EE 311 - Lecture 1

- Administrative
- Introductions
- Why study EM?
- Overview of course

Assigned reading: Secs 1.1-1.2 of Ulaby for basic overview material, incl SI system

Why study EM?

- All electrical engineering is based on the use of electricity and/or magnetism to accomplish tasks
- Both are fundamental properties of matter: useful because charges and currents can exert forces
- In general this can be quite complicated, but we try to set up devices that simplify interactions: for example, circuits
- You have studied circuit theory; basis of a majority of EE work
- However, it is a simplified version of reality and often fails to describe nature accurately.
- A good EE is able to go back to fundamental physics (i.e. EM) to get around limitations of cct theory

Examples

- A very long wire!
- Resistor, capacitor, inductor
- High speed (RF) circuits
- Optical fibers
- Antennas and wireless communications

Overview of Course

- We’ll start off studying transmission lines (i.e. long wires). We’ll do this by starting from the circuit theory you already know.
- We’ll learn a lot about how long wires behave both for sinusoidal and pulsed excitations.
- We’ll learn about reflections on wires, the “Smith chart”, and impedance matching networks.
- Then we’ll delve deeper into the real nature of electric fields and the associated resistive and capacitive circuit elements.
- Finally we’ll study the real nature of magnetic fields and the associated inductor element.

Continuing in EE 312 you’ll study the behavior of electromagnetic waves including propagation, reflection, and refraction, along with antennas and communications systems.
When is a wire a transmission line?

- From the examples we’ve talked about earlier, you’ve probably gathered that the length of a wire is an issue.
- For a line of length $l$, it takes $\delta t = l/c$ time (where $c$ is the speed of light $3 \times 10^8$ m/sec) to reach the end of the line.
- If this is an appreciable time delay then we can’t think of the line as a wire.
- Whether or not the delay is appreciable depends on how fast the circuit is operating, i.e. the operating frequency $\omega$ in an AC circuit.
- If $\omega l/c$ is large, the delay is important. We will learn later than $\omega/c = 2\pi/\lambda$ where $\lambda$ is the electromagnetic wavelength.
- Thus if $l/\lambda$ is appreciable for a sinusoidal circuit, we need to think of wires as transmission lines. If it is not appreciable, it is ok to think in terms of wires.

Examples/other transmission line effects

- A few examples:
  - 60 Hz: $\lambda = 5000$ km
  - 1 MHz: $\lambda = 300$ m
  - 1 GHz: $\lambda = 30$ cm
  - 10 GHz: $\lambda = 3$ cm
- Ulaby also tells us about the possibility of reflections on transmission lines as well as dispersion and power loss.
- Reflections occur due to time delay effects: at first the generator knows nothing about the load; load information obtained from a time delayed reflected signal.
- Dispersion occurs for non-AC signals when different frequency components propagate at different speeds. “Smearing” of original signal results.
Circuit model for a transmission line

- The “circuit” representation of a transmission line is simply two parallel wires, regardless of what the line actually looks like.
- To model a transmission line, we think about using circuit theory to describe a short piece of the line. Since this piece is small compared to λ circuit theory should work.
- However, we’ve got to be careful about the real behavior of a short pair of wires; real wires have resistance, capacitance, and inductance.
- A realistic model for a short pair of wires is below:

\[ L_0 \text{ is the inductance per unit length of a short piece of line (H/m); wires carry current therefore have inductance} \]

\[ C_0 \text{ is the capacitance per unit length between the two wires (F/m)} \]

\[ R_0 \text{ is the series resistance of the wire (ohms/m)} \]

\[ G_0 \text{ is the conductance for currents between the two wires. (mhos/m)} \]

- To make a good line we try to minimize \( R_0 \) and \( G_0 \); we want power transfer, not loss in the line.
- The circuit model describes only a section of length \( \Delta z \); these are cascaded to create the final total line length; we are going to work things out as \( \Delta z \) goes to zero.
- The \( R', L', G', \) and \( C' \) parameters are functions of the structure of the line; we will study later.

