Numerical Studies of Backscattering from Time Evolving Sea Surfaces: Comparison of Hydrodynamic Models

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Abstract—Results from a Monte Carlo simulation of L-Band backscattered Doppler spectra at 10 degrees grazing incidence obtained from one dimensional time evolving sea surface models are reported. A numerical method is used to calculate backscattered returns from surface profiles, which are initialized as realizations of a Pierson-Moskowitz spectrum at windspeed 2 m/s and then stepped in time through numerical hydrodynamic solutions. Results obtained from three distinct methods for time evolution of the surface profile are compared, and yield differing predictions of Doppler spectra for the case considered.

1. INTRODUCTION

Time evolution of the sea surface causes backscattered radar returns to be spread into a Doppler frequency spectrum. Scattering predictions based on the first order small perturbation method and a linear model of sea surface evolution produce a single Doppler frequency component at the temporal frequency of the Bragg wave in the surface spectrum. Measured microwave Doppler spectra however show more features than a single Bragg line, and also exhibit complex incidence angle and polarization dependencies. More realistic models for backscattered Doppler spectra require both more accurate electromagnetic scattering models and more accurate simulations of ocean surface dynamics, which are inherently non-linear.

In this paper, studies of 1.3 GHz backscattered Doppler spectra obtained from one dimensional time evolving surfaces are presented. A numerical electromagnetic scattering code is applied to avoid any scattering approximations, and ocean surface dynamics are computed numerically under three distinct hydrodynamic models.

2. SCATTERING AND SURFACE MODELS

Doppler spectra are determined from backscattered fields computed with "time frozen" surfaces, repeated for a series of points in time as the surfaces evolve. A Monte Carlo simulation is used to obtain average Doppler spectra, so that repeated calculations are necessary both in surface realizations and in time steps. The large number of computations involved makes use of an efficient numerical method imperative; the forward-backward method [1] with a spectral acceleration [2] is well suited for this purpose. Parallel Monte Carlo simulations performed with the IBM SP/2 at the Maui High Performance Computing Center (*www.mhpcc.edu*) further enable large scale simulations to be completed in a reasonable time.

To model sea surface backscattered Doppler spectra accurately, the time evolution of the surface should be captured in detail. A linear hydrodynamic approximation yields a simple set of sinusoidal waves which can be propagated analytically using a dispersion relation, taken to be the deep water gravity wave equation for this paper. More complete hydrodynamic theories can be applied to predict interaction effects between these sinusoidal waves, but the non-linear equations obtained make numerical solution computationally expensive. Efficient approaches which approximate the non-linear dynamics of sea surface evolution are therefore of interest. One method previously applied in scattering studies is based on the approximation of Creamer et al [3], which performs a canonical transformation of the hydrodynamic equations to eliminate the leading non-linear term and approximates the transformed system as linear. Time evolution of the surface is then performed analytically since the underlying system propagates linearly; the transformation back to the physical surface however requires an $O(N^2)$ operation, where N is the number of surface profile points.

A second method is the Watson-West method [4], which solves the non-linear equations directly but expands the

