interaction Between Turbulent Air Flow and Sea Surface Waves, with Application to Ocean Surface Wave Turbulence

Greg R. Baker and Kurt M. Berger

Department of Mathematics
The Ohio State University
231 W. 18th Avenue
Columbus, OH 43210

Joel T. Johnson

Department of Electrical Engineering and ElectroScience Laboratory
The Ohio State University
205 Dreese Laboratory
2015 Neil Ave
Columbus, OH 43210

2001 ITR/AP Proposal

Project Summary

The generation of sea surface waves by winds is one of the fundamental air-sea interaction processes which affect the global climate. Although this topic has been studied extensively through both physical modeling and experimental measurements, understanding of the physics involved remains limited due to the nonlinear phenomena implicit in both air flow and sea surface wave evolution. Numerical simulations offer a means to improve understanding of the complex interactions of this problem, but only recently have computing resources improved to make sufficiently large scale simulations possible. Recent studies have begun to apply computational fluid dynamics methods to simulate turbulent air flow over sea waves, but to date the sea surface models applied have been based on waves of permanent form, such as Stokes' waves. The neglect of nonlinear processes in the sea surface evolution results in wave growth estimates which are constant in time, clearly incorrect if an equilibrium between the air and water fields is to be obtained. Nonlinear hydrodynamic models have also been investigated previously, but air-forcing effects are included in these models through empirical estimates only, without regard to the effect of the sea surface profile on wind forcing. Results from these hydrodynamic-only models dramatically underestimate short sea wave modulation effects observed in remotely sensed sea images at microwave frequencies.

The proposed project develops a numerical model for turbulent air flow over nonlinearly evolving water surfaces which overcomes the limitations of previous studies. Efficient computational algorithms for modeling air flow, efficient hydrodynamic methods to evolve the sea surface boundary, and parallel computing techniques will be combined to make statistical analyses of realistic problems in both two and three dimensions possible. Scientific visualization methods will be applied to analyze results and to improve understanding of the nonlinear interactions between wind and waves that are important for sea surface growth and dissipation. Studies will be performed to determine the conditions under which "decoupling" the interacting complex systems into air-flow-only or hydrodynamic-only simulations is possible by including appropriate forcing terms; the form of these forcing terms, when applicable, will be compared with existing empirical and analytical models to clarify the important physical processes. Decoupled hydrodynamic-only simulations will be pursued to investigate wave-wave energy transfer effects in larger scale problems than those possible in the coupled air-water model, and resulting forms of the equilibrium sea surface spectrum will be obtained. Commonly applied approximate hydrodynamic models will also be considered to assess their performance, and wave spectra obtained from numerical studies will be extrapolated using applicable approximate models to include breaking-wave effects not captured in the full numerical simulations. Results of the project will be significant for studies of the global climate, sea wave forecasting, and interpretation of direct and remotely sensed oceanographic data. More generally, project results will clarify important physical effects that can occur in a system of coupled nonlinear turbulent systems.

The proposed project represents an interdisciplinary collaboration between applied mathematics and engineering researchers, with the former leading model and code development and testing and the latter leading result interpretation efforts along with applications to the oceanographic remote-sensing community. The proposed project will serve as the primary mechanism to support this collaboration. Educational efforts also comprise a principal objective of the project, including postdoctoral research training and graduate and undergraduate education and research. All participants in the project will gain information technology (IT) experience through algorithm and code development. Project results will be communicated to the external community through conference and journal publications and through use of the world-wide-web; project web resources will also be used in classes taught by the research team to introduce students to scientific applications of IT research.

I. Introduction

Wind over the ocean is one of the primary mechanisms by which sea surface waves are generated. The presence of sea surface waves influences almost all commercial activities related to the oceans, making wave forecasting of vital importance for marine operations. Almost all aspects of oceanography are influenced by sea waves, and accurately parameterizing the interaction between sea and atmosphere is important for both local and global climate modeling and forecasting. Although numerous studies of sea wave generation by winds have been performed previously, understanding of the fundamental physical processes involved remains limited at present. The difficulties of the problem stem from the nonlinear interactions inherent in both turbulent air flow and sea wave evolution, which make interpretation of measured data difficult, even under highly controlled conditions.

It is commonly accepted that the growth of surface waves on the ocean depends on three major factors: the forcing of the wind, wave-wave interactions, and wave dissipation (by both wave breaking and the effects of viscosity.) A standard assumption used both by experimental oceanographers and those who study the mathematical aspects of wave turbulence is that the air and wave fields can be decoupled, allowing standard hydrodynamic models—augmented by an approximate wind forcing term—to be used for wave predictions ("water-only" models). State-of-the-art terms for forcing of waves by winds use a simple exponential growth parameter obtained empirically from wave-tank measurements during the initial growth period of waves [1]-[2]. Approximate models for this parameter based on a perturbative solution of the air-flow equations [3]-[9] typically underpredict even these measurements by significant factors [6]. Although oceanographic measurements with satellite- or air-borne microwave remote sensing systems are improving the scale and size of sea wave observations [10], a comparison of sensor data with approximate hydrodynamic theories using empirical wind forcing models again shows poor agreement [11].

