Announcement: The midterm examination will occur Friday, Sept 28th in class. The exam will cover Chapters 1-6 of the notes and Problem Sets 1-3, and will be open book.

Problem 1

A plane wave with \( \mathbf{E}^{\text{inc}} = (2\hat{x} + \hat{y} + 2\hat{z})e^{-j2\pi(x-z)} \) V/m is incident from a lossless, non-magnetic medium with \( \varepsilon = 5\varepsilon_0 \) onto a planar boundary in the xy plane with free space.

(a) Find \( \theta_i \) and the frequency.

(b) Find \( k_z^{\text{trans}} \).

(c) Write the reflected electric field, \( \mathbf{E}^{\text{ref}} \), including numerical values for field amplitude.

(d) Write the transmitted magnetic field, \( \mathbf{H}^{\text{trans}} \), including numerical values for field amplitude.

(e) If a receiver in the free space region receives all the transmitted power in a 1 square meter area and requires \( 10^{-6} \) Watts in order to function, how far away from the boundary can the receiver be moved and still function? Assume that \( \langle \tilde{S}(t) \rangle = \frac{10}{\eta_0} e^{-2Im(\kappa z^{\text{trans}})z} \) W/m² in the transmitted region.

Problem 2

The relative permittivity of the atmosphere as a function of height, \( z \), is found to be

\[
\varepsilon(z) = (3.24 \times 10^{-8})z^2 - (3.6 \times 10^{-4})z + (1.002)^2 = (1.002 - 1.8 \times 10^{-4} z)^2
\]

for altitudes less than 10 km with \( z \) measured in km.

(a) Sketch \( \varepsilon(z) - 1 \) for altitudes from 0 to 10 km.

(b) Sketch \( n(z) - 1 \) in N-units for altitudes from 0 to 10 km. What is \( \frac{dn}{dh} \) for this plot?

(c) Sketch \( m(z) - 1 \) in M-units for altitudes from 0 to 10 km. What is \( \frac{dm}{dh} \) for this plot?

(d) Find the effective earth radius multiplier, \( \kappa \). Classify this type of atmospheric condition.

(e) Is ducting possible in this atmosphere? Explain your decision based on the plots in parts (b) and (c) and also on the earth radius multiplier.

(f) A transmitter located on the ground launches a ray at 2 degrees above the horizon. Find the angle this ray makes with the local horizontal direction when it reaches altitude 10 km above a spherical earth.
Problem 3

(a) Consider propagation in a surface based duct with the program ray.m, which traces rays for specified M-profiles. Assume a transmitter is located 10 m above the ground in a surface based duct. The M(h) profile of the atmosphere is such that M linearly decreases at 40 M-units/km from a value of 350 M-units at the surface to a value of 346.8 M-units at 80 m height. Above 80 m, M increases at 118 M-units/km. Generate some rays that result for varying initial ray elevation angles. Be careful to avoid spurious numerical reflections off the top of your input profiles.

(b) Find the maximum positive and negative values of the elevation angle which result in ducting propagation.

(c) A theory of ducting propagation specifies the angle of part (b) as \( \psi = (2\Delta M \times 10^{-6})^{\frac{1}{2}} \) in radians, where \( \Delta M \) is the difference between M at the transmitter height and the minimum value of M. Do your results from part (b) agree with this theory?

(d) Provide a brief derivation of this “critical angle” phenomenon for ducting propagation.