Circuit equations

Let’s take a closer look at one section of our line model:

\[ v(z + \Delta z, t) = v(z, t) - R' \Delta z \frac{\partial i(z, t)}{\partial t} \]

\[ i(z + \Delta z, t) = i(z, t) - G' \Delta z v(z + \Delta z, t) - C' \Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} \]

Simplify

Rearranging these

\[ \begin{cases} v(z + \Delta z, t) - v(z, t) \\ i(z + \Delta z, t) - i(z, t) \end{cases} = - \left( R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t} \right) \]

Or as \( \Delta z \) goes to zero,

\[ \frac{\partial v(z, t)}{\partial z} = - \left( R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t} \right) \]

\[ \frac{\partial i(z, t)}{\partial z} = - \left( G' v(z, t) + C' \frac{\partial v(z, t)}{\partial t} \right) \]

- These are the “Telegrapher’s Equations” in the time domain.
- Presence of both time and space derivatives makes these somewhat complicated; use phasor analysis to simplify!
EE 311 - Lecture 3

• Phasors
• Complex numbers
• Using phasors for circuits

Assigned reading: Sec 1.5 of Ulaby for Phasor review

Complex numbers

• Complex number \( z \) is written as \( x + jy \), where \( x \) is the real part of \( z \) (Re\{z\}), \( y \) is the imaginary part of \( z \) (Im\{z\}), and \( j = \sqrt{-1} \)

• The above form is the “rectangular form” of a complex number. We can also use a polar form:

\[
z = |z| e^{j\theta} = |z| \angle \theta
\]

where \( |z| \) indicates the magnitude of \( z \) and \( \theta \) is the phase of \( z \)

• Rectangular and polar forms are related similar to rectangular and polar coordinates. Careful when computing \( \tan^{-1} \)

![Diagram of complex number representation](image)

\[
x = |z| \cos \theta \\
y = |z| \sin \theta \\
|z| = \sqrt{x^2 + y^2} \\
\theta = \tan^{-1} \left( \frac{y}{x} \right)
\]

Phasors

• Phasors are used to simplify the analysis of time harmonic signals (\( A \cos(\omega t + \phi) \) for example)

• For a sinusoidal signal with a fixed angular frequency \( \omega \), the amplitude \( A \) and phase \( \phi \) parameters completely describe the signal

• For linear system, sinusoidal excitation at frequency \( \omega \) causes all outputs to be sinusoidal at frequency \( \omega \) also

• The circuits you’ve studied so far are linear: phasors used in AC circuit analysis to describe voltage and current amplitudes and phases

• The use of complex numbers makes phasor analysis particularly simple: complex numbers are ideal because they describe an amplitude and phase, just as we need for our phasors.

• Let’s review complex numbers and their associated operations....

Complex numbers: other basics

• Euler’s identify can be very useful:

\[
e^{j\theta} = \cos \theta + j \sin \theta
\]

• Note \( |e^{j\theta}| = 1 \)

• The complex conjugate of a complex number is given by \( z^* = x - jy \), i.e. the sign of the imaginary part is changed. Corresponds to reflection across the real axis in complex plane.

• The magnitude of a complex number can also be found from:

\[
|z| = \sqrt{zz^*} = \sqrt{|z|e^{j\theta} |z|e^{-j\theta}}
\]

• Useful identities:

\[
e^{j\pi} = -1 = 1 \angle \pi
\]

\[
e^{j\pi/2} = \pm j = 1 \angle \pm \pi/2
\]

\[
\sqrt{j} = \pm e^{j\pi/4} = \pm \frac{1 + j}{\sqrt{2}}
\]
Complex number operations

Operations on complex numbers include: \( z_1 = x_1 + jy_1 = |z_1| e^{j\theta_1}, \)
\( z_2 = x_2 + jy_2 = |z_2| e^{j\theta_2} \)
- Addition: \( z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \)
- Subtraction: \( z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \)
- Multiplication: \( z_1 z_2 = |z_1| |z_2| e^{j(\theta_1 + \theta_2)} \)
- Division: \( z_1/z_2 = (|z_1|/|z_2|) e^{j(\theta_1 - \theta_2)} \)

Note addition and subtraction are most easily performed with rectangular form, while multiplication and division are easiest with polar form.

Your calculator should perform these operations easily; see examples p. 24 in book for practice.