Waves on the surface of the ocean generated by winds can be considered a turbulent system, in which energy, momentum, and other conserved quantities can cascade from large scales to small scales (direct cascade) and vice versa (inverse cascade). Description of sea surface waves is generally provided through a second order statistic, such as $n(\mathbf{k},t) =< a(\mathbf{k},t)a^*(\mathbf{k},t) >$ (the sea surface "spectrum"), in which $a(\mathbf{k},t)$ is the complex amplitude of the wave of (vector) wavenumber $\mathbf{k}$. Accurate prediction of the sea surface spectrum is of crucial importance in many scientific disciplines. However, several recent models of the ocean surface wave spectrum based on both empirical data and "decoupled" models for wind forcing effects [12]-[15] show poor agreement in general, illustrating the limited current understanding of the underlying wave generation process. Asymptotic "water-only" studies with simplified forcing and dissipation effects [16]-[19] suggest that an equilibrium spectrum with a power law behavior should be obtained, but the use of closure hypotheses is required to obtain these results. More recent numerical studies [20]-[21] again confirm that an equilibrium spectrum should exist, but models for wave forcing by winds were not included.

Recent work has begun to address improved models for wind-wave interactions through the use of numerical methods for fluid (air) flow above a "water wave" boundary [22]-[29]. The water wave considered in these studies, however, was a simple two-component Stokes' wave approximation, assumed to propagate at a constant velocity without evolving under the action of wind forcing. Although these studies have contributed to an understanding of air-flow effects, a means for extrapolating their results to a coupled wind-wave system in which both wind and waves evolve in reaction to each other is not clear at present. Furthermore, most of these these "air-flow-only" studies model turbulent airflow effects through the use of one of several closure hypotheses [22]-[28]; results show that the choice of closure model can significantly influence final results. Only Sullivan, McWilliams, and Moeng [29] have performed a direct numerical simulation (DNS) of three-dimensional turbulent airflow above a Stokes' wave boundary; the results of this work demonstrate significant variations in both mean and turbulent air flow properties that can occur as the water wave boundary is modified.

The proposed project extends the studies of Sullivan *et al* to include a water surface boundary that evolves in response to air-flow forcing effects, representing a coupled air-water turbulent system. Water wave effects will be incorporated into the numerical model by including the water region as part of the computational domain and by enforcing appropriate boundary conditions at the air-water interface. Both two-dimensional and three-dimensional numerical models will be developed and analyzed statistically, with the two-dimensional model expected to allow larger geometries and sets of realizations, while the three-dimensional model will allow directional effects to be studied for more moderate-sized geometries. Inclusion of hydrodynamic evolution will result in a substantial increase in computational requirements compared to air-flow-only studies, so effective use of information technology methods will be required to make the solution of practical problems possible. Both parallel computing methods and recent advances in numerical algorithms for fluid flow will be applied to address this need. Data sets obtained from the simulation will be analyzed through scientific visualization methods so that a detailed understanding of the wind-wave interaction process can be obtained.

Results will also be analyzed to determine the conditions under which the coupled air-water model can be decoupled into air-only and water-only portions. When appropriate, the nature of the air-forcing effects that enter into the hydrodynamic equations will be determined, and hydrodynamic-only simulations [30] investigated to determine the limitations of commonly applied approximate models for hydrodynamic evolution [31]-[35]. Forms for the sea surface equilibrium directional spectrum that result from time and ensemble averages will also be examined and compared with those predicted in the literature [12]-[15]. Long wave–short wave interaction effects important for oceanographic remote sensing will also be studied and compared with observations from the literature [11],[36]-[37]. The final products of the project will be verified models for wind forcing of sea waves and the resulting sea surface spectrum, which will be reported to the scientific community for application across many disciplines.

Educational aspects are also an important part of the project, through direct involvement of a postdoctoral researcher, graduate, and undergraduate students and through inclusion of project issues in course teaching efforts of the project team.

The following sections of this document detail the project objectives and proposed research and education plans, followed by a discussion of the project's significance and societal impact, relationship to current knowledge, and relationship to the PIs' long term goals.

II. Objectives

1. To develop an efficient parallel numerical simulation of the physical equations governing coupled evolution of air flow over water waves. The use of this full simulation will
   - provide understanding and clarification of the important physical aspects of this highly nonlinear process
   - allow improved wind-forcing models to be developed for use in separate ocean wave turbulence studies
2. To determine the turbulent ocean wave spectrum that results in response to the improved wind-forcing models developed, by
   - using numerical or approximate hydrodynamic models to compute the resulting statistically steady energy spectrum of a large number of interacting surface waves
   - verifying the spectra obtained through comparison with standard models and with direct or remotely sensed oceanographic data
3. To communicate the verified wind forcing and sea spectral models to the oceanographic and atmospheric science communities for use in sea and climate forecasting studies.
4. To educate undergraduate and graduate students and postdoctoral researchers in current methods of scientific IT research, by
   - communicating the revolutionary power of information technology for improving scientific understanding, inspiring student participation in IT research and applications
   - providing a complete research experience in start-of-the-art scientific IT methods to benefit their careers in the academic, scientific, and industrial communities
5. To initiate an interdisciplinary collaboration between mathematics and engineering researchers which will strengthen the long-term research efforts of both areas.

III. Proposed plan

A. Fundamental equations

Consider a region of space that contains air for $y > h(x,z,t)$ and water for $y < h(x,z,t)$. The coordinate system (x,y,z) here is defined so that the x-axis lies in a horizontal plane pointing in the direction of initial (horizontal) air flow, the y-axis points vertically upwards, and the z-axis lies in the horizontal plane, but transverse to the direction of initial air flow. The symbols ρ, $\mathbf{u}$ and p are used for the fluid density, velocity and pressure, respectively. Quantities in air and water are denoted by the superscripts (a) and (w), respectively, and the gravitational constant and the dynamic viscosities of the fluids are denoted by g (m/sec^2) and μ (kg/m/sec), respectively.

