Numerical Studies of Backscattering from Time Evolving Sea Surfaces: Comparison of Hydrodynamic Models

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MOTIVATION

- Time evolving sea surfaces cause backscattered returns to be spread into a Doppler spectrum; affects remote sensing and imaging radars
- Bragg scattering and linear hydrodynamics predicts a single Doppler frequency shift; composite models broaden this
- Non-linear hydrodynamic effects can produce more Doppler features; empirical models developed to describe
- Numerical electromagnetic and hydrodynamic models are improving to make studies of Doppler spectra feasible: Monte Carlo simulation with time evolving surfaces
- Comparing scattering results under differing hydrodynamic models can clarify differences between models and important scattering features
- Results with differing hydrodynamic (Creamer, Watson-West, Vortex Sheet) and scattering models (MOM, SSA) compared
- Study limited to low wind speeds at present

OUTLINE

- Review of hydrodynamic models:
  - Improved Linear Model (Creamer et al, J. Fluid Mech., 1989)
  - Watson-West (West et al, J. Geophys. Res., 1987)
  - Vortex Sheet (Baker et al, J. Fluid Mech., 1982)
- Surface comparisons
- Review of scattering models
- Doppler results
- Conclusion

HYDRODYNAMIC MODELS

- Consider a two-dimensional incompressible, irrotational fluid; equations of surface motion (Eulerian) are then

$$\frac{\partial h}{\partial t} = \frac{\partial \phi}{\partial z}\left[1+\left(\frac{\partial h}{\partial x}\right)^2\right] - \frac{\partial h}{\partial x}\frac{\partial \phi}{\partial x}$$

$$\frac{\partial \phi}{\partial t} = -gh - \frac{1}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \phi}{\partial z}\right)^2\left[1+\left(\frac{\partial h}{\partial x}\right)^2\right]$$

  where h is the surface height, ϕ is the velocity potential evaluated at the surface, and g is the acceleration of gravity

- Equations are a canonical pair derivable from a single Hamiltonian

- Some methods can include surface tension, as well as approximate viscosity and wind forcing terms

- A system of non-linear PDE's: many potential problems in time-stepping solutions! $\frac{\partial \phi}{\partial z}$ also a problem since ϕ is known only on the surface

LINEAR AND IMPROVED LINEAR MODELS

- If non-linear terms are eliminated in equations of motion, surface waves become independent of one another and evolve via the linear dispersion relation
- Evolution is analytic: numerical time stepping not required
- Creamer et al use a canonical transformation of the equations of motion to eliminate the leading non-linear term for gravity waves
- Equations in new variables are approximated as linear: numerical time-stepping not required
- Transform back to physical surface is an $O(N^2)$ operation
- Reproduces lowest order non-linear effects, "does well" at higher orders but cannot capture any surface horizontal asymmetry
- Method previously applied in Doppler spectrum studies (Toporkov et al, IEEE TGRS, 2000)

WATSON-WEST MODEL

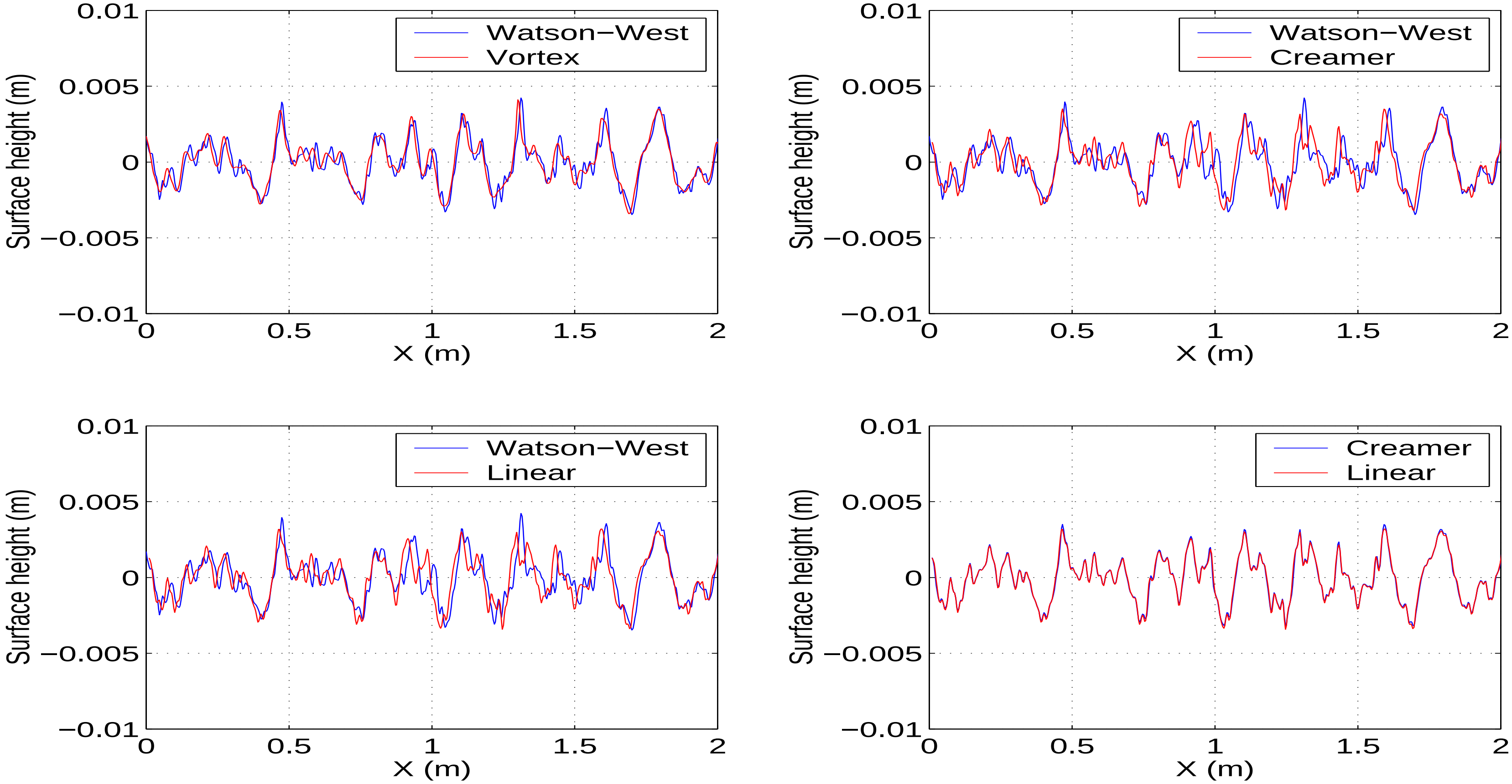
- Based on numerically time stepping non-linear equations of motion: predictor-corrector approach used
- Fixed, uniformly spaced grid: cannot capture overturning features
- Watson-West expansion comes in expression for $\frac{\partial \phi}{\partial z}$: expanded in a series in surface slope
- Series terms obtained from FFT operations on h and ϕ; involve higher order powers. Method is $O(N \log_2 N)$
- High order products in ϕ limit spectral content: level of oversampling needed increases with number of series terms (order of method)
- Method breaks down if slopes become too large: wind speed $\leq$ 2 m/s (Pierson-Moskowitz initial surface) for current study
- Initial "ramp-up" of non-linear terms used in some cases to transition between linear and non-linear dynamics

