

# A geometrical optics model of three dimensional scattering from a rough surface over a planar surface

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**Abstract**—A fast analytical method is described for predicting the bistatic normalized radar cross section of a rough homogeneous layer made up of a rough surface over a flat surface. The model starts from the iteration of the Kirchhoff approximation to calculate the fields scattered by a rough layer, and is reduced to the high-frequency limit in order to rapidly obtain numerical results. The shadowing effect, significant under grazing angles, is taken into account. The model is applicable for upper surface roughnesses having small to moderate slopes, for lossless and lossy inner media. It was validated for a two-dimensional problem (with 1-D surfaces) in a preceding contribution. Here, the extension of the model to 2-D surfaces is developed, and numerical results are presented to validate the asymptotic model by comparison with a numerical reference method.

**Index Terms**—Electromagnetic scattering by rough surfaces, Multilayered media, Physical optics, Multistatic scattering

## I. INTRODUCTION

SCATTERING from dielectric homogeneous layers has many applications in remote sensing, including the sensing of ocean ice, sand cover of arid regions, or oil slicks on the ocean, as well as in optics, including optical studies of thin films/coated surfaces and treatment of antireflection coatings. The use of fast asymptotic models can be very useful to predict the scattered signal of such systems, as such models provide fast numerical results compared to 'exact' numerical methods (for instance, see [1]–[3] and references therein), while retaining accuracy for specific classes of surfaces.

Many asymptotic models of electromagnetic scattering from a single rough interface have been developed over the last years. By contrast, few asymptotic models have been developed for rough layers separating homogeneous media, see mainly Refs [4]–[9]. Most of the available asymptotic models for rough layers are difficult to implement numerically and demand extensive computing time. To our knowledge, the specific case of rough layers with rms heights that may be large compared to the electromagnetic wavelength (while retaining moderate slopes) has not been treated before, and is the subject of this paper. The aim of this paper is to extend the Kirchhoff approximation to the case of a rough layer (with a rough upper

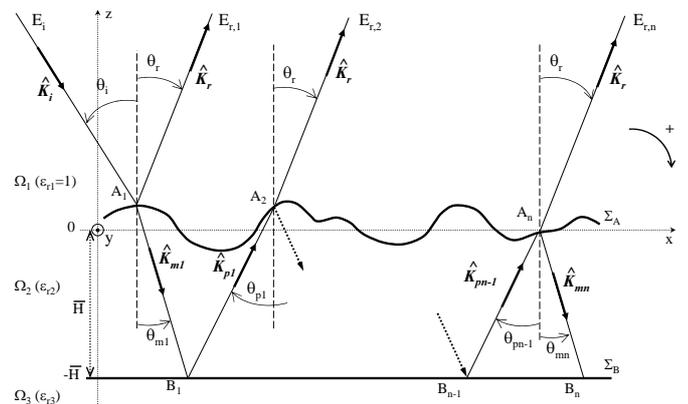


Fig. 1. Multiple scattering from a rough layer with a rough upper interface and a flat lower interface, represented in the plane  $(\hat{x}, \hat{z})$ . The points on the upper surface  $\Sigma_A$  are denoted as  $\{A_1, A_2, \dots, A_n\}$ , whereas the points on the lower surface  $\Sigma_B$  are denoted as  $\{B_1, B_2, \dots, B_{n-1}\}$ .  $\theta_i$  is the elevation incidence angle, and  $\theta_r$  is the elevation scattering angle in reflection, measured with respect to the vertical axis  $\hat{z}$ . The positive sense is defined as clockwise.

interface and a perfectly flat lower interface) and to obtain a formulation of the bistatic normalized radar cross section (NRCS) in the high-frequency limit. The model has been described in recent publications [10], [11] for one-dimensional (1-D) rough layers, in which the surface shadowing effect is taken into account [12], [13]. Here, the extension of the model to two-dimensional (2-D) rough layers is developed in order to include general three-dimensional (3-D) problems and to study scattering in cross-polarizations. Numerical results are presented and compared with a reference numerical method to validate the model.

## II. CALCULATION OF THE SCATTERED FIELDS DERIVED WITH THE KA AND THE MSP

### A. Problem presentation

The studied system (see Fig. 1) is composed of a rough layer (with a rough upper interface  $\Sigma_A$  and a perfectly flat lower interface  $\Sigma_B$ ), separating homogeneous media  $\Omega_\alpha$ , with  $\alpha = \{1, 2, 3\}$ . The three media  $\Omega_\alpha$ , with relative permittivity  $\epsilon_{r\alpha}$ , are assumed to be non magnetic (relative permeability  $\mu_r = 1$ ). Let  $\mathbf{E}_i$  be the incident field of polarization  $\hat{\mathbf{e}}_i$  inside the medium  $\Omega_1$  in the direction  $\hat{\mathbf{K}}_i = (k_{ix}, k_{iy}, k_{iz})/|k_1| =$

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$(\hat{k}_{ix}, \hat{k}_{iy}, \hat{k}_{iz})$  (the elevation angle, with respect to the vertical axis  $\hat{\mathbf{z}}$ , being  $\theta_i$ , and the azimuth angle, with respect to the axis  $\hat{\mathbf{x}}$ , being  $\phi_i$ ). The incident field on the upper surface  $\Sigma_A$  at the point  $A_1$  is given by  $\mathbf{E}_i(\mathbf{r}_{A_1}) = E_0 e^{ik_1 \hat{\mathbf{K}}_i \cdot \mathbf{R}_{A_1}} \hat{\mathbf{e}}_i$  (the term  $e^{-i\omega t}$  is omitted) where  $\mathbf{R}_{A_1} = x_{A_1} \hat{\mathbf{x}} + y_{A_1} \hat{\mathbf{y}} + z_{A_1} \hat{\mathbf{z}}$ , with  $x_{A_1}$  and  $y_{A_1}$  the abscissa, and  $z_{A_1}$  the elevation of the point  $A_1$ .  $k_1$  is the wave number inside  $\Omega_1$  (relative permittivity  $\epsilon_{r1}$ ).