Since the flow of air and water separated by a sharp interface is considered in this project, fluid flows near the interface are of primary importance. It is therefore reasonable to follow standard practice and to approximate the densities of both fluids as constants in this limited region of space. This is consistent with viewing flows as incompressible due to the moderate wind speeds that will be simulated.

The equations of motion for the air or water are then given by the standard Navier-Stokes equations [38]:

$$\rho\,[\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u}] = -\nabla p + \mu\nabla^2\mathbf{u} - \rho g\mathbf{j}\,, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0\,. \tag{2}$$

where the subscript t indicates differentiation with time and $\mathbf{j}$ is a unit vector pointing vertically upward.

At the air-water interface, continuity of the velocity is required:

$$\mathbf{u}^{(a)} = \mathbf{u}^{(w)}\,, \tag{3}$$

along with continuity of the normal component of the deviatoric tangential stress:

$$\sigma_{nt}^{(a)} = \sigma_{nt}^{(w)}\,, \tag{4}$$

while the jump in the normal component of the deviatoric normal stress must balance surface tension forces:

$$p^{(a)} + \sigma_{nn}^{(a)} - p^{(w)} - \sigma_{nn}^{(w)} = T\,\kappa\,, \tag{5}$$

where T is the surface tension coefficient and has units of Newtons per meter and κ is the local curvature of the interface. These surface conditions are more difficult to treat numerically than the standard ones for the flow past a solid boundary; one of the primary project objectives is to develop reliable numerical methods to track the motion of the surface subject to these conditions.

An alternate formulation of the equations of motion (equations (1) and (2)) provides one way a numerical method can be developed. By taking the curl of equation (1) and using equation (2), the pressure is eliminated and an evolution equation for the vorticity

$$\boldsymbol{\omega} = \nabla\times\mathbf{u} \tag{6}$$

is obtained:

$$\rho\,[\boldsymbol{\omega}_t + (\mathbf{u}\cdot\nabla)\boldsymbol{\omega}] = \rho(\boldsymbol{\omega}\cdot\nabla)\mathbf{u} + \mu\nabla^2\boldsymbol{\omega}\,. \tag{7}$$

Equation (2) is solved by introducing the vector potential $\mathbf{A}$:

$$\mathbf{u} = \nabla\times\mathbf{A}\,. \tag{8}$$

The use of a vector identity [39] when equation (8) is substituted into the definition of vorticity (and invoking the choice of gauge, $\nabla\cdot\mathbf{A} = 0$) results in

$$\nabla^2\mathbf{A} = -\boldsymbol{\omega}\,. \tag{9}$$

This alternate choice of variables requires that the interfacial conditions be rewritten in terms of the vector potential and the vorticity. Before deriving these expressions, it is worthwhile to consider how the interfacial motion will be treated numerically; the interfacial conditions can then be expressed in a form appropriate for the chosen numerical approach.

There are several numerical approaches for evolving interfaces between incompressible fluids. Front-tracking [40],[41] and Volume-of-Fluid [42] methods generally represent the interface as straight line segments connecting points on a grid: they are not designed to capture the fine-scale boundary layers near the interface. Level-set methods [43] employ the machinery developed for propagating discontinuities such as shocks. Here the interface is represented as the level-set of some function, usually the distance from the interface, which is updated everywhere on the grid. For this method to succeed it must “smooth” the interface in the way that shocks are usually smoothed when propagated numerically through the grid. This smoothing will obviously mask the influence of physically important effects near the interface. Thus these methods do not appear optimal for an accurate study of the detailed interaction of air and water at the surface.

Finally there are mapping techniques [44], in which new coordinates, referred to as *logical coordinates*, are introduced so that the interface lies along a coordinate line. Furthermore, coordinate lines can be concentrated near the interface to ensure sufficient resolution of thin boundary layers. This approach appears to hold the highest promise for calculating the important effects of wind flow above the water surface.

A simple mapping is now used to illustrate the new form of the equations and interfacial conditions, and how they may be solved numerically. For ease of presentation, a two-dimensional flow is considered: all quantities are uniform in z, so that there is no flow in the z-direction. Both the vorticity and and the vector potential have only one component (parallel to the z-axis): $\boldsymbol{\omega} = \omega \mathbf{z}$, and $\mathbf{A} = \psi \mathbf{z}$, where ψ is referred to as the stream function. Components of the velocity are defined as $\mathbf{u} = (u, v)$.

Next introduce the logical coordinates,

$$X = x \, , \quad Y = y - h(x,t) \, , \quad \tau = t \tag{10}$$

where $y = h(x,t)$ defines the height of the water surface measured from the undisturbed water level. With subscripts indicating partial derivatives, the definitions of the vorticity (equation (6)) and the vector potential (equation (8)) produce

$$u = \psi_Y \, , \quad v = -\psi_X + h_X \psi_Y \, , \tag{11}$$

$$\omega = v_X - h_X v_Y - u_Y \tag{12}$$

The evolution of the vorticity from equation (7) is now written as

$$\rho \left[\omega_\tau - h_\tau \omega_Y + u \left(\omega_X - h_X \omega_Y \right) + v \omega_Y \right] = \mu \Big[\omega_{XX} + (1 + h_X^2) \omega_{YY} - 2 h_X \omega_{XY} - h_{XX} \omega_Y \Big] \tag{13}$$

and equation (9) becomes

$$\Big[\psi_{XX} + (1 + h_X^2) \psi_{YY} - 2 h_X \psi_{XY} - h_{XX} \psi_Y \Big] = -\omega \tag{14}$$