VORTEX SHEET METHOD

- Based on a Lagrangian formulation of equations of motion; predictor-corrector approach also used for time stepping
- Grid points not fixed or uniform: can capture more detailed surface features
- Equations in terms of dipole source on interface which produces velocity potential; no approximations to dynamics
- Integral equation for dipole source inverted iteratively: method is $O(N^2)$
- Current implementation limits spectral content: minimum factor of 2 oversampling required
- Only initial results at wind speed 0.5 m/s currently available; used only in hydrodynamic model comparison

COMPARISON OF SURFACES

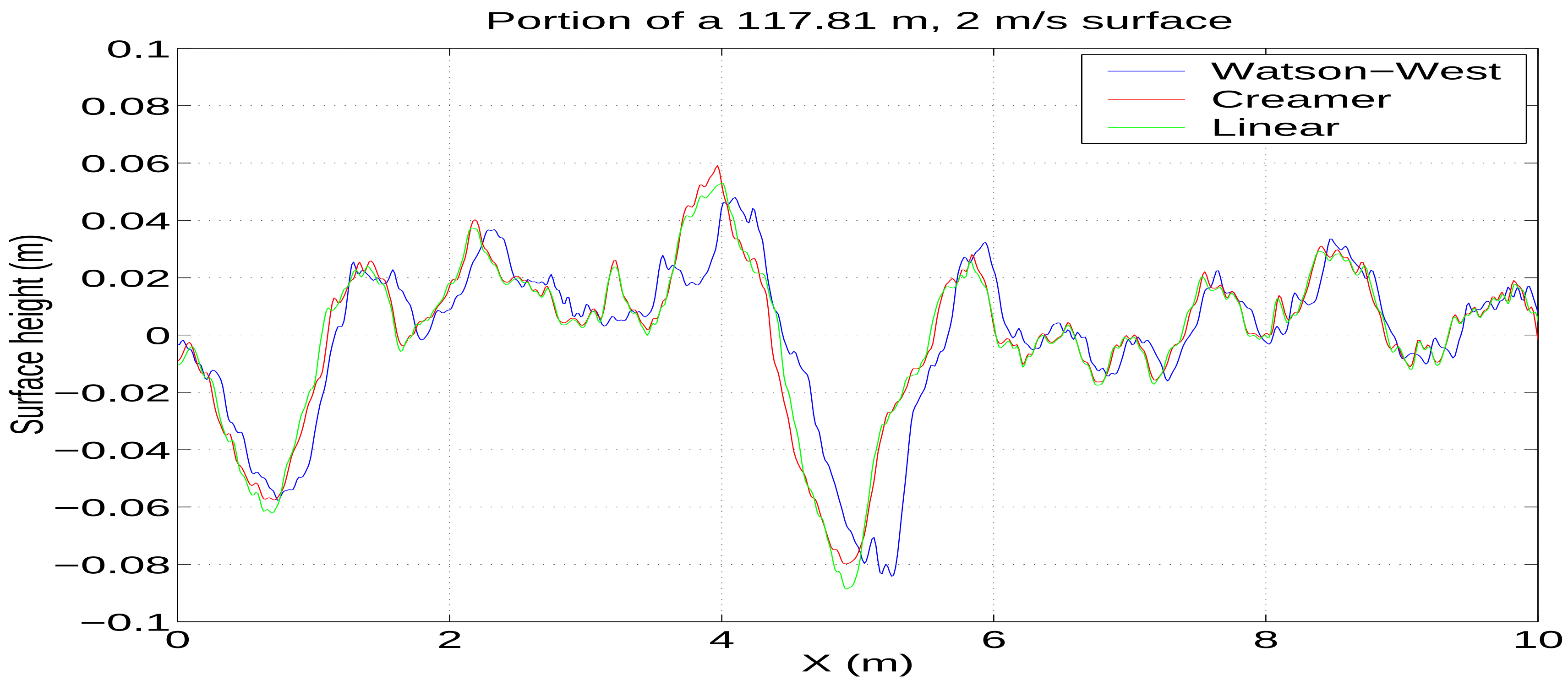
- Models run for 5.12 seconds: final profiles compared for one realization. Below is a 3.68 m, 0.5 m/s case



