Problem Set 2
ECE 312 Winter Quarter 2012

Assigned: Wed. Jan. 11th
Due: Wed. Jan. 18th in class
Instructor: Joel T. Johnson

Problem 1
A current density \( \vec{J} = \hat{z}(1 + r) \, A/m^2 \) flows through an infinitely long cylinder with radius 5 cm as shown, where \( r \) is the cylindrical coordinate. Assume that \( \mu = \mu_0 \) everywhere.

(a) Choose a coordinate system for representing the fields produced by this source. Based on the symmetry of the source, describe which field components should exist and which coordinates they should be a function of.

(b) Choose a circular contour of an arbitrary radius \( 0 < r_0 < 0.05 \) m; the contour lies in the \( x-y \) plane and is centered on the origin. Simplify the left hand side of Ampere’s Law (i.e. \( \oint (\vec{H} \cdot d\vec{l}) \)) based on your part (a) answer and your circular contour.

(c) Now find the total amount of current passing “through” the circular contour of radius \( r_0 \); this is the right hand side of Ampere’s Law.

(d) Equate the two sides of Ampere’s Law to determine the field intensity everywhere inside the cylinder. Provide a final answer for the magnetic field including magnitude and direction.

(e) Repeat parts (b)-(d) for \( r_0 > 0.05 \) m.

Problem 2
The ultra-high field Magnetic Resonance Imaging (MRI) system of OSU has one of the highest magnetic field strengths of any MRI in the world. The magnetic field in this system is produced by a large solenoid of radius \( a = 1 \) m which has \( N = 96000 \) turns of wire in a 3 m length. We will model this device as an infinite solenoid whose axis is aligned with the \( z \) direction of a cylindrical coordinate system. The currents of the solenoid flow in the \( \phi \) direction.

(a) The OSU system was designed to produce an 8 T flux density inside the magnet. Find the current that must flow in the solenoid’s wires to achieve this field strength.

(b) Write the magnetic flux density inside and outside the solenoid.

(c) A patient inside the solenoid has a small square loop of wire with side length 2 cm. The loop of wire carries current 0.5 A, and is oriented so that \( \vec{x} \) is normal to the loop in a right hand sense associated with the direction of current flow. Find the torque that results on the loop.

(d) This torque will cause the loop to rotate if it is not restrained. Describe this rotational motion.