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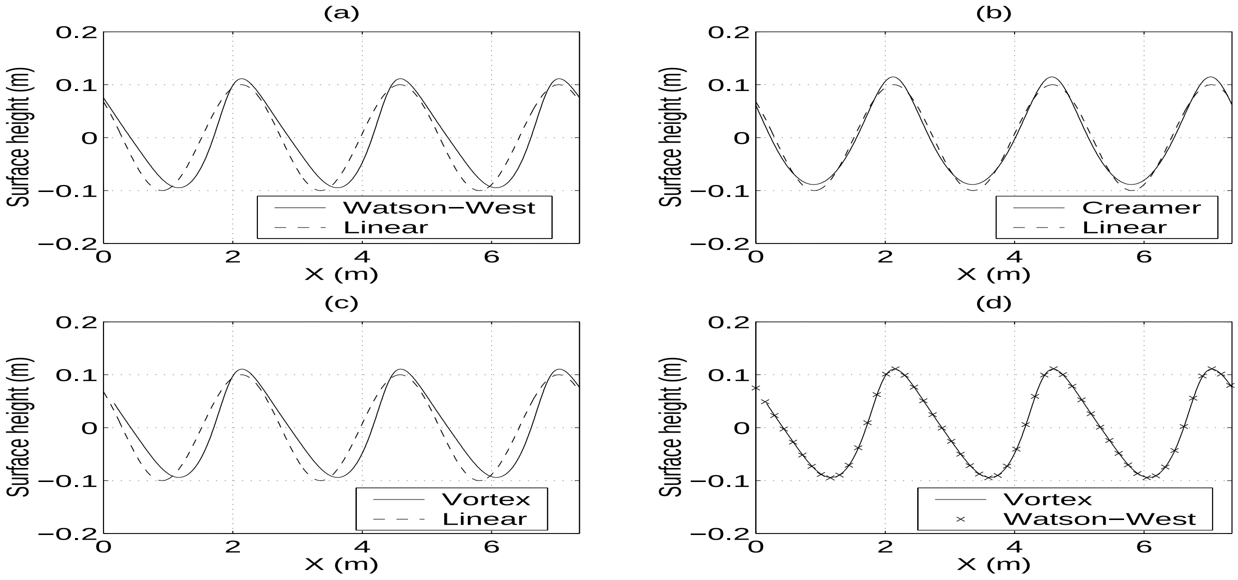

Figure 1: Sinusoidal surface after time evolution: comparison of hydrodynamic models

normal derivative operators in these equations into a translationally invariant series in the surface height times slope product. The resulting algorithm requires a numerical time stepping method, but is $O(N \log N)$ due to the expansion used. The number of series terms (also called the order of the method) needed increases as surface heights or slopes increase, so that the approach eventually fails as surface roughness increases. Although the Watson-West method can be expected to capture non-linear hydrodynamic effects more accurately than the Creamer method due to its direct equation solution, the numerical time stepping algorithm used requires a fine time step in order to remain stable, and the approach is limited to moderately rough surfaces only.

Figure 1 provides a simple illustration of some of the distinctions between hydrodynamic models. The plots are of a 2.45 m, 0.2 m peak to peak amplitude sinusoidal surface propagating in the $+x$ direction after a temporal evolution of 2 seconds. The surface was sampled in 1024 points, and time steps of 4 msec were used in all methods. Note the distinction between the linear and non-linear surfaces. The fourth order Watson-West method is observed to produce both vertical and horizontal asymmetry of the original profile, while the Creamer method provides only vertical asymmetry. A comparison between the Watson-West method and the "Vortex Sheet" hydrodynamic [5] method is also illustrated, and shows good agreement.

3. RESULTS

Figure 2 compares Doppler spectra obtained from Monte Carlo simulation of 1.3 GHz backscattering at 10 degrees grazing incidence for the Watson-West, Creamer, and linear hydrodynamic models. A surface size of 117.81 m ($\approx 512\lambda$) was used, with an impedance surface relative permittivity of $76+i53$. Initial surface profiles were generated from a 2 m/s Pierson-Moskowitz spectrum (spectral high frequency cutoff wavenumber 109 rads/m), and allowed to evolve for 5.12 seconds with backscattered fields calculated every 20 msec. The fourth order Watson-West method was used with hydrodynamic time step 4 msec and $16,384$ points in the surface profile, and a one second initial "ramp-up" period in which non-linear hydrodynamic effects were gradually included was applied before the scattering simulation began. Creamer and linear methods used time step 20 msec and $8,192$ points. Results are averaged over 82, 96, and 89 realizations for the Watson-West, Creamer, and linear cases, respectively.

