Terrain Reflection and Diffraction, Part One

1. UHF and VHF paths near the ground
2. Propagation over a plane Earth
3. Fresnel zones
I. UHF and VHF paths near the ground

- In many applications communications are influenced by reflections from the Earth surface.
- Particular problem when antenna gains are small and receivers are near the ground.
- In Chapter 7 we will study the direct + Earth reflections mechanism and also terrain diffraction.
- The Earth surface also plays a role in groundwave propagation (Chapter 9) but this applies mainly at lower than VHF frequencies.
- Since ionospheric propagation is also negligible for VHF and higher frequencies, direct + Earth reflections and terrain diffraction are the main mechanism for these frequencies over medium distances.
- Tropospheric scatter applies at very long distances.
- Consideration of “multipath environments” in Chapter 8.
• We will learn basic methods for predicting propagation over a specified terrain path
• Primarily applied to assess antenna heights needed to maintain communications for various values of $\kappa$
• Used in designing line-of-sight UHF and SHF point-to-point links
• Numerical methods are also now being applied here; we will learn a little about these as well
II. Propagation over a plane Earth

Begin by assuming the Earth surface to be planar (valid approximation for short distances and smooth terrain), so that

\[ R_2 = R'_2 + R''_2 \]  \hspace{1cm} (1)

\[ \tan \psi_2 = (h_1 + h_2)/d \]  \hspace{1cm} (2)

\[ \tan \psi_1 = (h_1 - h_2)/d \]  \hspace{1cm} (3)

\[ d = R_1 \cos \psi_1 = R_2 \cos \psi_2 \]  \hspace{1cm} (4)

We know the power density in the direct ray in free space is

\[ |\bar{S}_{dir}| = \frac{P_T G_T}{4\pi R_1^2} \]  \hspace{1cm} (5)

from which we can find

\[ |\bar{E}^{dir}| = \frac{\sqrt{60P_T G_T}}{R_1} \]  \hspace{1cm} (6)

\[ \approx \frac{\sqrt{60P_T G_T}}{d} \]  \hspace{1cm} (7)
We can re-write the direct field strength as

\[ |E^{dir}| = \frac{E_0}{d} \quad (8) \]

where

\[ E_0 = \alpha \sqrt{2P_T} \quad (9) \]

and

\[ \alpha = \sqrt{30G_T} \quad (10) \]

\( E_0 \) is known as the “unattenuated field intensity at unit distance”

<table>
<thead>
<tr>
<th>Antenna type</th>
<th>( \alpha ) for d in km</th>
<th>( \alpha ) for d in mi</th>
</tr>
</thead>
<tbody>
<tr>
<td>isotropic</td>
<td>173</td>
<td>108</td>
</tr>
<tr>
<td>short dipole</td>
<td>212</td>
<td>132</td>
</tr>
<tr>
<td>( \lambda/2 ) dipole</td>
<td>222</td>
<td>138</td>
</tr>
<tr>
<td>short monopole*</td>
<td>150</td>
<td>93.2</td>
</tr>
<tr>
<td>( \lambda/4 ) monopole*</td>
<td>157</td>
<td>97.5</td>
</tr>
</tbody>
</table>
The magnitude of the reflected ray can be found from

$$|E^{\text{ref}}| = \left| \frac{E_0}{d} \Gamma \right|$$

(11)

and the total field should be found by adding the direct and reflected fields in the proper phase relationship

$$|E^{\text{tot}}| = \frac{E_0}{d} \left| e^{-jk_0 R_1} + \Gamma e^{-jk_0 R_2} \right|,$$

(12)

Since distances are likely to be large in terms of $\lambda$, it is better to use

$$|E^{\text{tot}}| = \frac{E_0}{d} \left| 1 + \Gamma e^{-jk_0(R_2-R_1)} \right|$$