The field transmitted into the intermediate medium  $\Omega_2$  along the direction  $\hat{\mathbf{K}}_{m1}$  is reflected by the flat lower interface at the point  $B_1$  into the specular direction  $\hat{\mathbf{K}}_{p1}$ , and then reflected by the upper surface at the point  $A_2$ , and so on. Thus, multiple reflections of the field inside  $\Omega_2$  occur, and this system can be seen as a rough dielectric waveguide. The first two scattered fields in reflection,  $\mathbf{E}_{r,1}$  and  $\mathbf{E}_{r,2}$ , are treated in detail in this paper; higher orders can be expressed at any order in reflection in a similar manner to that described here.  $\mathbf{E}_{r,1}$ ,  $\mathbf{E}_{r,2}$  are observed in the direction  $\hat{\mathbf{K}}_r = (k_{rx}, k_{ry}, k_{rz})/|k_1| = (\hat{k}_{rx}, \hat{k}_{ry}, \hat{k}_{rz})$ . The unit wave vectors  $\hat{\mathbf{K}}_i, \hat{\mathbf{K}}_r, \hat{\mathbf{K}}_{m1}, \hat{\mathbf{K}}_{p1}$  are defined by

$$\hat{\mathbf{K}}_i = (\sin \theta_i \cos \phi_i, \sin \theta_i \sin \phi_i, -\cos \theta_i), \quad (1a)$$

$$\hat{\mathbf{K}}_r = (\sin \theta_r \cos \phi_r, \sin \theta_r \sin \phi_r, +\cos \theta_r), \quad (1b)$$

$$\hat{\mathbf{K}}_{m1} = (\sin \theta_{m1} \cos \phi_{m1}, \sin \theta_{m1} \sin \phi_{m1}, -\cos \theta_{m1}), \quad (1c)$$

$$\hat{\mathbf{K}}_{p1} = (\sin \theta_{p1} \cos \phi_{p1}, \sin \theta_{p1} \sin \phi_{p1}, +\cos \theta_{p1}). \quad (1d)$$

In order to calculate the fields  $\mathbf{E}_{r,1}$  and  $\mathbf{E}_{r,2}$ , the Kirchhoff approximation (KA) (which is sometimes also called physical optics approximation, or tangent plane approximation) is used for the (upper) rough interface, and the Weyl representation of the Green's function is used to describe the propagation from one scattering point to another. More precisely, the KA is iterated for each scattering inside the rough layer. That is why the method can be called iterated Kirchhoff approximation, and denoted IKA. The field at any point of the (upper) rough surface can then be approximated by the field that would be present on its infinite tangent plane. Thus, the Snell-Descartes laws and the Fresnel coefficients can be applied locally, at any point of the (upper) rough surface. This approach should be valid for near-specular scattering geometries at moderate incidence angles and for interfaces whose radii of curvature are much larger than the incident wavelength  $\lambda$ .

The formulation is further simplified by applying the method of stationary phase (MSP) for each scattering point inside the rough layer. The MSP is an approximation of the KA that assumes that the major contribution of the scattered field comes from regions of the rough surface in the vicinity of the (stationary phase) specular points of the rough surface, whose direction is given by the local normal to the surface and the incidence angle. Using these two approximations, simplified expressions of  $\mathbf{E}_{r,1}$  and  $\mathbf{E}_{r,2}$  can be obtained: in the case of 1D surfaces, with 1 integration for  $\mathbf{E}_{r,1}$ , and 3 fold integrations for  $\mathbf{E}_{r,2}$  [10]. Here, for the case of 2D surfaces, one will see that the number of numerical integrations is doubled.

### B. Fields scattered by the rough layer

The fields scattered in reflection  $\mathbf{E}_{r,1}$  inside  $\Omega_1$  and transmission  $\mathbf{E}_{m1}$  inside  $\Omega_2$  by the upper interface  $\Sigma_A$  at the

point  $A_1$  are obtained from the Kirchhoff-Helmholtz integral equations [14], [15]. Under the KA, these expressions can be written as

$$\mathbf{E}_{r,1}(\mathbf{R}) = +2 ik_1 (\bar{\mathbf{I}} - \hat{\mathbf{K}}_r \hat{\mathbf{K}}_r) \cdot \iint dx_{A_1} dy_{A_1} \quad (2a)$$

$$G_1(\mathbf{R}_{A_1}, \mathbf{R}) \mathbf{F}_r(\gamma_{A_1,x}, \gamma_{A_1,y}) E_i(\mathbf{R}_{A_1}) \Xi_r(\mathbf{R}_{A_1}),$$

$$\mathbf{E}_{m1}(\mathbf{R}) = -2 ik_2 (\bar{\mathbf{I}} - \hat{\mathbf{K}}_{m1} \hat{\mathbf{K}}_{m1}) \cdot \iint dx_{A_1} dy_{A_1} \quad (2b)$$

$$G_2(\mathbf{R}_{A_1}, \mathbf{R}) \mathbf{F}_{m1}(\gamma_{A_1,x}, \gamma_{A_1,y}) E_i(\mathbf{R}_{A_1}) \Xi_t(\mathbf{R}_{A_1}),$$

where  $\Xi_r$  and  $\Xi_t$  are the illumination function in reflection and transmission, respectively (i.e., with  $\Xi_{r,t} = 1$  if the point  $A_1$  corresponding to  $\mathbf{R}_{A_1}$  is illuminated, and the rays emanating from both the incident and the scattered waves do not cross the surface;  $\Xi_{r,t} = 0$  otherwise);  $x_{A_1} \in [-L_{Ax}/2; +L_{Ax}/2]$  and  $y_{A_1} \in [-L_{Ay}/2; +L_{Ay}/2]$  (the surface lengths  $L_{Ax}$  and  $L_{Ay}$  are assumed to be much greater than their correlation length  $L_{cAx}$ ,  $L_{cAy}$ , respectively, and than the wavelength  $\lambda$ ). In the above equations, the Weyl representation of the Green's function is used to describe the propagation of the scattered wave from a point  $\mathbf{R}_{A_n}$  (with  $n = 1$  here) of the upper interface to the point  $\mathbf{R}$  of considered medium  $\Omega_\alpha$ . Its expression is given by [8], [16], [17]

$$G_\alpha(\mathbf{R}, \mathbf{R}_{A_n}) = \frac{i}{2} \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_{A_n}) + if(\mathbf{k})|z - \zeta_{A_n}|}}{f(\mathbf{k})}, \quad (3)$$

where  $\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}}$  ( $k_x$  and  $k_y$  ranging  $] -\infty; +\infty[$ ) and  $\mathbf{r} = x \hat{\mathbf{x}} + y \hat{\mathbf{y}}$ , with

$$f(\mathbf{k}) = \begin{cases} \sqrt{k_\alpha^2 - \|\mathbf{k}\|^2} & \text{if } k_\alpha^2 \geq \|\mathbf{k}\|^2 \\ i\sqrt{\|\mathbf{k}\|^2 - k_\alpha^2} & \text{if } k_\alpha^2 < \|\mathbf{k}\|^2 \end{cases}. \quad (4)$$

If the point  $\mathbf{R}$  is in the far-field zone of the surface  $\Sigma_A$ , corresponding to inequality  $\|\mathbf{R}\| \gg \|\mathbf{R}_{A_n}\|$ , the Green's function can be approximated by

$$G_\alpha(\mathbf{R}, \mathbf{R}_{A_n}) \simeq \frac{\exp[i(k_\alpha R - \mathbf{K}_s \cdot \mathbf{R}_{A_n})]}{4\pi R}, \quad (5)$$

with  $\mathbf{K}_s = \mathbf{K}_r$  for  $\alpha = 1$ .