Using phasors for circuits

- We use phasors to analyze circuits excited by sinusoidal sources, because all voltages and currents in the circuit will have the same angular frequency as the source.
- The definition of a phasor \( \tilde{Z} \) corresponding to voltage or current \( z(t) \) is
  \[ z(t) = \text{Re} \left\{ \tilde{Z} e^{j\omega t} \right\} \]
\( \tilde{Z} \) is then the phasor corresponding to \( z(t) \)
- For example, given a voltage source \( v_s(t) = V_0 \cos(\omega t + \phi_0) \), we write:
  \[ V_0 \cos(\omega t + \phi_0) = \text{Re}\{\tilde{V} e^{j\omega t}\} \]
  \[ V_0 \cos(\omega t + \phi_0) = \text{Re}\{\tilde{V} e^{j\theta} e^{j\omega t}\} \]
  \[ V_0 \cos(\omega t + \phi_0) = |\tilde{V}| \cos(\omega t + \theta) \]
Thus \( |\tilde{V}| = V_0 \) and \( \theta = \phi_0 \), or \( \tilde{V} = V_0 e^{j\phi_0} \). Note the phasor represents the amplitude and phase of the source.

Using phasors for circuits: II

- A trick:
  \[ \text{Re}\{-j\tilde{V}' e^{j\omega t}\} = \text{Re}\{|\tilde{V}'| (\tilde{V}') e^{j\theta} e^{j\omega t}\} = |\tilde{V}'| \sin(\omega t + \theta) \]
  Thus to produce a sine function instead of cosine, multiply the phasor by \(-j\)
- We can use the same definition to convert phasors back into the time domain
- Because phasors all simply behave as \( e^{j\omega t} \) in time, time derivatives in equations become multiplication by \( j\omega \), time integrals become division by \( j\omega \)
- Complex impedances result for inductors (\( Z = j\omega L \)) and capacitors (\( Z = -j/(\omega C) \)); circuit solution as easy as DC circuits (i.e. algebraic instead of differential equations)
- Let’s try an example to help remind us...

Phasor circuit example

Given \( v_s(t) = V_0 \sin(\omega t + \phi_0) \), find \( i(t) \) in the circuit below
- The time domain circuit equation is:
  \[ R i(t) + \frac{1}{C} \int dt i(t) = v_s(t) \]
- Using phasors, \( v_s(t) \) becomes \( \tilde{V} = -jV_0 e^{j\phi_0} \), and the capacitor becomes an impedance \(-j/(\omega C)\)
Phasor circuit solution

- Solving the circuit:

\[
\begin{align*}
R\tilde{I} - \frac{j}{\omega C}\tilde{I} &= \tilde{V} \\
\tilde{I} &= \tilde{V} \left[ \frac{1}{R - j/(\omega C)} \right] = \tilde{V} \left[ \frac{\omega C}{\omega RC - j} \right] \\
\tilde{I} &= -jV_0e^{j\phi_0} \left[ \frac{\omega C}{\omega RC - j} \right]
\end{align*}
\]

- We can simplify this to

\[
\tilde{I} = V_0\omega Ce^{j\phi_0} \frac{1}{1 + j\omega RC}
\]

\[
\tilde{I} = \frac{V_0\omega C}{\sqrt{1 + \omega^2 RC^2}} e^{j(\phi_0 - \phi_1)}
\]

where \(\phi_1\) is the phase of \(1 + j\omega RC\)

- Finally

\[
\begin{align*}
i(t) &= \text{Re} \left\{ \tilde{I} e^{j\omega t} \right\} \\
i(t) &= \frac{V_0\omega C}{\sqrt{1 + \omega^2 RC^2}} \cos(\omega t + \phi_0 - \phi_1)
\end{align*}
\]

Phasor telegrapher’s equations

- Recall the time domain Telegrapher’s equations are

\[
\begin{align*}
\frac{\partial v(z,t)}{\partial z} &= - \left( R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t} \right) \\
\frac{\partial i(z,t)}{\partial z} &= - \left( G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t} \right)
\end{align*}
\]

where \(v(z,t)\) and \(i(z,t)\) are the voltage and current on a transmission line: functions of space and time

- We can introduce phasors through:

\[
\begin{align*}
v(z,t) &= \text{Re} \left\{ \tilde{V}(z)e^{j\omega t} \right\}, \quad i(z,t) &= \text{Re} \left\{ \tilde{I}(z)e^{j\omega t} \right\}
\end{align*}
\]

Phasors \(\tilde{V}(z)\) and \(\tilde{I}(z)\) are functions of space but not time now

- The telegrapher’s equations now become

\[
\begin{align*}
\frac{\partial \tilde{V}(z)}{\partial z} &= -(R' + j\omega L') \tilde{I}(z) \\
\frac{\partial \tilde{I}(z)}{\partial z} &= -(G' + j\omega C') \tilde{V}(z)
\end{align*}
\]