(13)
Examining the geometry, we can find

\[ R_2 = \left[ d^2 + (h_1 + h_2)^2 \right]^{1/2} \]  \hspace{1cm} (14)

\[ \approx d + \frac{1}{2d} (h_1 + h_2)^2 \]  \hspace{1cm} (15)

and

\[ R_1 \approx d + \frac{1}{2d} (h_1 - h_2)^2 \]  \hspace{1cm} (16)

so that

\[ R_2 - R_1 \approx \frac{2h_1 h_2}{d} \]  \hspace{1cm} (17)

if we are near grazing, to obtain

\[ |\overline{E}_\text{tot}| = \frac{E_0}{d} \left| 1 + \frac{\Gamma}{d} e^{-j2k_0 h_1 h_2/d} \right| \]  \hspace{1cm} (18)

If \( |\Gamma| \approx 1 \) and \( h_1 h_2 < 12 \frac{d_{\text{km}}}{f_{\text{GHz}}} \), we obtain

\[ |\overline{E}_\text{tot}| = \frac{E_0}{d} \frac{4\pi h_1 h_2}{\lambda d} \]  \hspace{1cm} (19)
Example: 1 GHz source 10 m above a perfectly conducting plane, plot received power as a function of height relative to free space for varying receiver heights at 1 km range.

- PEC, horizontal polarization: $\Gamma = -1$
- Thus, $|\overline{E}^{\text{tot}}| = \frac{E_0}{d} |1 - e^{-j2k_0h_1h_2/d}|$
- This will be zero when $2k_0h_1h_2/d = 2n\pi$
- Solve to find $h_2 = 15n$ m
- Field strength relative to free space is given by term in brackets
- Maximum value: 6 dB, minimum: $-\infty$ dB
Received power relative to free space (dB) vs. Receiver Height (m)

- $\Gamma = -1$
- $\Gamma = -0.5$
- Low antenna

1 GHz, 1 km distance, Transmitter height 10 m
1 GHz, Transmitter/Receiver heights 10 m

$\Gamma = -1$

$\Gamma = -0.5$

Low antenna
III. Fresnel zones

- Fresnel zones are defined as the locus of points around a direct ray that produce a specified phase delay if a reflection occurs there.
- Convention is to define the $n$th Fresnel zone as the set of points at which reflection would produce an excess path length of $n\lambda/2$.
- Analytic geometry shows that these are ellipsoids of revolution centered on the direct ray, axes increase with increasing $n$.
- When considered in a two dimensional plane, these become ellipses.
- Since $\Gamma \approx -1$ at grazing incidence, a signal reflected from the first Fresnel zone will be received in phase with the direct signal.
- Thus, odd Fresnel zone reflections produce constructive interference, even Fresnel zone reflections produce destructive interference.
To derive the Fresnel zone radii at a specified distance $d_1$ from one end of the path use

$$r_1 + r_2 = d_1 + d_2 + n\lambda/2$$

(20)

$$\sqrt{d_1^2 + F_n^2} + \sqrt{d_2^2 + F_n^2} = d_1 + d_2 + n\lambda/2$$

(21)

Approximating yields

$$d_1 + \frac{F_n^2}{2d_1} + d_2 + \frac{F_n^2}{2d_2} = d_1 + d_2 + n\lambda/2$$

(22)

and

$$\frac{F_n^2}{2} \left[ \frac{1}{d_1} + \frac{1}{d_2} \right] = F_n^2 \frac{d_1 + d_2}{2d_1 d_2} = n\lambda/2$$

(23)

so

$$F_n = \sqrt{\frac{n\lambda d_1 d_2}{D}}$$

(24)

$$= 17.3 \sqrt{\frac{nd_1, km d_2, km}{f_{GHz} D_{km}}}$$

(25)

with $F_n$ in m
Revisit our planar Earth direct plus reflected model in terms of Fresnel zones: transmit/receive heights $h_1$ & $h_2$, distance $d$

First, write an equation for a line between transmitter image and receiver; intersection with plane is the reflection point.

Next, write an equation for a line between transmitter and receiver (the “direct path”); we can determine the height of this line above the reflection point.

As derived in Section 7.3.1 of the book, this clearance $h$ divided by the first Fresnel zone radius at the reflection point $F_1$ is

$$\frac{h}{F_1} = 2\sqrt{\frac{h_1 h_2}{\lambda d}}$$

When $h/F_1$ is unity, constructive interference occurs, when $h/F_1 = \sqrt{2}$ destructive interference occurs, etc.