Substituting the expression of the far-field Green's function (5) in equation (2a), in the far-field zone the first-order scattered field  $\mathbf{E}_{r,1}^\infty(\mathbf{R})$  can be expressed under the MSP by

$$\mathbf{E}_{r,1}^\infty(\mathbf{R}) = + \frac{ik_1 E_0 e^{ik_1 R}}{2\pi R} (\bar{\mathbf{I}} - \hat{\mathbf{K}}_r \hat{\mathbf{K}}_r) \cdot \mathbf{F}_r(\gamma_{A_1}^0) \int d\mathbf{r}_{A_1} e^{i(\mathbf{K}_1 - \mathbf{K}_r) \cdot \mathbf{R}_{A_1}} \Xi_r(\mathbf{R}_{A_1}), \quad (6)$$

where  $\int d\mathbf{r}_{A_1} \equiv \iint dx_{A_1} dy_{A_1}$ , and  $\mathbf{F}_r(\gamma_{A_1}^0)$  is given by half the equation (2.1.55a) of [18], with  $\gamma_{A_1}^0 = (\gamma_{A_1,x}^0, \gamma_{A_1,y}^0)$  given by

$$\gamma_{A_1,x}^0 \equiv \gamma_{A_1,x}^{0(r)} = -(k_{rx} - k_{ix})/(k_{rz} - k_{iz}), \quad (7)$$

$$\gamma_{A_1,y}^0 \equiv \gamma_{A_1,y}^{0(r)} = -(k_{ry} - k_{iy})/(k_{rz} - k_{iz}). \quad (8)$$

Similarly, the higher-order scattered fields ( $\mathbf{E}_{r,2}, \mathbf{E}_{r,3}$ , etc.) are obtained from equation (2b) by first using the Weyl representation of the Green function which describes the propagation of the wave from  $\mathbf{R}_{A_n}$  to the point  $\mathbf{R}_{A_{n+1}}$  of the upper interface  $\Sigma_A$ , after reflection onto the flat lower

interface  $\Sigma_B$  at the point  $B_n$  of elevation  $z_B = -\bar{H}$ . It is given by [16], [19]

$$G_{2,r}(\mathbf{R}_{A_{n+1}}, \mathbf{R}_{A_n}) = \frac{i}{2} \int \frac{d\mathbf{k}}{(2\pi)^2} r(\mathbf{k}) \frac{e^{i\mathbf{k} \cdot (\mathbf{r}_{A_{n+1}} - \mathbf{r}_{A_n}) + if(\mathbf{k})(\zeta_{A_{n+1}} - \zeta_{A_n} - 2z_B)}}{f(\mathbf{k})}, \quad (9)$$

with  $r(\mathbf{k})$  the Fresnel reflection coefficient of the lower interface.

Then, the scattering in reflection and transmission at the point  $A_2$  is described by the Kirchhoff-Helmholtz integral equation under the KA, expressed in reflection  $\mathbf{E}_{m2}$  inside  $\Omega_2$  and in transmission  $\mathbf{E}_{r,2}$  inside  $\Omega_3$  by

$$\mathbf{E}_{m2}(\mathbf{R}) = +2ik_2 (\bar{\mathbf{I}} - \hat{\mathbf{K}}_{m2} \hat{\mathbf{K}}_{m2}) \cdot \iint dx_{A_2} dy_{A_2} \quad (10a)$$

$$G_2(\mathbf{R}_{A_2}, \mathbf{R}) \mathbf{F}_{m2}(\gamma_{A_2}) E_{p1}(\mathbf{R}_{A_2}) \Xi_r(\mathbf{R}_{A_2}),$$

$$\mathbf{E}_{r,2}(\mathbf{R}) = -2ik_1 (\bar{\mathbf{I}} - \hat{\mathbf{K}}_r \hat{\mathbf{K}}_r) \cdot \iint dx_{A_2} dy_{A_2} \quad (10b)$$

$$G_1(\mathbf{R}_{A_2}, \mathbf{R}) \mathbf{F}_t(\gamma_{A_2}) E_{p1}(\mathbf{R}_{A_2}) \Xi_t(\mathbf{R}_{A_2}),$$

respectively, with  $x_{A_2} \in [-L_{Ax}/2; +L_{Ax}/2]$  and  $y_{A_2} \in [-L_{Ay}/2; +L_{Ay}/2]$ , and the field inside  $\Omega_2$  incident on the upper interface at the point  $A_2$ ,  $E_{p1}(\mathbf{R}_{A_2})$ , being obtained from the relation

$$E_{p1}(\mathbf{R}_{A_2}) = G_{2,r}(\mathbf{R}_{A_2}, \mathbf{R}_{A_1}) E_{m1}(\mathbf{R}_{A_1}). \quad (11)$$

Thus, under the MSP and by neglecting the evanescent wave contribution, by using the far-field Green function (5) at the point  $A_n = A_2$ ,  $\mathbf{E}_{r,2}^\infty$  is expressed by

$$\frac{\mathbf{E}_{r,2}^\infty(\mathbf{R})}{E_0} = \frac{ik_1 k_2 e^{ik_1 R}}{(2\pi)^3 R} \int \frac{d\mathbf{k}_{m1}}{-k_{m1z}} d\mathbf{r}_{A_1} d\mathbf{r}_{A_2} \mathbf{F}_{m1}(\gamma_{A_1}^0) \times \bar{r}_{23}(\theta_{m1}) \times \mathbf{F}_r(\gamma_{A_2}^0) e^{i(\mathbf{K}_i \cdot \mathbf{R}_{A_1} + \mathbf{K}_{m1} \cdot \mathbf{R}_{A_1 B_1} + \mathbf{K}_{p1} \cdot \mathbf{R}_{B_1 A_2} - \mathbf{K}_r \cdot \mathbf{R}_{A_2})} \delta(\hat{\mathbf{k}}_{p1} - \hat{\mathbf{k}}_{m1}) \Xi_t(\mathbf{R}_{A_1}) \Xi_r(\mathbf{R}_{B_1}) \Xi_t(\mathbf{R}_{A_2}), \quad (12)$$

with  $\mathbf{R}_{B_1 A_2} = \mathbf{R}_{A_2} - \mathbf{R}_{B_1}$ . The calculation of  $\mathbf{E}_{r,2}^\infty$  then implies  $2 \times 3$  fold numerical integrations.

By using the same principle for the higher orders, i.e. by iterating the KA at each scattering point (that is why the model can be called iterated Kirchhoff approximation, and denoted IKA), and using the MSP, it is possible to obtain the expressions of the scattered fields in reflection  $\mathbf{E}_{r,n}^\infty$  at any order  $n \in \mathbb{N}^*$ . Nevertheless, their expression is long and is consequently not given here.

### III. NRCS IN THE HIGH-FREQUENCY LIMIT

As for 1-D surfaces, the  $n$ -th order total power  $P_{r,n}^{tot} = \langle |E_{r,n}^{tot}|^2 \rangle / 2\eta_1$  scattered by the rough layer is given by equation (14) of [10], and the  $n$ -th order incoherent total power  $P_{r,n}^{tot,inc}$  is given by  $P_{r,n}^{tot,inc} = [\langle |E_{r,n}^{tot}|^2 \rangle - \langle E_{r,n}^{tot} \rangle^2] / 2\eta_1$ , with  $\eta_1$  the wave impedance inside  $\Omega_1$ .