- All methods similar, but Vortex method closest to Watson-West, Creamer only slightly deviated from linear at 0.5 m/s

COMPARISON OF SURFACES

- Profiles from three methods compared for 117.81 m, 2 m/s case

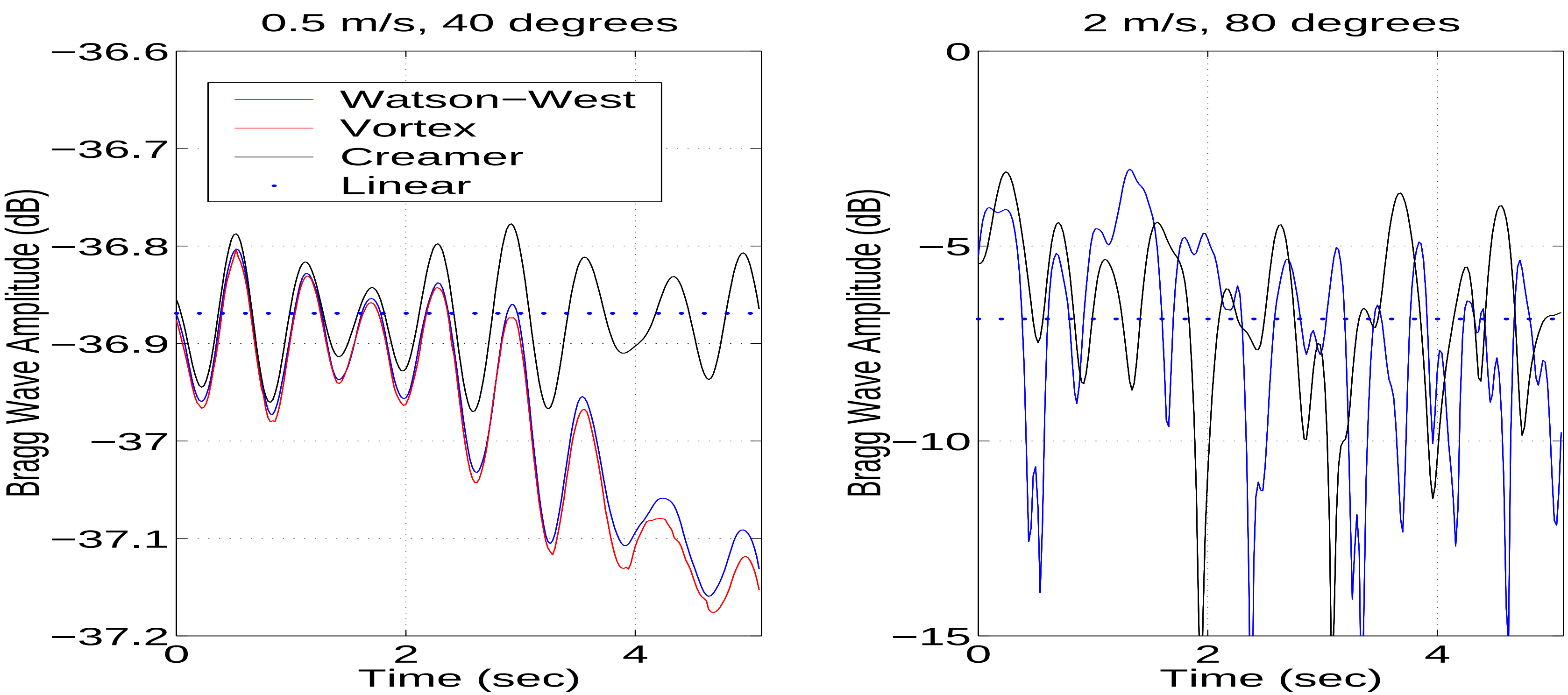


- Methods again differ; Creamer shows some vertical but no horizontal deviation from linear profile