Also it turns out that $\pi \left( \frac{h}{F_1} \right)^2$ is the phase difference between the direct and reflected rays at the receiver.

As the receiver height increases, the lowest height at which a free space equivalent power occurs is when $h/F_1 = 0.577$. 

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Terrain Reflection and Diffraction, Part Two

1. Path profile construction
2. Calculation of Earth “bulge”
3. Height gain curves
4. Obstructed region
5. Reflection region
I. Path profile construction

- Predicting propagation over a particular path requires knowledge of the terrain between Tx and Rx
- Such information can be obtained from topographical maps or from mapping agencies, although accuracy may be questionable
- Ground or aerial surveys can be used to obtain more accurate information, but these can get expensive
- On longer paths, Earth curvature and refraction become more important and we need to consider these
We’ll use our Chapter 6 models: either straight rays-modified Earth radius or curved rays-flat Earth.

The former is more useful if we want to consider several antenna heights for a given \( \kappa \) value, the latter if we want to consider several different refractive conditions for fixed antennas.

Note when we draw paths the horizontal scale will usually be much larger than the vertical; this makes angles appear incorrect.
II. Calculation of Earth “bulge”

Using the figure and a theorem for chords in a circle, we have

\[
AD \cdot DC = BD \cdot DE \quad (27)
\]

\[
AD \approx d_1 \quad (28)
\]

\[
DC \approx d_2 \quad (29)
\]

\[
BD \approx b \quad (30)
\]

\[
ED \approx 2R_e = 2\kappa a \quad (31)
\]

so that

\[
d_1 d_2 = (2b)(\kappa a) \quad (32)
\]

and

\[
b = \frac{d_1 d_2}{2\kappa a} \quad (33)
\]
\[ d_1 \quad B \quad d_2 \]

\[ A \quad AD \quad D \quad DC \quad C \]

\[ R_e = AO \quad R_e - b = DO \quad R_e = CO \]
If we use $d$ in km and $b$ in m we find

$$b = \frac{d_1 d_2}{12.74 \kappa}$$

(34)

- Maximum value occurs at center of path, where $b = \frac{d^2}{4(12.7 \kappa)}$
- Note that the Earth bulge becomes larger as the antennas are spread further apart
- Note this “bulge” is not relative to the horizon, it is relative to the direct ray (through the Earth) between Tx and Rx
- This occurs because as Rx moves further from Tx it decreases in height so that it appears the Earth surface is “bulging”
For ground based links:

- We want to minimize costs so we should minimize the number of stations by spacing them far apart.
- If they are too far apart the Earth “bulge” will block the line of sight under certain refractive index variations.
- Even if we are not blocked, reflections from the ground can produce nulls in our pattern that change as refractive indices vary.
- This would require a reflection coefficient magnitude near unity.

The aim of microwave link design is to assure proper path clearance and to avoid interference due to reflections by choosing paths and antenna heights.
Distance to the “radio horizon”:

- Given the Earth bulge, we can find the distance to the “radio horizon” by drawing lines between specified transmitters and receivers and trying to determine whether the Earth bulge is intersected.
- Derivation in Sect. 7.5.1 of the book finds maximum distance at which line between transmitter and receiver transitions from not touching to touching Earth bulge:

\[ d_{\text{max}} = \sqrt{2\kappa a}h_1 + \sqrt{2\kappa a}h_2 \]  

(35)

- For a receiver on the ground this is:

\[ d_{\text{horizon}} = \sqrt{2\kappa a}h_1 = \sqrt{2h_{1,\text{ft}}} \]  

(36)

if the distance is in miles in the last equation.