Phasor wave equation

- We have two coupled differential equations, but we can take \(\frac{\partial}{\partial z}\), then substitute one into the other to eliminate one unknown:

\[
\begin{align*}
\frac{\partial^2 \tilde{V}(z)}{\partial z^2} &= -(R' + j\omega L') \frac{\partial \tilde{I}(z)}{\partial z} \\
\frac{\partial^2 \tilde{I}(z)}{\partial z^2} &= (R' + j\omega L')(G' + j\omega C') \tilde{V}(z)
\end{align*}
\]

- Simplifying we have

\[
\frac{\partial^2 \tilde{V}(z)}{\partial z^2} - \gamma^2 \tilde{V}(z) = 0
\]

where \(\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\) is the “complex propagation constant” of the line.

- \(\gamma = \alpha + j\beta\) with \(\alpha\) called the “attenuation constant” of the line (Np/m) and \(\beta\) the phase constant of the line (rad/m); we’ll see why later

- We can derive exactly the same equation for \(\tilde{I}(z)\) on the line also, just replace \(\tilde{V}\) with \(\tilde{I}\)
Solution of phasor wave equation

- The phasor wave equation is a simple second order differential equation: differentiate a function twice, get a constant times the function.
- Solutions are exponentials:
  \[ \hat{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \]
  where the quantities \( V_0^+ \) and \( V_0^- \) are unknown constants.
- If we take the voltage solution and substitute it back into the telegrapher’s equations to find \( I(z) \), we get
  \[ \frac{\partial \hat{V}(z)}{\partial z} = -(R' + j\omega L') \hat{I}(z) \]
  \[ \hat{I}(z) = \frac{\gamma}{R' + j\omega L'} (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \]
  \[ \hat{I}(z) = \frac{1}{Z_0} (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \]
  where \( Z_0 = \frac{R' + j\omega L'}{\sqrt{\frac{R' + j\omega L'}{\omega C}}} \) is the “characteristic impedance” of the line (Ohms).

Time domain voltage behavior

- Now that we’ve found the phasor voltage, we can look at it in the time domain:
  \[ v(z, t) = \text{Re} \left\{ (V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}) e^{j\omega t} \right\} \]
  \[ v(z, t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi_+) + |V_0^-| e^{\alpha z} \cos(\omega t + \beta z + \phi_-) \]
- Two somewhat complicated terms added together. Notice:
  - Each has an amplitude term that varies in \( z \): \( |V_0^+| e^{-\alpha z} \) and \( |V_0^-| e^{\alpha z} \). The definition of \( \alpha \) as an “attenuation” constant is now clear.
  - Each has a sinusoidal dependence on both time and space: \( \cos(\omega t - \beta z + \phi_+) \) and \( \cos(\omega t + \beta z + \phi_-) \). The definition of \( \beta \) as a “phase” constant is now clear.
- The combination of time and space variables \( \omega t \pm \beta z \) results in a “propagating wave” behavior.

Line Properties

- The characteristic impedance of a line is a fundamental property.
- However be sure to note that the characteristic impedance of the line is NOT the ratio of total voltage to total current, unless \( V_0^- = 0 \).
- If \( R' = G' = 0 \) (lossless line), \( Z_0 = \sqrt{\frac{L'}{C}} \) and the characteristic impedance is purely real.
- If \( R' \) or \( G' \) is not zero, then the characteristic impedance will usually be complex.
- The complex propagation constant \( \gamma \) of a line is another fundamental property.
- If \( R' = G' = 0 \) (lossless line), \( \gamma = j\omega \sqrt{CL} \) is purely imaginary \((\alpha = 0)\).
- If \( R' \) or \( G' \) is not zero, \( \alpha \) will not be zero.