In order to determine the NRCS in the high-frequency limit, the geometric optics approximation (GOA) is applied on the (upper) rough surface in order to simplify the calculation. Valid for very rough surfaces, it assumes that the main contribution

to the power scattered by the rough surface  $\Sigma_A$  comes from closely-located correlated surface points  $M$  and  $M'$ . can be neglected. Moreover, the height difference  $\zeta_M - \zeta_{M'}$  can be expressed by the approximate expression  $\gamma_{M,x}(x_M - x_{M'}) + \gamma_{M,y}(y_M - y_{M'})$ , with  $\gamma_M = (\gamma_{M,x}, \gamma_{M,y})$  the rough surface slope at the point  $M$ .

Then, the incoherent total NRCS (equal to the total NRCS  $\sigma_{r,n}^{tot}$  under the GOA) of a 2-D target (for a 3-D problem) can be determined more easily. It is given by

$$\sigma_{r,n}^{tot}(\mathbf{K}_r, \mathbf{K}_i) = 2\eta_1 \frac{R^2 P_{r,n}^{tot}}{L_{Ax} L_{Ay} \cos \theta_i |E_i|^2}, \quad (13)$$

where  $R$  is the distance of the target, and  $L_{Ax}$ ,  $L_{Ay}$  the upper surface length along direction  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{y}}$ , respectively. In the above equation, for cases  $n = \{1, 2\}$ , one has the relations

$$P_{r,1}^{tot} = p_{r,1} \quad \text{and} \quad P_{r,2}^{tot} = p_{r,1} + p_{r,2}, \quad (14)$$

with

$$\begin{cases} p_{r,1} = \frac{1}{2\eta_1} \langle |E_{r,1}|^2 \rangle \\ p_{r,2} = \frac{1}{2\eta_1} \left[ \langle |E_{r,2}|^2 \rangle + 2 \Re \left( \langle E_{r,1} E_{r,2}^* \rangle \right) \right] \end{cases} \quad (15)$$

A NRCS corresponding to the contribution of each scattered power can then be defined. Thus, for  $n = 1$ ,  $\sigma_{r,1}$  corresponds to the contribution from  $p_{r,1}$ . For  $n = 2$ ,  $\sigma_{r,1}$  corresponds to the contribution from  $p_{r,1}$  and  $\sigma_{r,2}$  to the contribution from  $p_{r,2}$ . Under the GOA and with shadowing effect, one can show that the interference term  $\langle E_{r,1} E_{r,2}^* \rangle$  equals zero, which is in agreement with the latter approximation. In this model, the shadowing and masking effects in reflection [12] and in transmission [13] are taken into account. Indeed, for low grazing incidence and/or scattering angles, a part of the (upper) rough surface is not illuminated by the local incident wave and/or not seen by the local scattered wave. This phenomenon must be taken into account in order not to overestimate the NRCS.

#### A. Expressions of the first three contributions

The first-order NRCS in reflection  $\bar{\sigma}_{r,1}(\mathbf{K}_r, \mathbf{K}_i)$  corresponds to the NRCS in reflection from a single rough interface. Under the KA and the MSP, and reduced to the GOA, it is well-known [8], [18] that the NRCS can be expressed by

$$\bar{\sigma}_{r,1}(\mathbf{K}_r, \mathbf{K}_i) = \frac{1}{\cos \theta_i} \left| \bar{F}_r(\mathbf{K}_r, \mathbf{K}_i) \right|^2 \frac{p_s(\gamma_{A_1}^{0(r)})}{|\hat{k}_{rz} - \hat{k}_{iz}|^2} S_{11}(\mathbf{K}_i, \mathbf{K}_r | \gamma_{A_1}^{0(r)}), \quad (16)$$

with  $\bar{F}_r(\mathbf{K}_i, \mathbf{K}_r) = \hat{\mathbf{a}}_r \cdot \mathbf{F}_b(\gamma_{A_1}^{0(r)})$  being a  $2 \times 2$  matrix which depends on the polarizations of the incident and the scattered waves, and  $S_{11}(\mathbf{K}_i, \mathbf{K}_r | \gamma_{A_1}^{0(r)})$  given by

$$S_{11} = \begin{cases} \frac{1}{1+\Lambda(\hat{\mathbf{K}}_r)} & \text{if } \{\phi_r = \phi_i + \pi, \theta_r \geq \theta_i\} \\ \frac{1}{1+\Lambda(\hat{\mathbf{K}}_i)} & \text{if } \{\phi_r = \phi_i + \pi, \theta_r < \theta_i\} \\ \frac{1}{1+\Lambda(\hat{\mathbf{K}}_i + \Lambda(\hat{\mathbf{K}}_r))} & \text{if } \{\phi_r \neq \phi_i + \pi\} \end{cases} \quad (17)$$

For the second-order contribution, the principle is the same as for 1-D surfaces (see subsection 3.1.1 of [10]). Extending

the method from 1-D surfaces to 2-D surfaces does not raise any major problem in the calculation. However, this is true only for the coincidental case. The anti-coincidental case, which may contribute around the backscattering direction, was rather hard to quantify for 1-D surfaces. It is then complex to quantify for 2-D surfaces. Then, in this paper we will choose configurations for which this contribution (which may occur only around the backscattering direction) can be neglected: the condition of its contribution is given by equation (36) of [11]. Thus, the second-order NRCS  $\bar{\sigma}_{r,2}$  can be written as

$$\bar{\sigma}_{r,2}(\mathbf{K}_r, \mathbf{K}_i) = \frac{1}{\cos \theta_i} \int \sin \theta_{m1} d\theta_{m1} d\phi_{m1} \left| \bar{F}_{t,12}(\mathbf{K}_i, \mathbf{K}_{m1}) \times \bar{r}_{23}(\theta_{m1}) \times \bar{F}_{t,21}(\mathbf{K}_{p1}, \mathbf{K}_r) \right|^2 \frac{p_s(\gamma_{\mathbf{A}_1}^{0(t)})}{|\hat{k}_{m1z} - \frac{k_1}{k_2} \hat{k}_{iz}|^2} S_{12}(\mathbf{K}_i, \mathbf{K}_{m1} | \gamma_{\mathbf{A}_1}^{0(t)}) \frac{p_s(\gamma_{\mathbf{A}_2}^{0(t)})}{|\hat{k}_{rz} - \frac{k_2}{k_1} \hat{k}_{p1z}|^2} S_{21}(\mathbf{K}_{p1}, \mathbf{K}_r | \gamma_{\mathbf{A}_2}^{0(t)}). \quad (18)$$

with  $\gamma_{\mathbf{A}_1}^{0(t)} = (\gamma_{A_1,x}^{0(t)}, \gamma_{A_1,y}^{0(t)})$  and  $\gamma_{\mathbf{A}_2}^{0(t)} = (\gamma_{A_2,x}^{0(t)}, \gamma_{A_2,y}^{0(t)})$  given by

$$\gamma_{A_1,x}^{0(t)} = -(k_{m1x,y} - k_{ix,y}) / (k_{m1z} - k_{iz}), \quad (19)$$

$$\gamma_{A_2,x}^{0(t)} = -(k_{rx,y} - k_{p1x,y}) / (k_{rz} - k_{p1z}). \quad (20)$$