- Propagation at this distance is not necessarily good!
III. Height gain curves

- Predictions are usually presented in terms of plots of “excess propagation loss” relative to free space versus one antenna height with all other variables held fixed, as in our planar Earth example.
- These are known as “height gain” curves.
- Using the straight ray-effective Earth radius model, we plot the path terrain height for a specified value of $\kappa$.
- Note that topographical maps, surveys, etc. give heights relative to sea level; sea level is on the surface of a sphere!
- Thus, we first plot a reference height above sea level as the modified radius curved earth surface (the “bulge”) and then plot terrain heights relative to this.
- This just amounts to adding the Earth bulge to terrain heights along the path.
IV. Obstructed region (Terrain diffraction)

- If on our terrain plots we find rays that are blocked by the terrain, it is clear the path is “obstructed”
- However there must be some more gradual transition between the non-obstructed and obstructed cases as we will see
- It turns out that signals are still received when the path is obstructed: this is the terrain diffraction mechanism
- Solving a diffraction problem is usually difficult and possible analytically only for a few simple shapes; no solutions for exact terrain obstacles
- We will study the two canonical shape solutions to get some insight into expected field behaviors near obstruction
- Our two cases: a sphere and a knife edge
Spherical case:

- First find path clearances: either positive or negative
- Compute path clearance relative to $F_1$ vs distance; minimum value of this is a key parameter in propagation modeling
- Spherical earth diffraction can be computed from Van der Pol and Bremmer solution we will use again in Chapter 9
- Plotted vs. minimum $h/F_1$; curve depends on more than just this parameter but $h/F_1$ is primary influence
- Note that signal is still present even with obstructed path, although it falls off rapidly as the path becomes more obstructed
- Free space signal levels obtained when minimum $h/F_1 \approx 0.6$, i.e. the path clearance is 60% of the first Fresnel zone radius at the “obstruction” location
- As path clearance increases we see reflection region behavior with constructive and destructive interference; note Fresnel zones
A: Spherical Earth
B: Knife Edge
C: Reduced Reflection
Knife edge case:

- Diffraction by an infinite PEC halfplane of zero thickness, first solved by Sommerfeld
- Solution available in terms of a universal curve versus $h/F_1$
- Pure ray theory would give no field intensity in shadow zone, but diffraction produces fields
- Solution is curve $B$ in Figure, note that again free space value is obtained at $h \approx 0.6F_1$
- This turns out to be true often for general propagation paths, so it is a useful rule of thumb: free space field strength values occur when obstacles are cleared by approximately $0.6F_1$
- Knife edge is a more appropriate model for sharp obstacles such as mountains
A \{ \begin{align*} h &> 0 \quad \text{half-plane} \\ h &< 0 \end{align*} \} B C
Summary of obstructed region:

- Paths which clear an obstacle by more than $0.6F_1$ can be considered not obstructed; this will depend on the atmosphere!
- This requires higher antenna heights than just requiring rays not to be blocked; curves show that zero clearance gives $-19$ dB for the sphere, $-6$ dB for the knife edge. Some empirical formulas available from ITU-R as well.
- Subrefractive conditions $\kappa < 1$ give the worst problem so usually we design for a given clearance at the minimum expected $\kappa$ value for a location.
- In analyzing measurements, graphical methods based on the fact that free space values occur when obstacles are cleared by $0.6F_1$ can be used to identify obstacles.
- Codes exist for producing propagation predictions by combining the sphere and knife edge models.
V. Reflection region

- When paths are not obstructed, it is possible to obtain interference between direct and reflected rays.
- We want to avoid destructive interference!
- Note significant reflections are obtained only if $|\Gamma| \approx 1$, this is often the case since we are near grazing.
- However, large local terrain height variations on a wavelength scale reduce $\Gamma$ (reflection from a rough surface), as do vegetation, so reflections can usually be neglected on these paths.
- Determining reflection points from analysis can be difficult; instead we will focus on finding reflection points given measurements.
- If we can locate reflection points, sometimes it is possible to do something about them and eliminate the possibility of destructive interference.
Our direct + ground reflected formulas in terms of \( h/F_1 \) are

\[
|E| = \frac{|E_0|}{d} |1 + \Gamma e^{-j\pi \left( \frac{h}{F_1} \right)^2}|
\]  

(37)

where \( h \) is the path clearance at the reflection point.

This fits the sphere result well for clearing paths, plot also includes a reduced \( \Gamma = -0.5 \) case.