Results in Figure 2, plots (a) and (b) compare normalized HH Doppler spectra from the Watson-West method with the Creamer and linear methods respectively, while plots (c) and (d) illustrate the same comparisons for VV polarization. Doppler spectral widths are found to gradually increase from the linear to Creamer to Watson-West methods, indicating an increasing level of hydrodynamic interactions included. HH and VV Doppler spectra are observed to have only small differences at this relatively low windspeed. The dashed lines in the figure indicate the location of the Bragg Doppler frequency; all three spectra show distinct peaks at this location, while the Watson-West method captures a small component at the negative Doppler frequency as well. Figure 3 compares averaged curvature spectra obtained from surface profiles at the end of the time evolution period for the three methods;

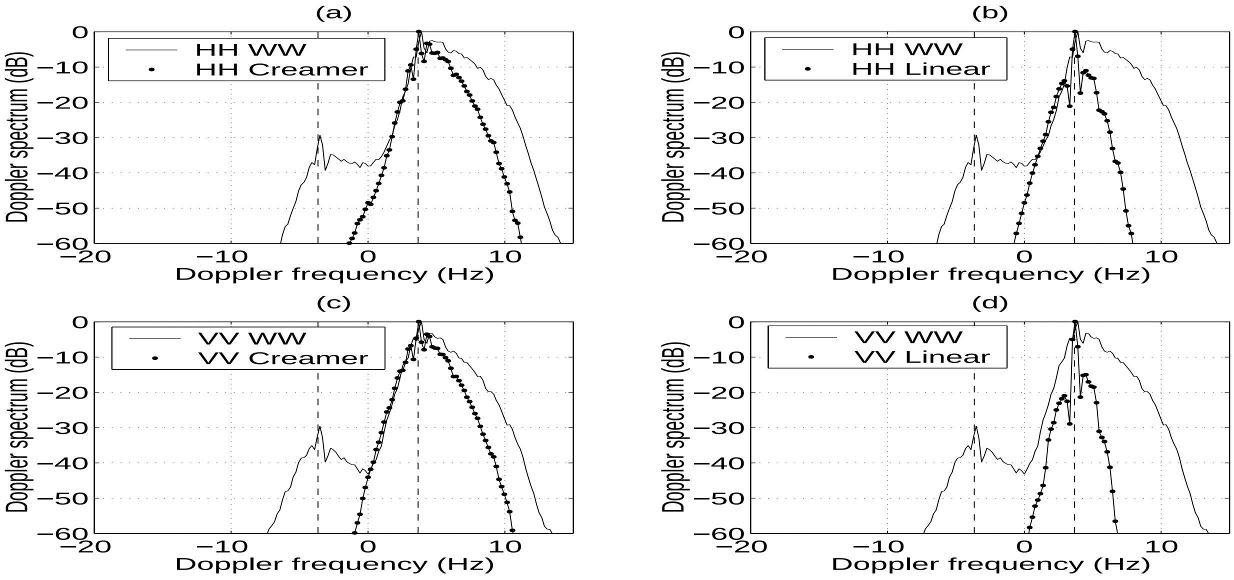

Figure 2: Doppler spectra obtained from Monte Carlo simulation

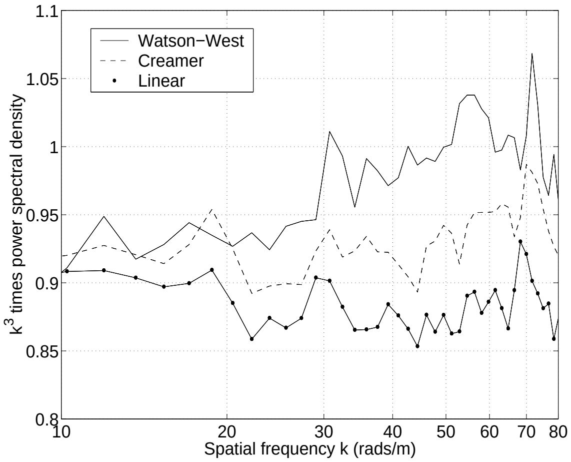

Figure 3: Average curvature spectra after time evolution: comparison of hydrodynamic models

again the same trend of increases in curvature spectra (particularly at higher frequencies) are observed for the three methods.

These results demonstrate the importance of modeling non-linear hydrodynamic effects for studies of sea surface backscattered Doppler spectra. Detailed comparisons of Doppler spectra obtained under differing hydrodynamic models and obtained from measurements can also help to identify important hydrodynamic and scattering effects at near grazing incidence angles. Further studies with the Creamer and Watson-West models are reported in [6] and [7], respectively. Studies of Doppler spectra obtained from the Vortex Sheet method are currently in progress.

4. REFERENCES

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