The success of this formulation depends on the ability to rewrite the interface conditions (equations (3)-(5)) in terms of the vorticity and stream function. A discussion of appropriate boundary conditions for solid boundaries is provided in [45]. First, equation (3) implies when $Y = 0$,

$$\psi^{(a)} = \psi^{(w)}, \quad \psi^{(a)}_Y = \psi^{(w)}_Y . \tag{15}$$

The continuity of tangential stress from equation (4) can be related to the jump in vorticity:

$$(1 + h_X^2)\left(\mu^{(a)}\omega^{(a)} - \mu^{(w)}\omega^{(w)}\right) = 2(\mu^{(a)} - \mu^{(w)})(v_X - h_X u_X) \tag{16}$$

where $u = u^{(a)} = u^{(w)}$ and $v = v^{(a)} = v^{(w)}$ indicate the velocities at the interface. Equation (16) contains jumps in μ and ω only since all other quantities are continuous across the interface. The fourth interfacial condition is derived by evaluating equation (1) on either side of the interface and requiring the jump in pressure to satisfy equation (5). After much algebra, the result is

$$\begin{aligned}
&(1 + h_x^2)(\mu^{(w)}\omega^{(w)}_Y - \mu^{(a)}\omega^{(a)}_Y) - \frac{2h_X^2 h_{XX}}{1 + h_X^2}(\mu^{(w)}\omega^{(w)} - \mu^{(a)}\omega^{(a)}) \\
&\quad + (1 - h_X)(\mu^{(w)}\omega^{(w)}_X - \mu^{(a)}\omega^{(a)}_X) \\
&= \quad 2(\mu^{(w)} - \mu^{(a)})\left[\frac{h_X h_{XX}}{1 + h_X^2}(u_X - h_X v_X) - u_{XX}\right] \\
&\quad - (\rho^{(w)} - \rho^{(a)})(u_\tau + h_X v_\tau - h_\tau v_X + u u_X + v v_X - g h_X) - T\kappa_X
\end{aligned} \tag{17}$$

This condition relates the jump in the Y-derivatives of the vorticity with the jump in the vorticity. Finally, the motion of the interface is determined by

$$h_\tau + u h_X = v \tag{18}$$

If solutions periodic in X are assumed, the question is whether the interfacial conditions (equations (15)-(17)) and appropriate far-field conditions are sufficient to determine a solution to equations (13), (14), and (18). The mathematical problem involves two diffusion equations for the vorticity (equation (13)), one in air and the other in water. The behavior of the vorticity in the far-field must also be specified, giving two conditions, and equations (16) and (17) provide a further two conditions, enough to determine a solution to the diffusion equations. There are also two elliptic equations for the stream function (equation (14)), one in each region. With two far-field conditions, the interfacial conditions (equation (15)) should be sufficient to determine a solution. Of course, these equations and interfacial conditions are all coupled, but the counting of equations and boundary conditions do balance, suggesting that solutions exist.

Note that this formulation explicitly models boundary layers in both the air and water regions. However, the relationships between the densities of air and water, $\rho^{(a)} = 1.2$ kg/m^3 and $\rho^{(w)} = 10^3$ kg/m^3, and their dynamic viscosities, $\mu^{(a)} = 1.8 \times 10^{-5}$ kg/(m sec) and $\mu^{(w)} = 1.1 \times 10^{-3}$ kg/(m sec), suggest that the boundary layer in air will have larger velocity gradients and extend further than in the boundary layer in water. Therefore, it is believed that reasonable solutions may still be obtained if the viscosity of water is neglected. In cases in which water viscosity effects are negligible, the vorticity created at the surface (equation (17)) no longer diffuses into the interior but remains a vortex sheet. Thus the fluid flow equations in the water region simplify and it is possible to construct solutions that involve information at the interface only. This is advantageous because the resulting combined air-water model should have smaller computational requirements. The combined model formulation for this case is described below.

The surface is parameterized as $(x(\xi,t), y(\xi,t))$ so that surface motion is described by

$$x_t(\xi,t) = u(\xi,t)\,, \quad y_t(\xi,t) = v(\xi,t)\,. \tag{19}$$

The time derivatives are Lagrangian, that is, ξ is kept fixed. If there is no initial vorticity in the water, then $\omega^{(w)} = 0$ always and the velocity potential $\mathbf{u} = \nabla\phi$ may be used. Consequently, Bernoulli's equation provides the evolution of the potential at the surface.

$$\rho^{(w)}\phi_t(\xi,t) = \frac{\rho^{(w)}}{2}(u^2+v^2) - p^{(w)} - \rho^{(w)}gy \tag{20}$$

The velocity at the surface may be expressed in terms of the potential and the stream function,

$$u(\xi,t) = \frac{x_\xi\phi_\xi + y_\xi\psi_\xi}{x_\xi^2 + y_\xi^2}\,, \quad v(\xi,t) = \frac{y_\xi\phi_\xi - x_\xi\psi_\xi}{x_\xi^2 + y_\xi^2}\,. \tag{21}$$

Finally, the potential and stream function can be connected through a dipole distribution Γ along the surface (representing the jump in circulation across the surface):

$$\phi(\xi) = \frac{1}{4\pi}\oint\!\!\!\!\!\!-\; \Gamma(\xi')\frac{y_\xi(\xi')\sin\left(x(\xi)-x(\xi')\right) - x_\xi(\xi')\sinh\left(y(\xi)-y(\xi')\right)}{\cosh\left(y(\xi)-y(\xi')\right) - \cos\left(x(\xi)-y(\xi')\right)}\,d\xi' + \frac{\Gamma(\xi)}{2} \tag{22}$$

$$\psi(\xi) = \frac{1}{4\pi}\oint\!\!\!\!\!\!-\; \Gamma(\xi')\frac{x_\xi(\xi')\sin\left(x(\xi)-x(\xi')\right) + y_\xi(\xi')\sinh\left(y(\xi)-y(\xi')\right)}{\cosh\left(y(\xi)-y(\xi')\right) - \cos\left(x(\xi)-y(\xi')\right)}\,d\xi' \tag{23}$$

where the integrals are principal-valued ones.