To locate reflection points on paths from measurements, first plot rays for paths which result in transmission minima (these are usually sharper and easier to identify than maxima).

For \( \Gamma = -1 \), it is clear from the formula that minima should be obtained when the reflection point is cleared by an even number Fresnel zone radius.
In the planar Earth problem, finding the reflection point is easy; we can also solve the problem for a sphere, but it is more complicated.

Locating reflection points for real paths is often not easy because there can be several reflection points (or even reflection “regions”) so that path intersections are not clear.

Changing terrain slopes can also cause variations in reflection point effects.

In practice, reflection points usually arise from particularly flat parts of the terrain on a path, for example ponds or asphalt surfaces.

It is therefore important to map these out if reflections are a problem.
Terrain Reflection and Diffraction, Part Three

1. Path analysis examples
2. Numerical methods
I. Path analysis examples

- Consider the 36 km profile on the next frame. A 600 MHz Tx is located on the left side at height 20 m above terrain.
- What is the minimum receiver height required to avoid obstruction (i.e. lowest height that achieves free space propagation level)?
- First elevate profile by Earth bulge (given assumed $\kappa = 4/3$), then by $0.6F_1$.
- Find line from transmitter that intersects terrain + bulge + $0.6F_1$ in only one point.
- Line height at end of path determines required Rx height.
- If we assume that reflection point is located at same place as the “last” obstruction, a similar procedure raising terrain by bulge + $F_2$ can be used to predict height of first null.
- Location of reflection points generally hard to predict a-priori.
Sometimes a-priori determination of primary obstructions can be difficult given absence of trees, buildings, etc. in many sources of topographic information.

If measurements at various transmitter and receiver altitudes are available (unusual), then paths providing a free space propagation level can be examined together.

Intersection points common among these paths likely indicate obstruction location.

A similar procedure can be used to try to locate reflection points if datasets with varying transmit and receive heights are available.

When trying to locate reflection points, nulls in the height gain pattern are better points of reference as compared to maxima.
Another example: propagation measurements performed by MIT Lincoln Laboratories at 167 MHz with a receiver on a helicopter (allows measurement of height-gain pattern.)

15.2 km profile example on next frame; transmitter is 18.3 m above local terrain at left.

Using our procedure to locate the minimum receiver height for free space received power (i.e. elevate terrain by bulge + 0.6\(F_1\)), a receiver height of 25 m above the local terrain is determined.

Compared with data from experiment in next frame; not too far off!

Measurement also shows local minima at 71.6 and 113 m height; use these to try to locate reflection point.

Using terrain elevated by \(F_2\) and \(F_4\) shows intersection around 13.2 km.

A much stronger minimum occurs around 490 m; a similar analysis for this minimum suggests a reflection point around 0.5 km.
II. Numerical methods for path loss analysis

- Our analysis of obstructed paths and reflections has been based on simple analytical models.
- Real paths do not necessarily fit these models, so our insights may not apply.
- Computers can be used to solve Maxwell’s equations with general boundary conditions, allowing us to get more exact answers for path predictions.
- However, numerical methods for electromagnetics usually require discretization on the scale of $\lambda$, if paths are very large this becomes a very large problem!
- Computers and numerical methods are improving to enable us to solve these problems!
- “Advanced refractive effects prediction system” (AREPS) code of US Navy is a good example.
Possible numerical methods:

- **Method of moments (MOM):** discretizes an integral equation for the boundary conditions into a matrix equation, requires sampling on scale of $\lambda$. Usually does not include atmosphere.

- **Finite difference-time domain (FDTD):** discretizes differential form of Maxwell’s equations throughout space, also requires sampling on scale of $\lambda$ but can run several frequencies simultaneously, could include atmosphere at higher discretization cost.

- **Parabolic wave equation (PWE):** based on parabolic approximation to wave equation, does not require sampling on scale of $\lambda$ and so is very efficient compared to MOM and FDTD. Easy to include atmosphere.

Each of these methods has its advantages and disadvantages - we also have our old analytical techniques as well that can be programmed and should be the most efficient of all.