The bistatic shadowing functions in transmission are given by  $S_{12}(\mathbf{K}_i, \mathbf{K}_{m1} | \gamma_{\mathbf{A}_1}^{0(t)}) = B[1 + \Lambda(\mathbf{K}_i), 1 + \Lambda(\mathbf{K}_{m1})]$ , and  $S_{21}(\mathbf{K}_{p1}, \mathbf{K}_r | \gamma_{\mathbf{A}_2}^{0(t)}) = B[1 + \Lambda(\mathbf{K}_{p1}), 1 + \Lambda(\mathbf{K}_r)]$ . The second-order NRCS  $\bar{\sigma}_{r,2}$  is expressed in the form of a square matrix of dimension 2, in which each term depends on the polarization of the incident and the scattered waves. The general polarization term  $\bar{F}_{s,\alpha\beta}(\mathbf{K}_1, \mathbf{K}_s)$  being also a matrix, the NRCS cannot rigorously be split up into a product of elementary NRCSs as for the 2D case (with 1-D surfaces), corresponding to each scattering point inside the dielectric waveguide. As a general rule,  $\bar{\sigma}_{r,2}$  and  $\bar{F}_{s,\alpha\beta}(\mathbf{K}_1, \mathbf{K}_s)$  can be expressed as

$$\bar{\sigma}_{r,2} = \begin{bmatrix} \sigma_{h_r h_i}^{r,2}(\mathbf{K}_r, \mathbf{K}_i) & \sigma_{v_r h_i}^{r,2}(\mathbf{K}_r, \mathbf{K}_i) \\ \sigma_{h_r v_i}^{r,2}(\mathbf{K}_r, \mathbf{K}_i) & \sigma_{v_r v_i}^{r,2}(\mathbf{K}_r, \mathbf{K}_i) \end{bmatrix}, \quad (21)$$

$$\bar{F}_{s,\alpha\beta}(\mathbf{K}_1, \mathbf{K}_s) = \begin{bmatrix} F_{h_s h_1}(\mathbf{K}_1, \mathbf{K}_s) & F_{v_s h_1}(\mathbf{K}_1, \mathbf{K}_s) \\ F_{h_s v_1}(\mathbf{K}_1, \mathbf{K}_s) & F_{v_s v_1}(\mathbf{K}_1, \mathbf{K}_s) \end{bmatrix}, \quad (22)$$

where the first subscript in the terms inside the matrix represents the polarization of the scattered wave, and the second subscript the polarization of the incident wave. The general polarization term in reflection (for  $s \equiv r$ , implying  $\mathbf{K}_s = \mathbf{K}_2$ )  $F_{r,\alpha\beta}(\mathbf{K}_1, \mathbf{K}_2) = F_r(\mathbf{K}_1, \mathbf{K}_2)$  is defined by

$$F_r(\mathbf{K}_1, \mathbf{K}_2) = \frac{\|\hat{\mathbf{K}}_2 - \hat{\mathbf{K}}_1\|^2}{\|\hat{\mathbf{K}}_1 \wedge \hat{\mathbf{K}}_2\|^2 |\hat{k}_{2z} - \hat{k}_{1z}|} \left\{ \frac{r_h(\chi_1^o)}{2} [(\hat{\mathbf{b}}_1 \wedge \hat{\mathbf{K}}_1) \cdot \hat{\mathbf{K}}_2] [(\hat{\mathbf{a}}_2 \wedge \hat{\mathbf{K}}_2) \cdot \hat{\mathbf{K}}_1] + \frac{r_v(\chi_1^o)}{2} (\hat{\mathbf{b}}_1 \cdot \hat{\mathbf{K}}_2)(\hat{\mathbf{a}}_2 \cdot \hat{\mathbf{K}}_1) \right\}, \quad (23)$$

and the polarization general term in transmission (for  $s \equiv t$ , implying  $\mathbf{K}_s = \mathbf{K}_3$ )  $F_{t,\alpha\beta}(\mathbf{K}_1, \mathbf{K}_3)$  is defined by

$$F_{t,\alpha\beta}(\mathbf{K}_1, \mathbf{K}_3) = \frac{2 \|\hat{\mathbf{K}}_3 - k_\alpha/k_\beta \hat{\mathbf{K}}_1\| |(\hat{\mathbf{N}}^{0(t)} \cdot \hat{\mathbf{K}}_3)|}{\|\hat{\mathbf{K}}_1 \wedge \hat{\mathbf{K}}_3\|^2 |\hat{k}_{3z} - k_\alpha/k_\beta \hat{k}_{1z}|} \left\{ \frac{t_h(\chi_1^o)}{2} [(\hat{\mathbf{b}}_1 \wedge \hat{\mathbf{K}}_1) \cdot \hat{\mathbf{K}}_3] [(\hat{\mathbf{a}}_3 \wedge \hat{\mathbf{K}}_3) \cdot \hat{\mathbf{K}}_1] + \frac{t_v(\chi_1^o)}{2} (\hat{\mathbf{b}}_1 \cdot \hat{\mathbf{K}}_3)(\hat{\mathbf{a}}_3 \cdot \hat{\mathbf{K}}_1) \right\}, \quad (24)$$

with  $\mathbf{K}_1$  the incident wave vector inside the medium  $\Omega_\alpha$  and  $\mathbf{K}_3$  the scattered wave vector in transmission inside the medium  $\Omega_\beta$ .

The contribution of the third-order NRCS  $\bar{\sigma}_{r,3}$  is given by

$$\bar{\sigma}_{r,3}(\mathbf{K}_r, \mathbf{K}_i) = \frac{1}{\cos \theta_i} \int \sin \theta_{m1} d\theta_{m1} d\phi_{m1} \sin \theta_{m2} d\theta_{m2} d\phi_{m2} \left| \bar{F}_{t,12}(\mathbf{K}_i, \mathbf{K}_{m1}) \times \bar{r}_{23}(\theta_{m1}) \times \bar{F}_r(\mathbf{K}_{p1}, \mathbf{K}_{m2}) \times \bar{r}_{23}(\theta_{m2}) \times \bar{F}_{t,21}(\mathbf{K}_{p2}, \mathbf{K}_r) \right|^2 \frac{p_s(\gamma_{\mathbf{A}_1}^{0(t)})}{|\hat{k}_{m1z} - \frac{k_1}{k_2} \hat{k}_{iz}|^2} S_{12}(\mathbf{K}_i, \mathbf{K}_{m1} | \gamma_{\mathbf{A}_1}^{0(t)}) \frac{p_s(\gamma_{\mathbf{A}_2}^{0(r)})}{|\hat{k}_{m2z} - \hat{k}_{p1z}|^2} S_{22}(\mathbf{K}_{p1}, \mathbf{K}_{m2} | \gamma_{\mathbf{A}_2}^{0(r)}) \frac{p_s(\gamma_{\mathbf{A}_3}^{0(t)})}{|\hat{k}_{rz} - \frac{k_2}{k_1} \hat{k}_{p2z}|^2} S_{21}(\mathbf{K}_{p2}, \mathbf{K}_r | \gamma_{\mathbf{A}_3}^{0(t)}), \quad (25)$$

and so on for the higher orders.