To include the dynamic influence of the air, consider $\phi(\xi)$, $x(\xi)$ and $y(\xi)$, and ω in the air region as known. Then equation (22) may be solved for Γ, and $\psi(\xi)$ subsequently determined from equation (23). Thus velocities at the interface can be determined from equation (21), followed by h_τ. Since the velocities at the interface are known, $\psi^{(a)}$ and $\psi_Y^{(a)}$ are known. Thus equations (13) and (14) can be solved for the air region as described in the next section. Since $\omega^{(w)}$ remains zero, equation (17) may be used to determine u_τ and v_τ at the interface as well as the arclength derivative of ϕ_t, so that both the interface and ϕ can be updated. Note that this coupled solution is very different from the procedure used in earlier studies in which the motion of the water surface is prescribed and the influence of the air pressure is not accounted for directly.

Since models with and without water viscosity will be available, tests will be conducted to clarify when an irrotational model in the water region is valid. Further studies will then be based on the appropriate choice. It is anticipated that water viscosity effects may be more important at early times (particularly if the water boundary is initially flat), but that pressure effects will dominate at later times. Thus a formulation in which the water viscosity effects are included initially and later neglected will also be considered.

Finally, note that if $p^{(w)}$ is known, Bernoulli's equation (20) may be used directly to update the potential. In classical water wave theory, $p^{(w)} = 0$, i.e., the dynamic influence of the air is neglected. One of project goals is to determine through statistical studies whether an empirical $p^{(w)}$ can be specified that models the dynamical influences of the air flow on the water surface. Obviously, $p^{(w)}$ will depend on several factors, such as wind speed, local wavenumber, amplitude of a water wave, etc., and many experiments will be needed to assess whether a functional can be found. If successful, numerical studies of the full nonlinear interaction of ocean waves over long distances and for long times will be possible through the application of the boundary integral method described above.

B. Numerical solution

The two important issues in designing an algorithm to solve the equations of motion of the coupled air-water system are accuracy and stability. Since important behavior occurs over a large range of spatial scales, the algorithm must execute with as high a resolution as possible. Parallel computing provides the only practical way to achieve high resolution at moderate execution times. Since a 64-processor Beowulf system is available, the potential for parallelization of the algorithm will be discussed where appropriate.

Since solutions are assumed to be periodic in X, all variables can be expanded in a Fourier series in X and their coefficients determined through the application of Fourier transforms. Furthermore, parallel implementations of the fast Fourier transform [46] are readily available for computing the necessary transforms; Fourier coefficients of products of two variables will be determined in parallel through pseudo-spectral techniques [47]. The flow equations (13), (14), and (18) then collapse into a system of differential equations in τ and Y for the Fourier coefficients Ω_k and Ψ_k of the vorticity and stream function respectively.

$$\rho(\Omega_k)_\tau + \rho F_k = \mu\left[(\Omega_k)_{YY} - k^2\Omega_k + G_k\right] \tag{24}$$

$$(\Psi_k)_{YY} - k^2\Psi_k + H_k + \Omega_k = 0 \tag{25}$$

where F_k, G_k and K_k are the Fourier coefficients of

$$\begin{aligned} F &= u(\omega_X - h_X\omega_Y) + (v - h_\tau)\omega_Y & (26)\\ G &= h_x^2\omega_{YY} - 2h_X\omega_{XY} - h_{XX}\omega_Y & (27)\\ K &= h_x^2\psi_{YY} - 2h_X\psi_{XY} - h_{XX}\psi_Y & (28) \end{aligned}$$

These equations are written in this form to facilitate the subsequent discussion on choices for the time-stepping procedure. Equations (24,25) must be solved subject to far-field conditions and the transformed interfacial conditions (15-18). In the far-field, Ω_k , $\Psi_k \rightarrow 0$, except for $\Psi_0 \rightarrow UY$ where U is the wind speed.

There are several choices for time-stepping and approximating derivatives in Y, but for purposes of illustration, simple choices will be made. Second-order finite differences will be used to approximate the derivatives in Y and fictitious points will be introduced at the interface to approximate the derivatives there ([48]). The next important issue is the numerical time-stepping of the method. Since vorticity will be created continuously at the interface and will diffuse into the interior it seems prudent to use an implicit method on the diffusion terms in equation (24). On the other hand, explicit methods for the motion of the interface have proved successful. Let $\Omega_{k,n}$, $\Psi_{k,n}$ and $H_{k,n}$ be the kth Fourier coefficient of ω, ψ and h respectively, evaluated at time $\tau = n\triangle\tau$. Then the interface can be advanced by the third-order Adams-Bashforth method,

$$H_{k,n+1} = H_{k,n} + \frac{3}{2}\left(v - h_X u\right)_{k,n} - \frac{1}{2}\left(v - h_X u\right)_{k,n-1} \tag{29}$$