- Important scattering features can be difficult to resolve in these plots

BRAGG WAVE DYNAMICS

- Time variation of single surface Fourier component compared



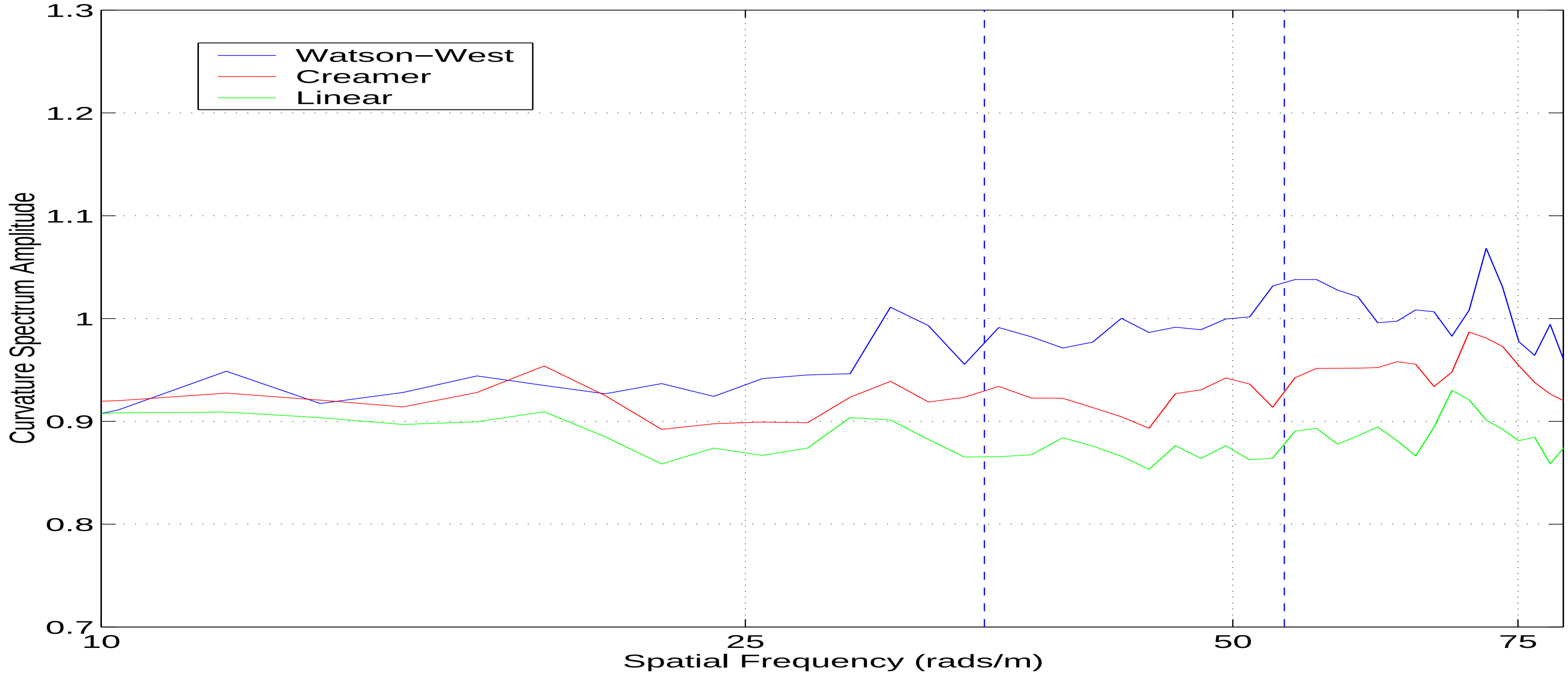
- Linear model has no Bragg amplitude variations; other models show similar variations

- Vortex model most closely matched by Watson-West

SURFACE STATISTICS

- 117.81 m Linear, Creamer, and Watson-West surfaces at 2 m/s

- Height histograms all approximately Gaussian; curvature spectra for 82, 96, and 89 realizations compared below