## B. Model validity domains and properties

The validity domains of the model are the same as for a 2D problem (with 1D surfaces): see subsection 2.C of [11]. Based on the iteration of the Kirchhoff approximation and then called iterated Kirchhoff approximation (and denoted IKA), it has the validity domain of the Kirchhoff approximation. Moreover, as reduced to the high-frequency limit, it uses the geometric optics approximation (denoted GOA). That is why this method can be denoted as IKA+GOA, and has the validity domain of the GOA.

One can notice that this asymptotic model, generalized to a 3D problem (with 2D surfaces), has the same general properties as for the 2D case (with 1D surfaces). Indeed, it is independent of the height statistics (when the anti-coincidental contribution can be neglected), as well as of the frequency or of the mean layer thickness (for lossless inner media). As for the 2D case, the model in itself, as based on the GOA, cannot deal with lossy media. Still, by using exactly the same approach as for the 2D case (see section 7 of [10]), taking lossy media into account does not raise any problem.

Some simulation results, not presented here, were used to validate the first-order contribution  $\sigma_{r,1}^{tot} = \sigma_{r,1}$  (corresponding to the scattering in reflection from a single surface) by comparison with results of the literature [20]. In next section, asymptotic model predictions of the first two order contributions of the NRCS  $\sigma_{r,1}^{tot} = \sigma_{r,1}$  and  $\sigma_{r,2}^{tot} = \sigma_{r,1} + \sigma_{r,2}$  are compared with a reference numerical method for validation.

#### IV. NUMERICAL RESULTS: ASYMPTOTIC MODEL VALIDATION

##### A. Numerical reference method

A numerical method based on the method of moments was developed for the configuration of Figure 1 following previous implementations for non-layered, penetrable rough surfaces [21]–[23] and for objects in the presence of a penetrable rough surface [24]–[26]. The latter problem can be directly extended to the case of interest by replacing the penetrable object previously considered [24] with a perfectly conducting surface. Although the code was implemented to be able to incorporate rough lower boundaries, here only flat lower interfaces were of interest.

The method discretizes both the upper and lower interfaces, the former in terms of the vector induced electric and magnetic current densities, while only induced electric current densities are required on the lower interface. To allow direct comparisons with the asymptotic approach, the combined matrix equation for induced currents on both interfaces was solved by iterating on successive couplings between the interfaces (i.e. the “orders” of the asymptotic approach); the implementation is described in detail in [24]. This requires that algorithms be implemented for determining induced current densities on each interface individually given an exciting field consisting of contributions from the incident field as well as radiation from the current densities found at lower orders. Solutions for the upper interface currents were also performed iteratively, and the canonical grid method [21]–[26] was used to accelerate the required matrix-vector multiply operations. No acceleration of computations of the coupling between layers was performed, so that overall algorithm is order  $N^2$  where  $N$  is the total number of unknowns on the upper interface. Improvements in efficiency would be possible if acceleration methods were adopted for this coupling.

Reduction of surface edge effects given finite surface size were achieved by using the “tapered” incident field described in [21], [22], as is common in rough surface scattering studies. A half power spot diameter of 5.6 wavelengths was used, and fields at surface edges were reduced by 54 dB. Comparisons of scattering from the upper surface with standard asymptotic theories was used to verify that this approach should provide reasonable results for co-pol scattering within approximately 70 degrees scattering angle. These comparisons also showed the tapered wave to increase cross-polarized scattered fields significantly above the very small cross-sections predicted by standard theories. However, the presence of the lower layer dramatically increases cross-pol results, so that reasonable predictions are apparently achieved for cross-polarization in the presence of the lower layer.

The results to be illustrated consider a layer of relative permittivity  $\epsilon_{r2} = 3$ , above a flat perfectly conducting boundary ( $\epsilon_{r3} = i\infty$ ). Surface sizes of 24 by 24 free space wavelengths were used, discretized into 256 by 256 points for a total of 393216 unknowns in the matrix equation (4 unknowns for each point on the upper interface, 2 on the lower.) The upper surface profiles were generated as realizations of a Gaussian stochastic process with an isotropic Gaussian correlation function.

Two roughness cases are considered: rms height  $0.25\lambda$  and correlation length  $1.768\lambda$  or rms height  $0.3\lambda$  and correlation length  $2.12\lambda$ . Both cases have rms slope 0.2. A distance of 2.41 free space wavelengths between the layers was used in the numerical method; simulations using other distances showed only a very weak dependence on this distance (likely due to tapered wave issues), consistent with the asymptotic method. While the asymptotic method should be applicable for surfaces with even larger rms heights, the canonical grid method acceleration technique used in the numerical algorithm is best suited for moderate rms height surfaces. A total of thirty-two surface realizations (sufficient to achieve mean NRCS estimates accurate to within approximately 2 dB) were used in each simulation, with the required computations performed on parallel computing resources at the Maui High Performance Computing Center. The parallel computing algorithm was developed to use groups of 8 processors for a single surface realization, so that required couplings between the upper and lower interfaces could be computed more rapidly. Results for a single surface realization using eight processors were achieved in approximately 5 hours of CPU time.

In the comparisons to be shown, the incident wave is linearly polarized with an incident elevation angle  $\theta_i$  of either 0 (normal incidence) or 15 degrees, and the incident azimuth angle is always  $\phi_i = 0^\circ$ . The numerical results present the total NRCS in reflection  $\sigma_{r,n}^{tot}$  for HH, HV, VH, VV polarizations (the first term representing the polarization of the scattered wave, and the second term the polarization of the incident wave) and in either the incident plane ( $\phi_r = 0$ ) or in scattering planes rotated azimuthally with respect to the plane of incidence ( $\phi_r$  not zero).

##### B. Simulation results

Figure 2 presents numerical results for scattering in the plane of incidence (azimuth angle  $\phi_r = 0^\circ$ ) for an incidence angle  $\theta_i = 0^\circ$ . For the numerical simulations of the IKA+GOA, the first-order contribution  $\sigma_{r,1}^{tot} = \sigma_{r,1}$  with shadowing effect is plotted as a black line with circles (the case without shadowing effect is not presented as no difference appears). The second-order contribution  $\sigma_{r,2}^{tot} = \sigma_{r,1} + \sigma_{r,2}$  with shadowing effect is plotted as a green dotted line with plus signs (the case without shadowing effect is not presented here for the sake of clarity of the figures). For the numerical simulations of the reference numerical method, results are presented in HH and VH polarizations only for this case, using surfaces of rms height  $0.25\lambda$  and correlation length  $1.768\lambda$ . The contribution from the upper interface alone obtained from the numerical method, corresponding to  $\sigma_{r,1}^{tot} = \sigma_{r,1}$ , is plotted as a black line. The result after one iteration of the method, corresponding to  $\sigma_{r,2}^{tot} = \sigma_{r,1} + \sigma_{r,2}$ , is plotted as a green dash-dot line, and the result after many iterations, corresponding to  $\sigma_{r,n}^{tot} \simeq \sigma_{r,\infty}^{tot}$ , is plotted as a blue dashed line.

