Further Numerical Studies of Backscattering from Time Evolving Non-linear Sea Surfaces

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Abstract—Previous studies have demonstrated that the West et al. model for non-linear hydrodynamic evolution of a sea surface realization produces significant features in calculated L-band backscattered Doppler spectra compared to a linear sea surface evolution model. Previous comparisons, however, were limited to a maximum wind speed of 2 m/s due to failure of the West et al. algorithm when steep short-wave features formed in the surface. In this paper, L-band Doppler spectra with the West et al. model are reported for wind speeds up to 4 m/s through the use of a suppression filter to reduce steep short-wave features. Results again show significant effects on Doppler spectra, including a strong polarization sensitivity at observation angle 80 degrees.

1. INTRODUCTION

Time evolution of the sea surface causes backscattered radar returns to be spread into a Doppler frequency spectrum; this effect has important implications for many sea remote sensing systems. Scattering predictions based on the first order small perturbation method and a linear model of sea surface evolution produce a single Doppler frequency component at the temporal frequency of the Bragg wave in the surface spectrum. Measured microwave Doppler spectra however show more features than a single Bragg line, and also exhibit complex incidence angle and polarization dependencies. More realistic models for backscattered Doppler spectra require both more accurate electromagnetic scattering models and more accurate simulations of ocean surface dynamics, which are inherently non-linear.

Combined numerical hydrodynamic and electromagnetic Monte Carlo simulations with one-dimensional (1-D) surfaces [1]-[2] have recently demonstrated a significant influence of non-linear hydrodynamics on L-band backscattering. Reference [2] employed the West et al. hydrodynamic model [3], and demonstrated strong Doppler spectrum broadening compared to a linear model, including some evidence of reverse traveling features in the sea surface. However, the West et al. model was also found to suffer from instabilities at wind speeds greater than 2 m/s in these simulations, due to the formation of steep short wave features that invalidate the slope expansion implicit in the formulation.

In this paper, further coupled hydrodynamic and electromagnetic simulations are described that extend the previous results up to wind speed 4 m/s. This is accomplished through inclusion of a steep wave suppression filter in the West et al. model. The basic hydrodynamic algorithm and the suppression filter are discussed in the next section, and sample hydrodynamic and Doppler spectra results presented in Section 3.

2. METHODOLOGY

Time evolution of an 1-D irrotational, incompressible fluid surface is governed by the following pair of non-linear partial differential equations:

$$\frac{\partial h}{\partial t} = \frac{\partial \phi}{\partial z}\left[1 + \left(\frac{\partial h}{\partial x}\right)^2\right] - \frac{\partial h}{\partial x}\frac{\partial \phi}{\partial x} \tag{1}$$

$$\frac{\partial \phi}{\partial t} = -gh - \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \phi}{\partial z}\right)^2\left[1 + \left(\frac{\partial h}{\partial x}\right)^2\right] \tag{2}$$

where $\phi(x, z = h(x,t), t)$ is the fluid velocity potential, $h(x,t)$ is the fluid surface profile, g denotes the gravitational acceleration, and x and t are the spatial and temporal coordinates, respectively. For a given initial condition, an exact solution to these equations is unknown, making a numerical solution in the form of a time-stepping process necessary to determine time evolution of h and ϕ. The primary difficulty in this process is evaluating the $\frac{\partial \phi}{\partial z}$ term, since ϕ is known only on the surface. The West et al. method addresses this problem through a perturbation series for the vertical component of the velocity. The resulting expansion is in terms of surface slope, and therefore experiences poor convergence in regions where local slopes become large.

