EE 816 - Extra Lecture

1. Midterm questions
2. Thermal noise
3. Brightness temperature
4. Uniform atmosphere
5. Layered atmosphere

III. Brightness temperature

- Because intensity is directly related to the physical temperature of a blackbody, it is more common to talk about the “brightness temperature” of a source (units of Kelvin), rather than the specific intensity radiated.
- Objects that are not blackbodies do not satisfy the Planck law. However, it is still used as a reference: the brightness temperature of an object is the temperature of a blackbody that would produce the same specific intensity as the real object.
- Emissivity is defined as the ratio of an object’s brightness temperature to its physical temperature:

\[ T_B = \varepsilon T_{\text{phys}} \]  
(3)

- Energy conservation arguments can be used to relate the emissivity of an object to its scattering properties: “Kirchhoff’s Law”

II. Thermal noise

- All objects at non-zero absolute temperature emit radiation over a wide range of frequencies: thermal noise.
- The standard for this emission is a “blackbody”: an object that perfectly absorbs all incident radiation.
- If it remains in thermal equilibrium, a blackbody must also emit radiation; this is thermal noise however, not reflected incident radiation.
- The Planck blackbody law describes the specific intensity radiated by a blackbody at Kelvin temperature \( T \):

\[ I = \frac{1}{c^2} \frac{h \nu^3}{e^{h \nu/kT} - 1} \]  
(1)

where \( h \) is Planck’s constant, \( \nu \) is the frequency, and \( k \) is Boltzmann’s constant \( 1.38 \times 10^{-23} \text{ J/K} \).
- In the microwave region, the exponent can be expanded to yield

\[ I = \frac{kT}{\lambda^2} \]  
(2)

Brightness temperature of a halfspace

- A simple argument can be used to find the brightness temperature of a halfspace medium at constant physical temperature \( T_{\text{phys}} \):

\[ T_{B,\phi}(\theta) = T_{\text{phys}} \left( 1 - |\Gamma_{\phi}(\theta)|^2 \right) \]  
(4)

- \( \beta \) is the polarization observed, while \( \Gamma \) is the halfspace reflection coefficient. The brightness temperature function of \( \beta \) and \( \theta \) comes from the reflection coefficient.
- The more reflective a boundary is, the “colder” it appears.
- Properties of a halfspace can be determined from thermal noise measurement: microwave radiometry.

![Radiometer Diagram](image-url)
IV. $T_B$ of a uniform atmosphere

- We can use radiative transfer theory to study $T_B$'s of absorbing and scattering media. $I$ is simply related to $T_B$.
- We have to add the emission source term to our RT equation:

$$\frac{dI(\mathbf{r}, \mathbf{s})}{ds} = -\rho \sigma_t I(\mathbf{r}, \mathbf{s}) + \frac{\rho \sigma_t}{4\pi} \int 4\pi d\omega \, p(\mathbf{s}', \mathbf{s}) I(\mathbf{r}, \mathbf{s}') + \rho \sigma_a \frac{K T}{\lambda^2}$$

<table>
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<th>Earth</th>
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<tbody>
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<td>z=0</td>
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<tr>
<td>z=d</td>
<td>Constant $\sigma_a$</td>
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V. $T_B$ of a “layered” atmosphere

- It is more realistic to have a temperature profile $T(z)$ in the atmosphere, as well as an absorption profile. Here define $\kappa_a(z) = (\rho \sigma_a)(z)$
- Solution of RT equation neglecting scattering remains easy. Note different coordinate system below.

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$T_B$ of a uniform atmosphere

- We can still divide into forward and reverse going intensities as before. However the Earth boundary is reflective, so we need:

$$I_s(z = d) = r I_s(z = d) + \frac{KT_{surf}}{\lambda^2} (1 - r)$$

where $T_{surf}$ is the physical temperature of the Earth and $r$ is the reflection coefficient at the boundary.
- In many cases, scattering in the atmosphere can be neglected. The RT solution is a straightforward 1st order DE solution. Write solution in terms of $T_B$ in space region:

$$T_B(\theta) = T(1 - e^{-\tau \sec \theta})(1 + re^{-\tau \sec \theta} + T_{surf} e^{-\tau \sec \theta})$$

- Here $\tau = \rho \sigma_a d$. There are three terms:
  - Direct upward emission by layer
  - Downward emission of layer reflected off boundary and attenuated
  - Direct upward surface emission attenuated by layer

$T_B$ of a “layered” atmosphere

- RT equation solution is:

$$T_B = \sec \theta \int_{-d}^{0} dz \, \kappa_a(z) T(z) \exp \left( -\int_{z}^{0} dz' \, \kappa_a(z') \sec \theta \right)$$

$$+ \sec \theta \, r e^{-\int_{-d}^{0} dz \, \kappa_a(z) \sec \theta} \int_{-d}^{0} dz \, \kappa_a(z) T(z) \exp \left( -\int_{z}^{0} dz' \, \kappa_a(z') \sec \theta \right)$$

$$+ T_{surf} (1 - r) e^{-\int_{-d}^{0} dz \, \kappa_a(z) \sec \theta}$$

- Terms are similar to before but include atmospheric profiles
- We could think of the first term only as:

$$T_B = \int_{-d}^{0} dz \, T(z) w(z)$$

where $w(z)$ is a “weighting function” connecting the atmospheric temperature profile to the observed brightness
- With a proper sensor design, it is possible to use multi-frequency thermal noise measurements to sense atmospheric temperature profiles.