In co-polarizations HH and VV, the two first-order contributions  $\sigma_{r,1}^{tot}$  and  $\sigma_{r,2}^{tot}$  of the IKA+GOA have the same basic properties as in the 2-D problem (see section 6 of [10] and subsection 3.C of [11]). The second-order NRCS  $\sigma_{r,2}^{tot}$  contributes primarily around the specular direction, where the NRCS is

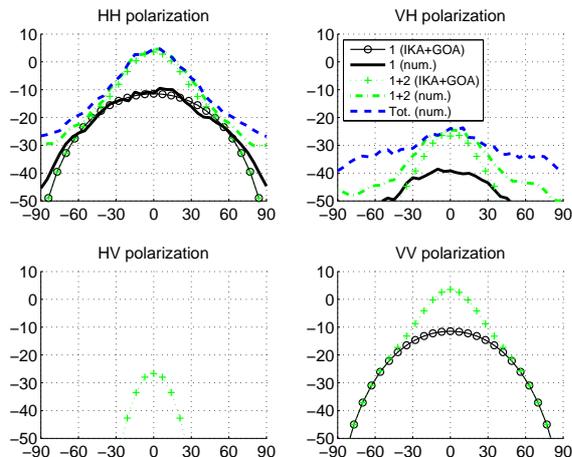


Fig. 2. Simulation of the first two total NRCSs  $\sigma_{r,1}^{tot}$  and  $\sigma_{r,2}^{tot}$  in dB scale, with respect to the observation angle  $\theta_r$  in the plane of incidence (azimuth angle  $\phi_r = 0^\circ$ ), for an incidence elevation angle  $\theta_i = 0^\circ$

much larger than that of the first-order contribution  $\sigma_{r,1}^{tot}$ . The case without shadowing effects (which is not represented here for the sake of clarity of the figure) diverges for observation angles  $|\theta_r| > 60^\circ$ : this highlights the relevance of taking shadowing into account. In cross-polarizations VH and HV, as expected by the first-order KA, the first-order NRCS  $\sigma_{r,1}^{tot}$  of the IKA+GOA has a negligible contribution compared to the second-order contribution  $\sigma_{r,2}^{tot}$ .

For HH polarization, the comparison with the reference numerical method shows a very good agreement for  $\sigma_{r,1}^{tot}$ , which corresponds to the scattering from the upper surface when no lower layer is present. The differences that appear for grazing angles,  $|\theta_r| \gtrsim 75^\circ$ , are likely impacted by the finite surface size as well as the limitations of the KA theory for this region. Very good agreement is also observed for the second-order contribution  $\sigma_{r,2}^{tot}$ ; significant differences are observed only for grazing angles,  $|\theta_r| \gtrsim 60^\circ$ , also are likely impacted by the finite surface size as well as limitations of the KA theory. More complete numerical simulations would be required to clarify these differences. The result of the numerical method for many iterations highlights that around the specular direction for these surfaces, there is no significant difference with the first iteration  $\sigma_{r,2}^{tot}$ , which means that  $\sigma_{r,2}^{tot}$  is sufficient to quantify the scattering process. This result is in agreement with observations made for a 2-D problem (see subsection 3.C of [11]). Thus, in co-polarization, the second-order contribution  $\sigma_{r,2}^{tot}$  of the IKA+GOA model can correctly quantify the scattering process around the specular direction, for observation angles  $|\theta_r| \lesssim 60^\circ$ .

For cross-polarization VH, the comparison of the IKA+GOA with the reference method highlights a good agreement for the second-order contribution  $\sigma_{r,2}^{tot}$  near the specular direction,  $|\theta_r| \lesssim 20^\circ$ . The differences that appear for higher  $|\theta_r|$  may be attributed to multiple scattering from the same interface effect or possibly to finite surface size effects,

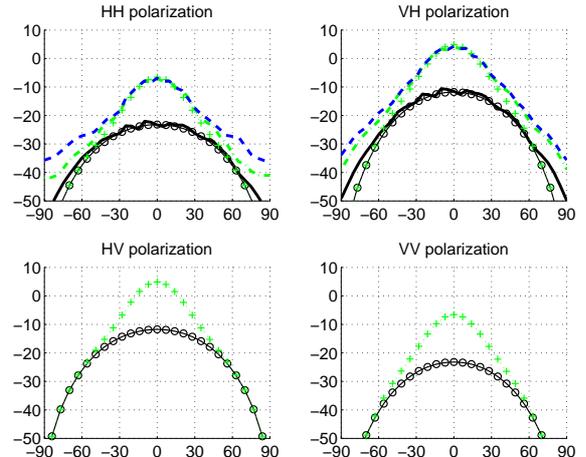


Fig. 3. Same simulations as in figure 2, but with an azimuth angle  $\phi_r = 75^\circ$

although such effects would likely not be major contributors for angles within 30 degrees scattering angle. The total scattering coefficient computed from the reference method shows larger contributions away from the specular direction. Thus, in cross-polarization, the second-order contribution  $\sigma_{r,2}^{tot}$  of the IKA+GOA model appears sufficient for these statistics to describe the scattering process near the specular direction, for observation angles  $|\theta_r| \lesssim 15^\circ$ . Note the low level first order (i.e. upper surface only with no interface) cross-polarization response obtained by the numerical reference method, which is likely an overestimate of the true value due to the impact of the tapered incidence field. However the much larger values for in-plane cross-polarized NRCS values in the presence of the layer reduces the impact of tapered wave effects.

Figure 3 presents results for the same parameters as in figure 2, but for out-of-plane scattering at azimuth angle  $\phi_r = 75^\circ$ . As  $\phi_r = 75^\circ$  is close to the quadrature angle ( $= 90^\circ$ ), the numerical results for co-polarizations are rather similar to those for cross-polarization obtained in figure 2 and vice-versa. Therefore, co-pol and cross-pol results here can largely be interpreted in the same manner as used for cross-pol and co-pol results, respectively, in the in-plane scattering case. The main distinctions of such an approach appear in co-polarization: due to the difference of  $15^\circ$  with respect to a perpendicular scattering plane, the first-order contribution  $\sigma_{r,1}^{tot}$  of the IKA+GOA cannot be neglected. Moreover, the second-order contribution  $\sigma_{r,2}^{tot}$  increases and reaches approximately  $-7$  dB in the specular direction  $\theta_r = 0^\circ$ . The agreement of the IKA+GOA with the reference method is somewhat improved in this comparison compared to that for the plane-of-incidence, since first-order scattering effects are more important in both polarizations.