Propagating waves

- A wave is a disturbance that moves in space as time progresses.
- Consider a function of the form \( y(x, t) = A \cos \left( \frac{2\pi}{T} t - \frac{2\pi x}{\lambda} \right) \).
  Note this is sinusoidal in time at a fixed value of \( x \); sinusoidal in space at a fixed value of time.
- At a fixed point in \( x \), the temporal period is found to be \( T \). \( \frac{2\pi}{T} \) corresponds to \( \omega \) for our voltage wave.
- The temporal frequency in Hz (1/sec) is then \( f = 1/T \). Our angular frequency \( \omega = 2\pi f \) (rads/sec).
- At a fixed time, the spatial period is found to be \( \lambda \). \( \frac{2\pi}{\lambda} \) corresponds to \( \beta \) for our voltage wave.
- How does this function behave as both space and time are varied?
Propagating waves II

Let’s plot the function versus space as time progresses:

![Diagram showing wave propagation](image)

Propagating waves III

- Notice the disturbance in space moved as time progressed, in the $+x$ direction.
- If we used $\cos \left( \frac{2\pi t}{T} + \frac{2\pi x}{\lambda} \right)$ instead of $\cos \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right)$ the wave would propagate in the $-x$ direction instead.
- “Phase velocity” $u_p$ is found from distance over time:
  \[ u_p = \frac{\lambda}{2T} = \frac{\lambda}{T} \frac{\omega}{\beta} \]
- Our voltage solution thus consists of two waves, one propagating in the $+z$ direction, the other in the $-z$ direction, both at speed $u_p = \frac{\beta}{2}$

Waves on transmission lines

- We have found the phasor voltage on a transmission line to be
  \[ \tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \]
  where $\gamma = \alpha + j\beta$ is the complex propagation constant of the line.
- In the time domain, this is:
  \[ v(z,t) = \left| V_0^+ \right| e^{-\alpha z} \cos(\omega t - \beta z + \phi_+) + \left| V_0^- \right| e^{\alpha z} \cos(\omega t + \beta z + \phi_-) \]
- The first term is a voltage wave propagating in the $+z$ direction, the second propagates in the $-z$ direction.
- Both propagate at velocity $u_p = \frac{\beta}{2}$; this is known because we know $\beta$; both attenuate according to $\alpha$.
- The total voltage is the sum of two phasors whose phase varies differently along $z$: leads to an interference pattern in $z$.
- This behavior is very different from a wire!
Effect of attenuation

- Let's consider the plus going wave only in the time domain:
  \[ |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi_+) \]
- If we plot this in space at a fixed time we see an attenuating oscillation.
- As time evolves, the oscillation propagates, but the “envelope” of the amplitude does not.

Aside: Attenuation in decibels per meter

- Consider a line having only a plus going wave
- Our attenuation constant \( \alpha \) is in units of Np/m due to the magnitude
  \[ |V_0^+| e^{-\alpha z} \]
- Note when \( z = 1 \) the wave amplitude is \( e^{-\alpha} \) of the amplitude at \( z = 0 \)
- To express a voltage ratio in decibels, take \( 20 \log_{10} \) of the ratio.
  \[ \frac{\tilde{V}(z = 1 \text{ m})}{\tilde{V}(z = 0 \text{ m})} = e^{-\alpha} \]
  \[ 20 \log_{10} \left[ \frac{\tilde{V}(z = 1 \text{ m})}{\tilde{V}(z = 0 \text{ m})} \right] = 20 \log_{10} e^{-\alpha} = -20 \alpha \log_{10} e \]
  \[ \alpha_{\text{dB/m}} = 8.6859 \alpha_{\text{Np/m}} \]
- It is much more common to express line attenuation in dB/m

Special case: A Lossless line

- For a lossless line, \( R' = G' = 0 \) (no lossy elements in line circuit).
- This leads to:
  \[ \gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \sqrt{-\omega^2 L'/C'} = j\omega \sqrt{L'/C'} \]
- Thus \( \alpha = 0 \) and \( \beta = \omega \sqrt{L'/C'} \). There is no wave attenuation on a lossless line.
- The phase velocity \( u_p = \omega/\beta = 1/\sqrt{L'/C'} \). We will see how this relates to the speed of light soon... Note this is independent of frequency, so a lossless line has no dispersion
- The characteristic impedance becomes \( Z_0 = \sqrt{L'/C'} \) (real)
- The wavelength on the line is (as always) \( \lambda = 2\pi/\beta \)
- Although real lines aren’t totally lossless, good lines will satisfy these equations approximately. Include effects of \( \alpha \neq 0 \) over longer distances