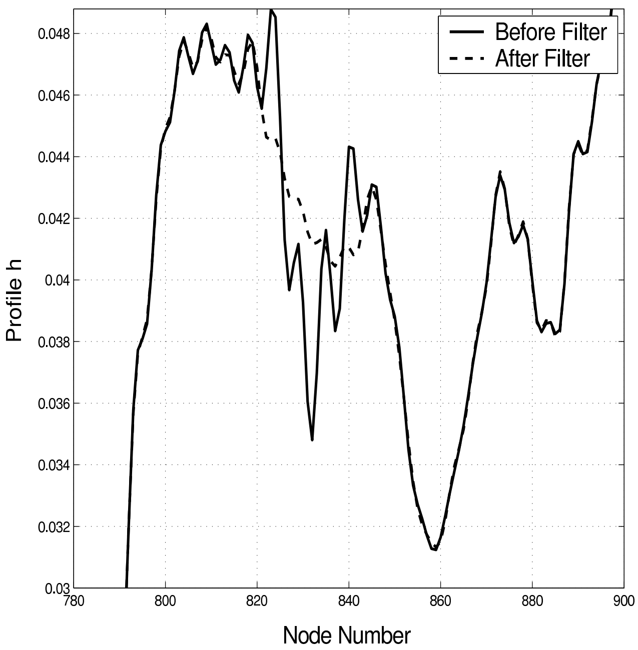

Figure 1: Sample portion of a 4 m/s surface profile before and after application of suppression filter

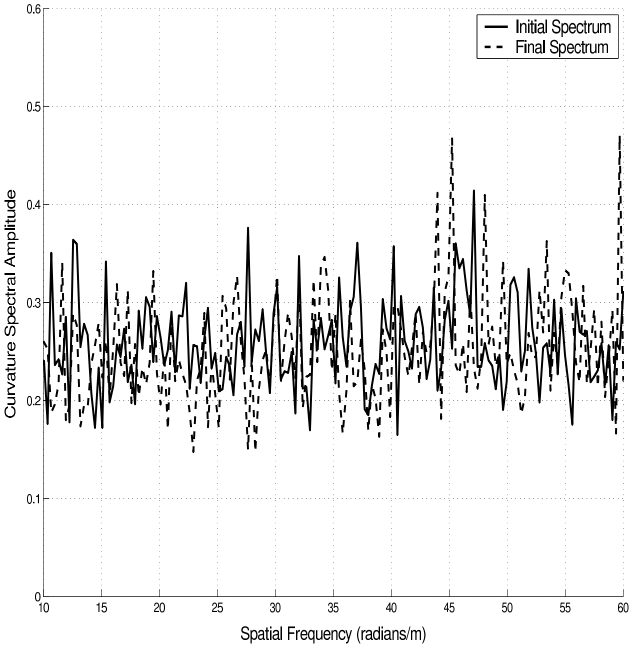

Figure 2: Comparison of initial and final average curvature spectra for wind speed 4 m/s

Methods for reducing this problem in the West et al. method have been considered in the literature. Reference [4] proposes a local curvature threshhold as a means of detecting potential problematic points in the surface profile. This approach is applied here by calculating the local curvature, κ, at each surface point, and then by multiplying κ by a, the maximum amplitude of the wave. For a 1-D surface $h(x,t)$, κ can be written as:

$$\kappa = \frac{|h''|}{\left[1 + (h')^2\right]^{3/2}} \tag{3}$$

where $'$ indicates differentiation with respect to x (implemented spectrally in the code developed). The threshhold value, $(\kappa a)_{\max}$, is a parameter of the detector. Observations of several surface profiles for the simulations described in Section 3 showed a value of 12 to be an effective indicator of points that would likely cause the time evolution to fail at future time steps. To suppress problems associated with these points, a region of $\pm Q$ points is removed from both the surface profile h and velocity potential ϕ surrounding each detected point. The deleted points are replaced with a spline interpolation to the remaining profile and velocity potential points, respectively. The process is performed for all detected points, beginning with the largest value of κa and proceeding to the smallest value. The value Q is another parameter of the filter; tests with several profiles for the simulations described in Section 3 showed a value of 10 points to be effective.

Assessing the influence of this filter on surface hydrodynamic evolution is difficult given the complexity of the non-linear equations. Of course, it is desirable to minimize the influence so that no strong non-physical effects are introduced. Doppler spectra results varying the values of $(\kappa a)_{\max}$ and Q can be used to provide some insight into filter influence, and are currently in progress. Statistics on the number of points modified and the frequency of filter operation can also provide some information, and are discussed below.