and the vorticity by the combined method,

$$\begin{aligned} &\rho s\Omega_{k,n+1} - \mu(\delta_Y^2\Omega_{k,n+1} - k^2\Omega_{k,n+1} + F_{k,n+1}) \\ &\quad = \quad \rho s\Omega_{k,n} + \mu(\delta_Y^2\Omega_{k,n} - k^2\Omega_{k,n} + F_{k,n}) + \frac{3\rho}{2}G_{k,n} - \frac{\rho}{2}G_{k,n-1} \end{aligned} \tag{30}$$

where $s = 1/(\triangle\tau)$ and δ_Y^2 is the standard second-order difference in the Y-direction. At the same time, the discrete version of equation (14) must be solved:

$$\delta_Y^2\Psi_{k,n+1} - k^2\Psi_{k,n+1} + K_{k,n+1} + \omega_{k,n+1} = 0 \tag{31}$$

To these equations must be add the transformed interfacial conditions (15-17) and the far-field conditions. Standard practice is to use a Neumann-Dirichlet map at finite height L above the undisturbed water surface (assumed to be at $Y = 0$). By linearizing the discrete equations (30,31) about the flow conditions in the far-field, homogeneous solutions of the form $\exp(-r_k Y)$ can be obtained. The appropriate condition at $Y = L$ is then $(\Omega_k)_Y + r_k\Omega_k = 0$ (which excludes the solution $\exp(r_k Y)$), and a similar condition for Ψ_k can be obtained. Similar expressions at $Y = -L$ can be derived.

Equations (30,31) and the interfacial and far-field conditions constitute a very large system of nonlinear algebraic equations. This system can be solved by Newton's method. However, the Jacobian matrix for the system will be full because of the terms $F_{k,n+1}$ and $K_{k,n+1}$ in (30) and (31) respectively, and a direct inversion will be too costly. Instead, the GMRES algorithm [49] will be applied to solve these equations iteratively. GMRES is best used in conjunction with a preconditioner, and a natural one emerges when $h = 0$. With this choice, the preconditioner is equivalent to computing the solution to

$$\rho s\Omega_{k,n+1} - \mu(\delta_Y^2\Omega_{k,n+1} - k^2\Omega_{k,n+1}) = R_1 \tag{32}$$

$$\delta_Y^2\Psi_{k,n+1} - k^2\Psi_{k,n+1} + \omega_{k,n+1} = R_2 \tag{33}$$

where R_1 and R_2 are the residuals passed back from the GMRES subroutine. They will contain, for example, the quantities $F_{k,n+1}$ and $K_{k,n+1}$ evaluated through pseudo-spectral methods using data at the current iteration. The system of equations (32) and (23) is a coupled tridiagonal system for each set of Fourier coefficients and may be solved in parallel for each choice of k. The operation count for the preconditioner solution is dominated by $\mathrm{O}(NM\log_2(N))$ where N and M are the number of points used in X and Y respectively. From previous experience in applying these ideas to the growth of crystal from a hot melt in a Bridgman device [50]-[51], it is anticipated that the GMRES algorithm should converge in 10-20 iterations.

There are many possible refinements to improve accuracy and/or stability. The interface location can also be updated implicitly if time-stepping instabilities are observed. Higher-order compact differencing or additional spectral methods [52] can be used to improve accuracy in Y. A modified mapping that concentrates more points near $Y = 0$ [44] can also be applied. Although the previous discussion has emphasized the two-dimensional case, the extension to three dimensions is similar in that the Fourier transform must be applied in both X and Z directions. This still leads to a system of the form (24) and (25).

C. Analysis of model

1. Validation of Mathematical Model

Use of the numerical models will begin with validation tests of the air-flow over a fixed boundary, and with water flow without wind forcing. Results will be compared with analytical solutions [3]-[9], results from previous numerical studies [22]-[29], and with standard hydrodynamic codes [30]-[35] to insure that a correct implementation is obtained in these limits. These tests will be performed for both the two-dimensional and three-dimensional codes.

The first validation of a coupled air-water case will be the examination of the initial growth rates of short waves as turbulent air forcing is introduced onto a slightly perturbed water boundary. Again, several analytical, numerical, and empirical results [1]-[2] exist for comparison. The temporal dependence of water wave growth will be carefully examined to determine if the expected exponential behavior is obtained over short times. In the three-dimensional case, directional effects (such as the "cat's paws" pattern observed on an initially perturbed surface) will be examined as well. To assess the importance of nonlinear hydrodynamic effects in the initial wave growth period, simulations will also be performed in which hydrodynamic evolution is approximated as linear; it is expected that nonlinear effects will show a significant contribution to the observed wave growth as time increases. As described previously, tests will also be performed to determine the importance of near surface water turbulence at longer time periods; if negligible, the boundary solution for the water region will be used to improve computational efficiency.

In all tests, scientific visualization resources will be applied to improve the understanding of detailed air and water turbulence structures. Effective public-domain visualization tools are currently available [53]-[54] which enable complex three-dimensional fluid flows to be displayed and manipulated on standard computing platforms. Careful analysis of the near surface air and water flows will be performed to classify important forcing effects on the water boundary. Note that the libraries of [54] are developed for the Java language, enabling platform-independent visualizations to be developed for the world-wide-web. Use of these visualizations will serve as an effective tool for communicating project goals and results to interested students and researchers.