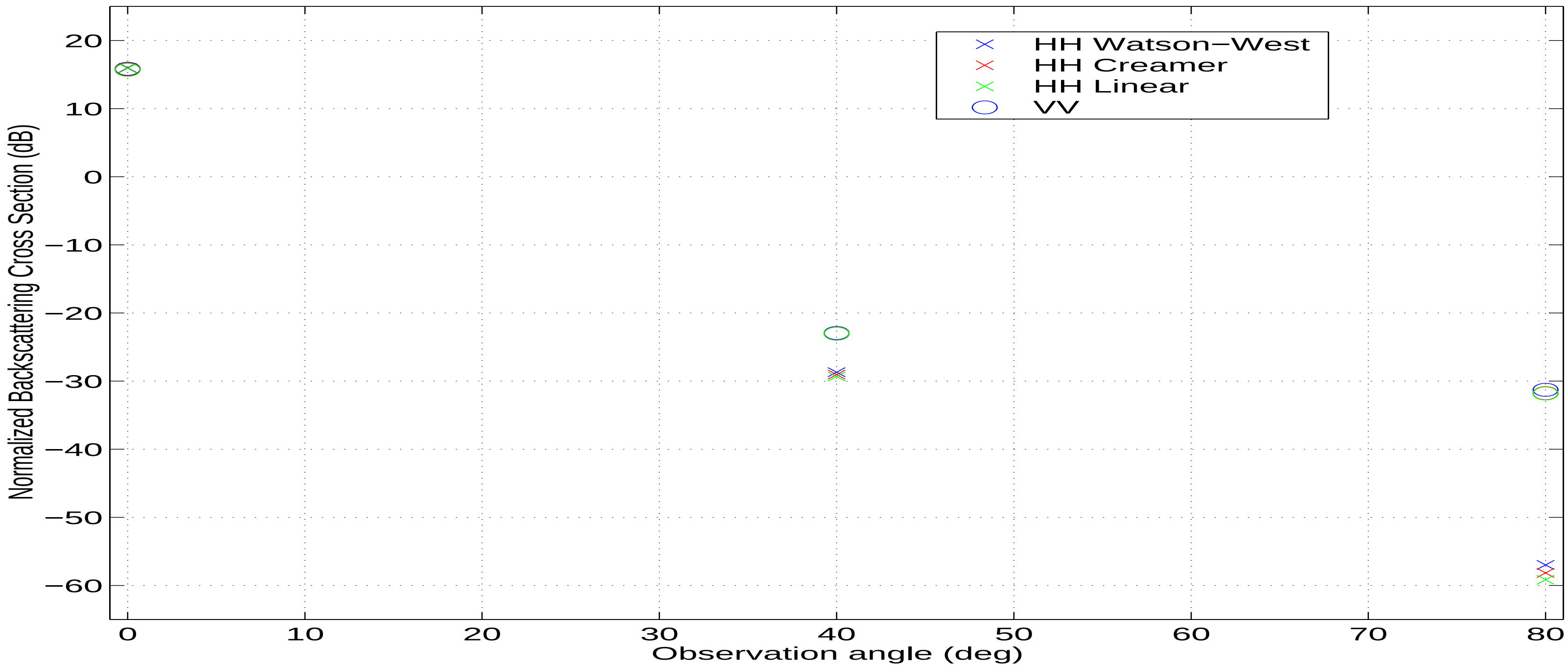
- Surfaces appear statistically similar, but dynamics are distinct

SCATTERING MODELS

- Scattering calculated at 1.3 GHz ($\lambda = 23$ cm) for impedance surfaces ($\epsilon = 76 + i53$) under tapered wave illumination
- Numerical scattering model is an iterative method of moments accelerated through a spectral technique
- Results compared with those from the zeroth and first order small slope approximation (SSA)
- Zeroth order SSA identical to "extended Kirchhoff approach" for backscattering; first order provides a polarization sensitive correction
- Computing times for numerical scattering and hydrodynamic models comparable
- Low wind speed surfaces are not very rough in terms of λ: Bragg wave dynamics typically of most importance