Other comparisons (not presented here) for various rotations of the scattering plane (i.e.  $\phi_r$  values) allowed a check on the symmetry of the NRCS of the IKA+GOA around  $\phi_r = 90^\circ$  and its  $180^\circ$ -periodicity (owing to the isotropy of the rough

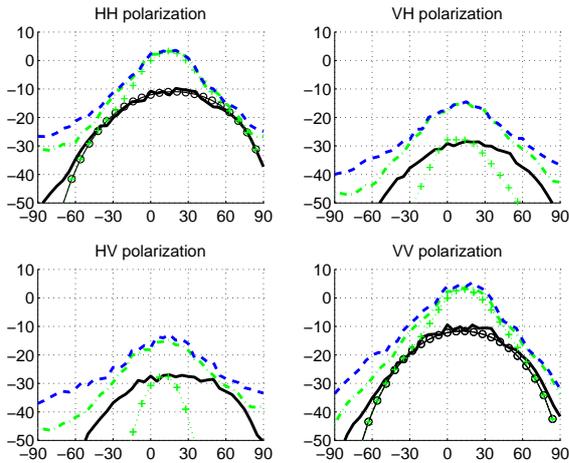


Fig. 4. Same simulations as in figure 2, but with an incidence elevation angle  $\theta_i = 15^\circ$

surface). For example, NRCS values for  $\phi_r = 180^\circ$  and  $\phi_r = 360^\circ$  are similar to those for  $\phi_r = 0^\circ$ , and the results for  $\phi_r = 105^\circ$ ,  $\phi_r = 220^\circ$  and  $\phi_r = 250^\circ$  are similar to those for  $\phi_r = 75^\circ$ . As the azimuth angle  $\phi_r$  moves away from  $0^\circ$  or  $90^\circ$ , the lower polarization contribution (VH or HH, respectively) increases and shows an increasingly good agreement of the IKA+GOA with the reference method around and away from the specular direction.

Figure 4 presents comparisons for the parameters of figure 2, but with an incidence angle  $\theta_i = 15^\circ$ . While the upper surface RMS slope  $\sigma_s$  remains unchanged, the reference numerical method used a surface RMS height  $\sigma_h = 0.3\lambda$ , and  $L_c = 2.12\lambda$ . The results show the same general behavior of the IKA+GOA as in the preceding configuration. The results for the case without shadowing effect (not presented here for the sake of clarity of the figure) again diverge for grazing  $\theta_r$ . The results of the IKA+GOA are compared with the reference numerical method for all polarizations (HH, VH, HV, VV). In co-polarizations HH and VV, the first-order contribution  $\sigma_{r,1}^{tot}$  highlights a good agreement of the IKA+GOA with the reference method. Again differences that appear for larger  $\theta_r$  values, and in particular for  $\theta_r < 0$ , can be attributed to the limitations of the KA method as well as tapered wave effects at the larger angles. The second-order contribution  $\sigma_{r,2}^{tot}$  highlights a good agreement of the IKA+GOA with the reference method in and around the specular direction, and in HH polarization for  $\theta_r$  up to approximately positive  $80^\circ$ . The differences that appear away from the specular direction, in particular for backward configuration  $\theta_r < 0$ , are likely primarily due to limitations of the KA approximation for non-specular observations. As in figure 2, results from the reference numerical method highlight that in and around the specular direction, the higher orders are negligible: the second-order  $\sigma_{r,2}^{tot}$  is enough to quantify the scattering phenomenon.

In cross-polarizations VH and HV, as in figure 2, results from the IKA+GOA confirm that  $\sigma_{r,1}^{tot}$  has a negligible contribution, while  $\sigma_{r,2}^{tot}$  which has a moderate contribution around

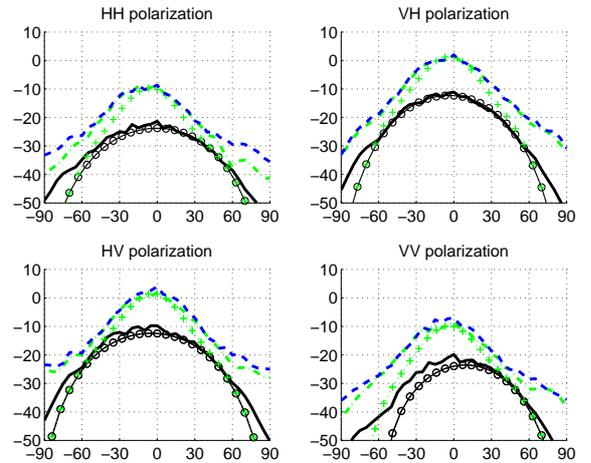


Fig. 5. Same simulations as in figure 4, but with an azimuth angle  $\phi_r = 105^\circ$

the specular direction. The reference method again shows appreciable contributions for  $\sigma_{r,1}^{tot}$  that are impacted by the tapered wave and are likely to be overestimates of the true scattering. In contrast to figure 2 where a good agreement was found for second order cross-polarized NRCS values in and near around the specular direction, the IKA+GOA here underestimates cross-polarized scattering. This is likely due to the impact of multiple scattering on the upper interface, which plays a significant role generally in cross-polarized scattering. The results of the reference method for higher orders confirm that the second-order contribution yield sufficient accuracy for these surface statistics around the specular direction.

Figure 5 presents numerical results for the same parameters as in figure 4, but for a rotated scattering plane at azimuth angle  $\phi_r = 105^\circ$ . As for the first case,  $\phi_r = 105^\circ$  is close to the quadrature angle ( $= 90^\circ$ ), and an interchange of co- and cross-polarized results can be used to help interpret NRCS properties. The main distinctions from such an interchange appear in co-polarization, again since the first order contribution is appreciable for this geometry. Overall, a good agreement is found with the reference method because multiple scattering effects are less important when compared to first order scattering processes. Moreover, the second-order contribution  $\sigma_{r,2}^{tot}$  of the IKA+GOA increases, so that the multiple scattering from the same interface effect has a less important contribution. Thus, there is a good agreement of the IKA+GOA with the reference method in and around the specular direction. Other comparisons (not presented here) for various values of  $\phi_r$  provide similar conclusions. Overall, these comparisons validate the IKA+GOA in its validity domain, and help to quantify limitations of the approach for non-specular scattering and for cross-polarized predictions in the plane of incidence at non-normal incidence angles.

## V. CONCLUSION

In conclusion, the IKA+GOA has been extended to a general 3D problem with 2D surfaces, allowing to deal with realistic

problems and to study the influence of cross-polarizations. Comparisons with a reference numerical method validated the IKA+GOA in its validity domain. The two different configurations confirmed that the model is independent of the RMS surface height. Results highlighted that in and around the specular direction, the second-order contribution  $\sigma_{r,2}^{tot}$  is sufficient to quantify the scattering process. Observed differences of the IKA+GOA with the reference method can be attributed primarily to multiple scattering effects on the upper interface.

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