The range of computational parameters will also be considered in the initial tests to help clarify the numerical requirements of the simulation. Parameters such as the domain size, the air and water mesh sizes, the time step, the initial state of the air and water, and the methods used to initialize the air-region turbulence will all be examined to ensure sufficient accuracy and efficiency in validation tests and in further studies. These tests will also clarify the computational limitations of the model and the Beowulf platforms in terms of domain size, wind speeds, and time duration. It is expected that domain sizes on the order of tens of meters and wind speeds up to several meters per second should be possible with the two-dimensional model, although three-dimensional studies likely will be limited to domain sizes of approximately one meter with wind speeds less than four to five meters per second. These parameters are sufficient to allow useful information on wave growth and surface spectral evolution to be determined.

2. Detailed simulations

The validated model will then be applied in longer simulations of air flow over water, with the goal of determining whether a statistical steady state is reached. Air flow will be initialized as laminar flow with small random perturbations introduced to initiate turbulence, while the water boundary will be modeled as either flat or of a specified form at time zero. The detailed air turbulence structure and growth of the interfacial boundary layer will be examined to investigate the applicability of approximate theories and closure models applied in earlier studies. If any closure models are found to be appropriate, the algorithm will be modified for their inclusion to allow more efficient calculations. Particular attention will be given to determination of the "effective roughness parameter" z_0 and the effective drag coefficient C_D of the air flow, and the dependence of these parameters on the underlying water surface. Fourier analysis will be used to consider "wave-induced" structures in the air flow (structures correlated with the surface wave phase [2],[55]) and to determine if such features can be identified with water surfaces more complex than Stokes' waves. Again scientific visualization resources will be applied to enable detailed analysis of simulation results, while the effects of the numerical model parameters will be carefully assessed to insure that model accuracy is retained.

Repeated simulations involving varying turbulence initializations and initial water profiles will also be performed to allow ensemble averages to be computed as well as temporal averages over long time computations. If a statistical equilibrium is reached, the form of the resulting water surface spectrum will be determined, as well as the rates of energy transfer both among differing surface wave length scales and between the air and water regions. Variations in the resulting surface wave spectrum with initial wind speed will be compared with results from the literature to determine if similar dependencies are obtained. These comparisons may be limited by the fact that wave breaking effects are not included in the coupled air-water model. Empirical dissipation terms which model breaking wave effects will be incorporated approximately in the studies described below.

3. Decoupling studies

A primary goal of this project is to understand to what extent the effects of wind can be decoupled from the evolution of water waves and thought of as providing simply a forcing input in terms of a surface pressure distribution. Clearly, if water viscosity is neglected, a pressure forcing term represents the only possible influence of the air on the water's evolution. Further studies will analyze the surface pressure distribution from the coupled model over time to clarify its temporal evolution and to determine if a statistical steady state is reached. If so, a model based on this steady distribution will be used as an input to a water-only model to determine if similar statistical results are obtained. Of particular interest is the wavenumber range over which wind forcing is significant, which could imply to what extent the generation of long waves from shorter waves occurs (as predicted by empirical forcing models). It is expected that more general *dynamic* statistical models of wind forcing will be necessary and appropriate forms for such models will be explored. For verification, predictions from the developed wind-forcing model will be compared with existing empirical or approximate models in the appropriate limits and surface statistics resulting from decoupled simulations under the forcing model will be compared to those from the coupled model. A new surface-dependent wave growth parameter would represent a dramatic improvement over typical sea-surface-independent forms now available and would be of vital importance in further studies of wind-wave interactions in many disciplines.

D. Hydrodynamic turbulence studies

Although the coupled wind-wave model will address many of the issues involved in ocean surface wave prediction, the computational complexity of the model will limit studies to moderate-sized geometries. The verified wind-forcing model extracted from the air-water numerical studies, however, will allow larger scale "water-only" computations to be performed on more realistic cases. Studies of wave turbulence using the wind-forcing model will be continued through use of analytical [16]-[19], approximate numerical [20]-[21],[31]-[35], and purely numerical [30] methods under appropriate conditions. Again both two- and three-dimensional geometries will be used, but domain sizes in the numerical studies will be significantly larger than in the coupled air-water study. These larger domain sizes will allow improved resolution of the low frequency portion of the sea spectrum (which is most accurately known experimentally), correspondingly higher wind speeds, and greater temporal and ensemble averaging. Energy spectra resulting from these studies will again be compared with those from the literature [12]-[15] to determine the significance of the new wind forcing model. Results will also be compared with direct and remotely-sensed oceanographic data for verification.

Note that since wind forcing and wave-wave interactions should be captured accurately in these simulations, wave breaking will remain as the only mechanism neglected. Breaking wave effects can be included approximately in hydrodynamic simulations by using certain dissipative factors available in the literature. Such calculations can be performed with several choices of the many available to determine the possible range of spectral variations that may be expected.

Final forms of for the sea surface spectrum will be classified and communicated to the applied scientific communities for use in oceanographic studies across many disciplines; improved understanding of and mathematical forms for the sea surface spectrum are of great importance in numerous applications.