AVERAGE BACKSCATTERING CROSS SECTIONS

- Average backscattering cross sections: 0, 40, 80 degrees

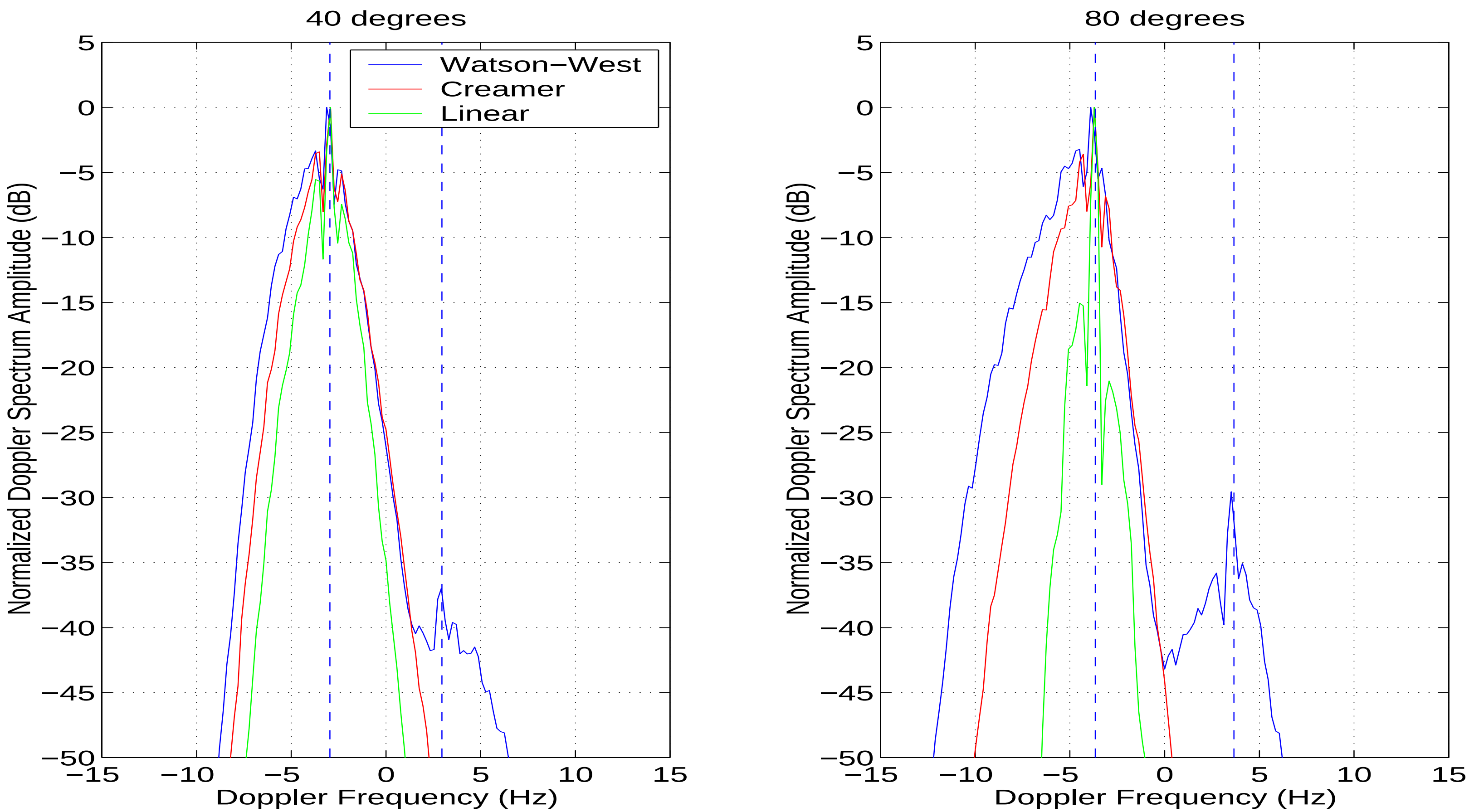


- Average scattering cross sections appear similar; differences at 80 degrees consistent with surface curvature spectra

- Backscattered field histograms appear to follow Rayleigh statistics well

DOPPLER SPECTRUM RESULTS

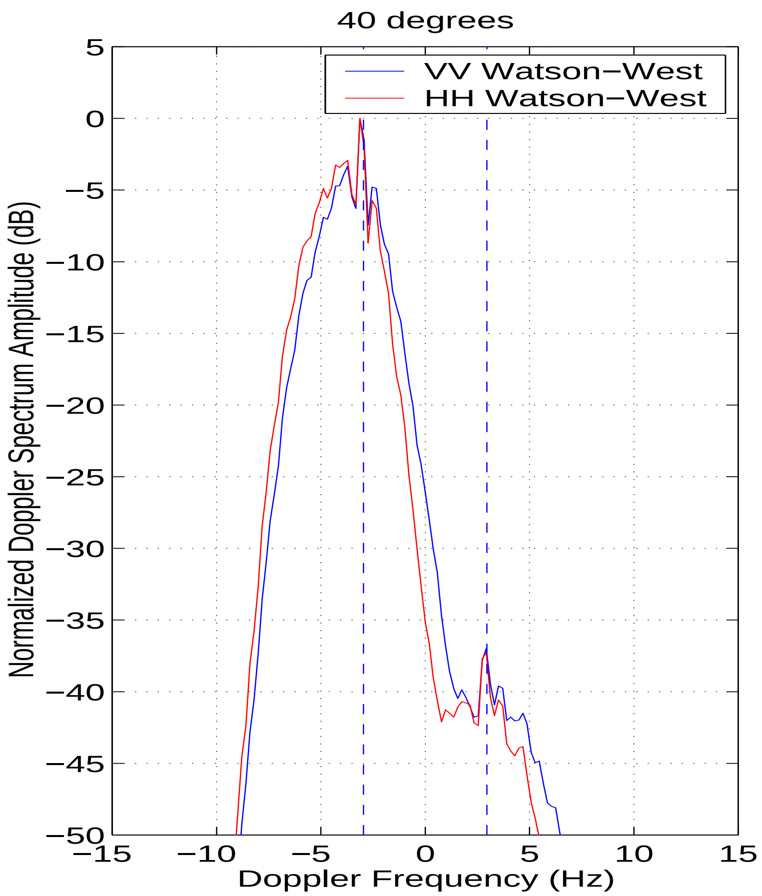
- MOM results at 2 m/s, three hydro methods: VV at 40° and 80°