3. RESULTS

The hydrodynamic evolution of equations (1) and (2) was initialized with a realization of a Pierson-Moskowitz (P-M) spectrum (as in [1]-[2]) and then advanced in time using a fourth-order Adams-Bashforth predictor-corrector scheme. An initial adjustment period was also included in order to gradually include non-linear terms so that discontinuities in the initial conditions are avoided [5]. Terms up to fourth order in the West et al. expansion were included. Electromagnetic scattering calculations again used the numerical procedure described in [2] to compute L-band (23 cm wavelength) backscattering under "tapered beam" illumination with an impedance boundary condition model of the sea water.

To investigate the hydrodynamic effects of the suppres-

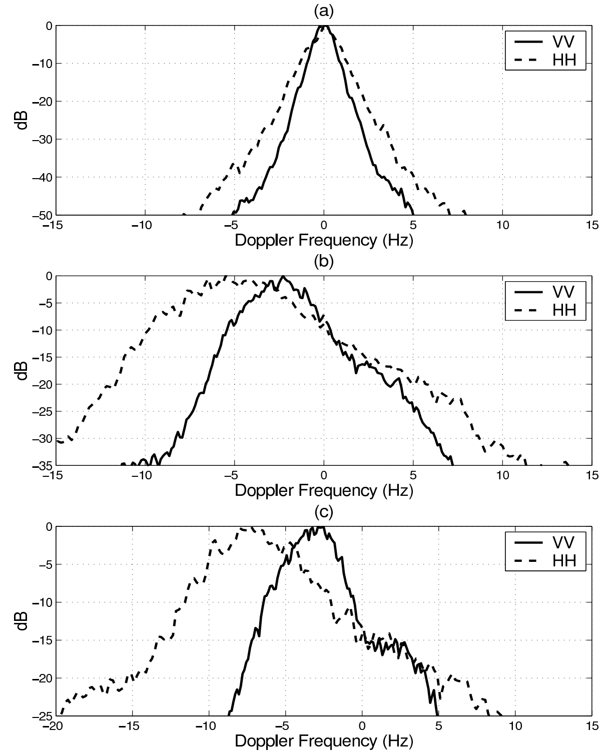

Figure 3: L-band normalized backscattered Doppler spectra at 4 m/s (a) 0°, (b) 40°, (c) 80°.

sion filter, an initial Monte Carlo simulation was performed with thirty 20 m surfaces sampled into 4096 points and time evolved for a period of 8.34 seconds in 2085 time steps of 4 msec. The P-M spectrum with wind speed 4 m/s was used to generate initial conditions. Figure 1 illustrates a portion of a sample surface realization before and after the suppression filter is applied, and shows the suppression of steep short wave features achieved by the filter. The suppression filter was found to activate only an average of 51 out of 2085 time steps (2.45%) for the thirty realizations, and several realizations occurred for which the filter activated only once or not at all. An average of 53 out of the 4096 surface points (1.29%) were interpolated on each filter activation. While these numbers are not negligible, their relatively small values suggest that the influence of the filter on average surface hydrodynamics and eventually on surface backscattered Doppler spectra may be only minor. As further evidence of limited filter effects, Figure 2 compares initial ($t = 0$) and final ($t = 8.34$ s) average curvature spectra for the thirty realizations. The initial P-M surface appears as approximately a straight line for this range of frequencies (i.e. a k^{-3} spectrum) with some residual stochastic variations. The final curvature spectrum is similar, indicating that the filter is not causing strong changes in the overall surface spectrum.

Backscattered field Doppler spectra were then computed at wind speed 4 m/s and observation angles of 0, 40, and 80 degrees from normal incidence. A surface of 117.81 m sampled into 16384 points was used for all angles to avoid diffraction effects at the largest observation angle and to avoid hydrodynamic aliasing problems in the Bragg spectral region. Doppler spectra were computed from 256 backscattered field time samples at time interval 20 msec for an ensemble of 50 surface realizations. Results in Figure 3 show Doppler spectra that are broader than those observed in the previous 2 m/s wind speed study [2]. Some evidence of reverse traveling features (i.e. the plateau at positive Doppler frequencies) in the surface is also evident in the 40 and 80 degree results, although these features are again broader than those observed at 2 m/s windspeed. Finally, significant polarization dependencies are observed, with HH polarization showing a much larger mean Doppler shift than VV. These results appear to capture some of the "fast scatterer" phenomena observed in measured sea backscattering data. Further investigations of the sources of these effects are currently in progress.

4. REFERENCES

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