E. Undergraduate and graduate education

Education is also a primary objective of this project, through both research training and graduate and undergraduate course efforts. The project will provide support for a postdoctoral researcher (KMB) in the Department of Mathematics, and experience obtained from participation in the project will contribute to the development of IT research expertise for use in a future faculty position. Three graduate students will be involved throughout the project in code development, testing, application, and interpretation of results to insure that a complete scientific IT research experience is provided. The interdisciplinary nature of the project will also provide these researchers with a broader perspective from which to perform future research. Courses taught by the project team at both the graduate and undergraduate level will incorporate project issues and results to provide project experiences for students and to inspire interest in scientific IT applications. The world-wide-web (WWW) will be used in part as a tool to communicate with students, by maintaining a project web page, which will include several examples of "impressive" project results and visualizations and which will describe the current research efforts of the project team. Course and WWW methods will also be used to recruit undergraduate researchers to the project team; Ohio State currently maintains an "honors thesis" program under which students can obtain a degree with distinction for participation in a research project. NSF REU grants will also be pursued as necessary to support undergraduate research participation. For both graduate and undergraduate participation, candidates from under-represented groups will be sought to improve their educational experience and to increase the pool of qualified scientific professionals.

F. Information technology aspects

The numerical models developed as part of this project will extend state-of-the-art algorithms for computational fluid dynamics in the presence of complex boundaries and will be applicable with only modest modifications to a range of problems in many other disciplines. The parallel computing methods to be applied are important in that they make large simulations possible; the experience obtained by junior project personnel in this area will enable them to utilize parallel computing resources in their future scientific studies. Scientific visualization methods to be used for understanding complex fluid flows will also be critical to project success; results from such visualizations will be utilized as an effective "marketing" tool to communicate project results and to encourage undergraduate student interest in IT research. Fundamentally, the entire project is possible due to the computational power and efficient scientific algorithms that have resulted from previous IT research. The project will continue this research and contribute to future improvements in scientific computing.

G. Presentation and dissemination of results

Project results will be communicated to the scientific community through annual presentations at scientific conferences, refereed journal publications, and through the use of the world-wide-web. Publications regarding code development and testing, results of simulations, parameterization of wind-forcing effects, hydrodynamic studies, and the implications of the model for oceanographic and atmospheric communities are expected. The final results of the project in terms of a verified improved model for ocean wave generation by wind will be provided to scientific organizations to insure that the model is used in future studies. The interdisciplinary nature of the project will insure effective communication of project results to both the theoretical and applied scientific communities.

IV. Significance and Impact

Due to the dramatic impact of ocean waves on sea commerce, transportation, and coastal regions, and the important role that air-sea interaction plays in long-term climate forecasting, understanding the process by which wind waves are generated is of great importance. Current forecasts of sea and climate conditions often contain significant errors [6] which can be reduced if more effective procedures based on improved physical understanding are developed. The proposed project will provide a conclusive, verified means for obtaining this understanding. The results of the project will also improve models for the sea surface directional spectrum and its dependence on atmospheric effects, and clarify the important physical processes that contribute to this spectrum. Accurate models for the sea surface spectrum are important in many disciplines. Project results will improve interpretation of direct and remotely sensed oceanographic data so that more effective studies of the sea can be performed; surface profiles from the current project can be applied directly in radar imaging calculations (see [56]-[59].) Scientific computing methods developed in the project will provide more efficient algorithms for fluid flow over complex evolving boundaries which can find application in a variety of disciplines. Educational efforts of the project will produce well trained faculty, graduate, and undergraduate researchers for future IT research, and will inspire other students to continue their studies of IT applications.

V. Relation to current knowledge in field

Current knowledge of the generation of wind-waves is based primarily on empirical models, due to the general failure of the predictions of approximate analytical models when compared with measured data. Even the measurements themselves are influenced by a number of physical factors whose effects are difficult to isolate due to the nonlinear dynamics of the problem. Recent numerical models for wind flow over water waves have neglected the nonlinear dynamics of the water surface, and therefore cannot be considered complete. The studies proposed in this document address these problems by including both air and water dynamics in a complete simulation; analysis of the results in the "controlled" numerical model should clarify important effects in the generation of wind-waves and illustrate when air-only or water-only models are appropriate. Models in the oceanographic and remote-sensing communities for sea forecasting and data interpretation are based on approximate hydrodynamic models which include empirical wind forcing terms approximately. The lack of realistic air flow effects in these models makes the accuracy of their predictions unclear. Current models of the sea surface directional spectrum are based on sea measured data combined with wave-tank data [12]-[15]; the dramatic differences between these and other models demonstrates the need for the proposed project.

VI. Relation to long term project team goals

The proposed study will serve as the initial support for an interdisciplinary collaboration between mathematics and engineering faculty at The Ohio State University. Expertise of the mathematics project members in model development will complement with that of the engineering project members in result assessment and application to produce an effective team. The collaboration will insure that mathematical models developed are communicated to and used in the applied scientific community, and that experience from the applied community will be used to assess model results and to guide continued model development. The collaboration is expected to continue well beyond the period of the current proposal if initial support is obtained, since further projects involving applications of the current project's wind-wave model should be possible. The project will also serve to support continued growth and development of the applied mathematics (Dept. of Mathematics) and remote sensing theory (Dept. of Electrical Engineering) programs at Ohio State, allowing improved student education and research efforts of these areas far into the future.

VII. Results from Prior NSF Support

Co-PI Gregory Baker has not been supported by the NSF during the last five years.

Co-PI Joel T. Johnson is supported by an NSF CAREER grant (Project ECS-9701678, $210,000 for 7/1/97 to 6/30/01) titled "Surface scattering effects in mine detection and remote sensing systems." Under this project, models and experiments to improve understanding of soil surface scattering effects on microwave sensing systems have been developed. Four graduate students were fully or partially supported by this project (one female), with two master's and one doctoral degree granted at present. The project produced 17 conference papers, six published or accepted journal articles, and three journal articles currently under review. The results of this project further emphasize the importance of accurate surface descriptions if realistic microwave scattering responses are to be simulated.

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