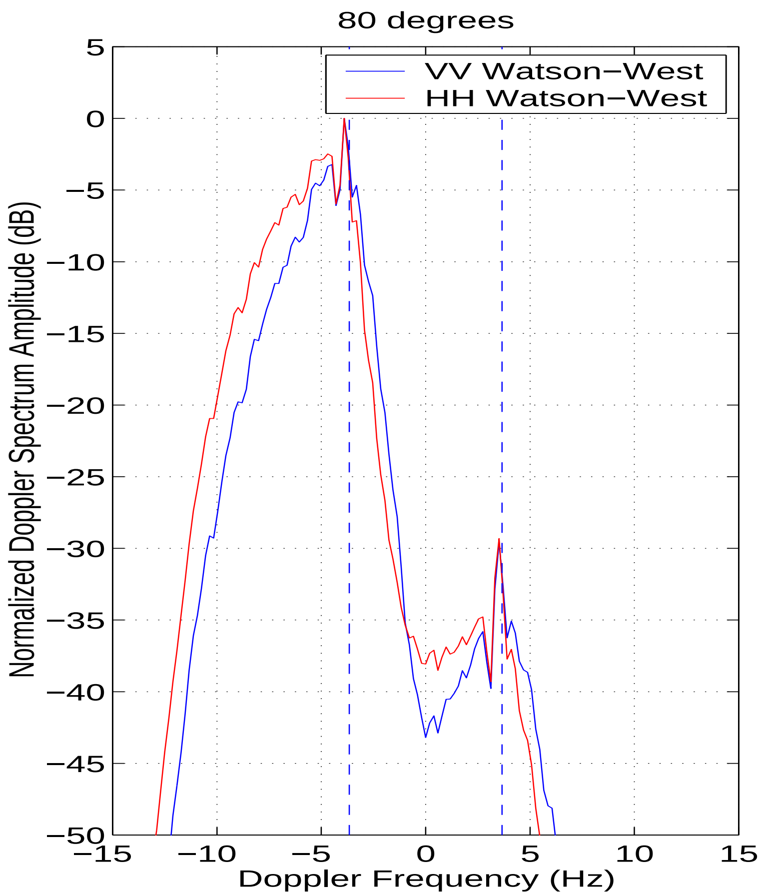


- Watson-West method produces broader spectrum; source of reverse peak unclear

INFLUENCE OF POLARIZATION

- MOM results at 2 m/s, Watson-West: 40° and 80°

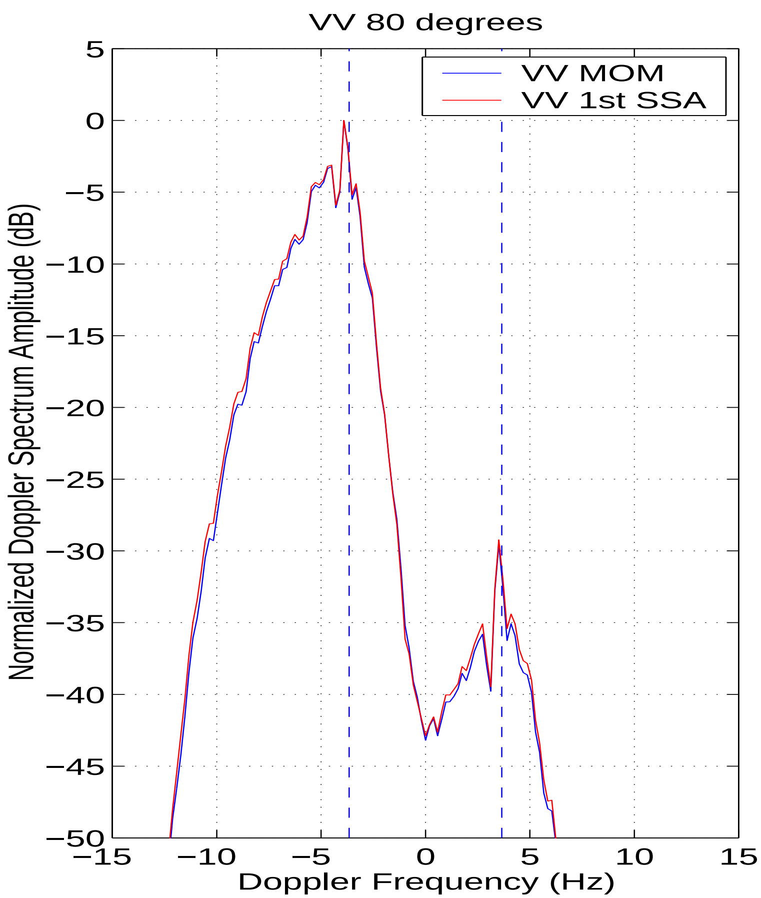


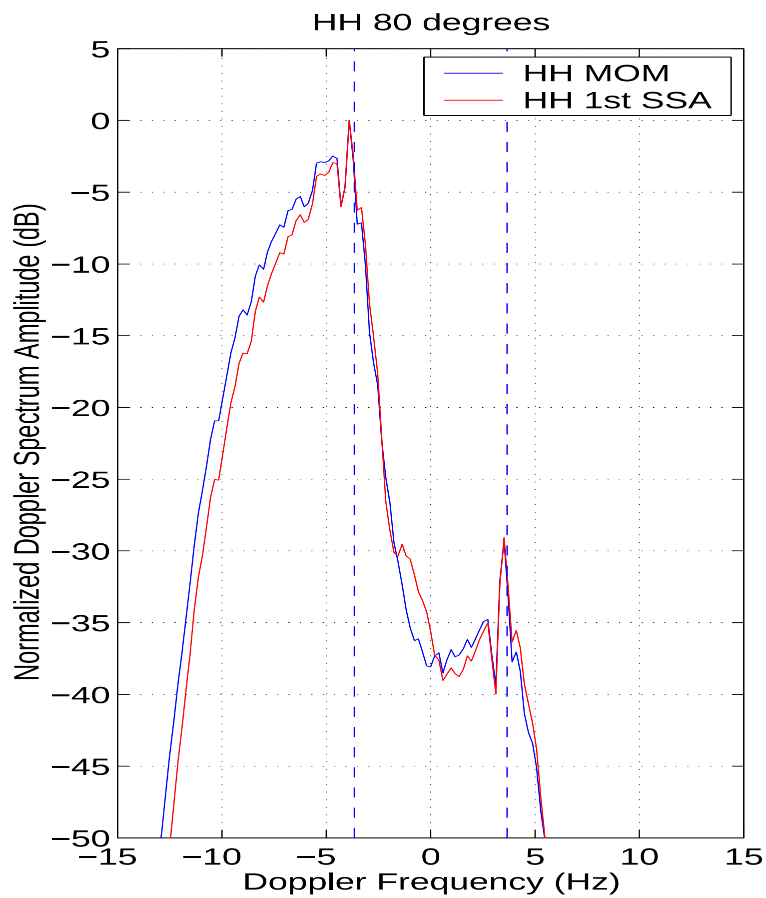


- VV and HH returns have distinct Doppler spectra, particularly at 80°

INFLUENCE OF SCATTERING MODEL

- Watson-West: 80 degrees

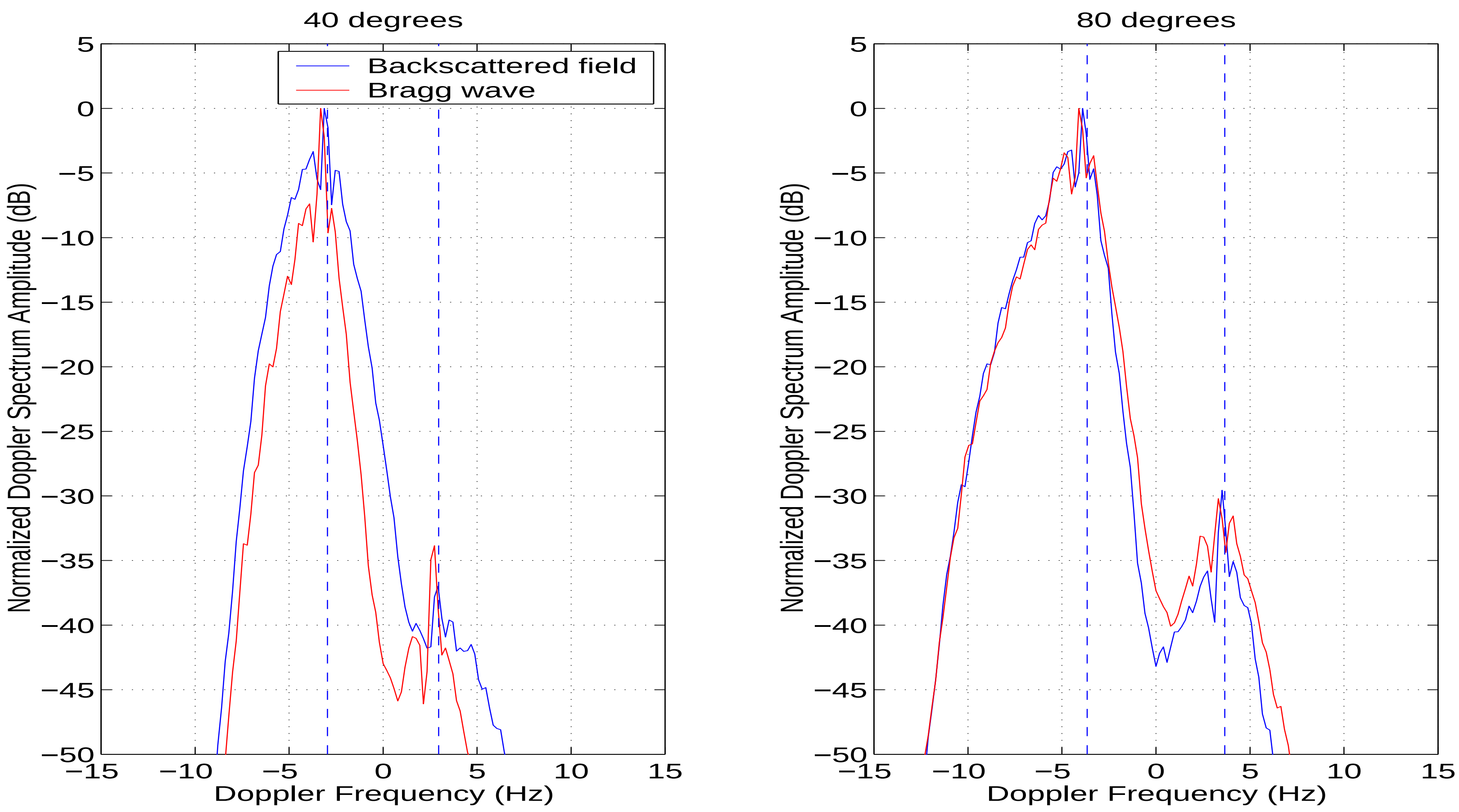




- First order SSA provides good prediction, some inaccuracies in HH

IMPORTANCE OF BRAGG WAVE DYNAMICS

- Watson-West: VV Pol vs. PSD of Bragg Fourier Component



- Combining profile and scattering data allows detailed studies of scattering effects

CONCLUSIONS

- Doppler simulations combining electromagnetic and hydrodynamic models are feasible
- Surface and field statistics appear similar for differing hydrodynamic models, but dynamics are distinct
- Watson-West model appears to capture more hydrodynamic effects; easy to include surface tension and approximate forcing terms
- Watson-West method however breaks down as roughness increases
- Vortex sheet method potentially can capture some breaking wave effects: investigations continuing
- First order SSA matches numerical model results well for cases considered; some problems at 80 degrees
- Combined surface and scattering studies can help to clarify sea surface scattering mechanisms