Special case: Distortionless line

- The general propagation velocity on a line is:
  \[ u_p = \omega/\beta = \omega/\left( \text{Im}\left\{ \sqrt{(R' + j\omega L')(G' + j\omega C')} \right\} \right) \]
- This has a complicated dependence on frequency, so a general lossy line has dispersion
- However there is one case where we can get rid of dispersion: choose
  \[ R'/L' = G'/C' \]
  so that
  \[ \sqrt{(R' + j\omega L')(G' + j\omega C')} = \sqrt{L'C'} \sqrt{(j\omega + R'/L')} = j\omega \sqrt{L'C'} + R' \sqrt{C'/L'} \]
- This is called a “distortionless line”. Here
  \[ \alpha = R' \sqrt{C'/L'} \quad \beta = \omega \sqrt{L'C'} \quad \alpha_p = 1/\sqrt{L'C'} \quad Z_0 = \sqrt{L'/C'} \]
- Although the line is lossy, the phase velocity is the same as for a lossless line, and we have no dispersion (or “distortion”). The characteristic impedance is also purely real
Types of lines

- So far we’ve been thinking of a transmission line as a general structure having two conductors and a uniform cross section

Other devices called “waveguides” (or “higher order transmission lines” according to Ulaby) cannot support TEM modes. They do not have to have two conductors.

Example waveguides are rectangular waveguide, optical fiber, etc.

It turns out that waveguides and higher order modes can propagate only when the operating frequency exceeds a specific “cutoff frequency”. TEM modes can propagate at any frequency

“TEM” and higher-order lines

- All transmission lines can propagate both “TEM” modes and higher-order modes. We are studying “TEM” modes only
- “TEM” means “transverse electric-magnetic”: we will learn more about this later. You have to have 2 conductors to support a TEM mode, so all our lines have two metal conductors.
- Important line material properties:
  - $\epsilon$: (units F/m) the “dielectric constant” of the material between the two conductors. For air, $\epsilon = \epsilon_0 = 8.854 \text{ pF/m}$. Other materials are often specified in terms of a relative permittivity (or “dielectric constant”) $\epsilon_r = \epsilon / \epsilon_0$
  - $\mu$: (units H/m) the “magnetic permeability” of the material between the two conductors. For air, $\mu = \mu_0 = 0.4\pi \text{ H/m}$. Most materials have $\mu = \mu_0$.
  - $\sigma$ (units mhos/m) the “electric conductivity” of the medium between the two wires. Typically this is very small because insulators are used.
  - $\sigma_c$ (units mhos/m) the electric conductivity of the line conductors (metal). Very large for most metals.
  - $\mu_c$: the magnetic permeability of the metal line conductors.

### EE 311 - Lecture 6

- Determining line parameters
- Coaxial line
- Parallel plate line
- Propagation velocities again

Assigned reading: Sec. 2.2 of Ulaby
Coaxial line

Coaxial line properties are determined by the inner radius $a$, the outer radius $b$, and the material properties of the conductors and insulator.

Coaxial line equations

The relationship between coaxial line properties and the transmission line parameters is

$$ R' = \frac{R_s}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right) \quad G' = \frac{2\pi \sigma}{\ln(b/a)} \quad R_s = \sqrt{\pi f \mu_c / \sigma_c} $$

$$ L' = \frac{\mu}{2\pi} \ln \left( \frac{b}{a} \right) \quad C' = \frac{2\pi \varepsilon}{\ln(b/a)} $$

- These equations are derived using electro- and magneto- static methods we will study later in the course.
- Note to make $R'$ small, we should make $R_s$ small, or $\sigma_c$ large. This means using high conductivity metals.
- To make $G'$ small, we should make $\varepsilon$ as small as possible; use good insulators between conductors.
- Note that $R'$ typically will be dominated by the inner conductor. Smaller $a$ means larger $R'$. Also, $R_s$ increases as $\sqrt{f}$.
- All parameters other than $R'$ depend on $b/a$; absolute size of line doesn’t matter for these.

Practical coaxial lines

- Coaxial lines are very commonly used in practice. Note both inner ("wire") and outer ("shield") connections are important for proper operation.
- If we consider a near lossles line, it turns out that
  $$ Z_0 = \frac{1}{2\pi} \sqrt{\mu / \varepsilon} \ln(b/a), \quad \alpha \approx \frac{R'}{2Z_0} = \frac{R_s}{4\pi \left( \frac{1}{a} + \frac{1}{b} \right) / \left( \frac{1}{2\pi} \sqrt{\mu / \varepsilon} \ln(b/a) \right)} $$
- For fixed $b, R_s, \mu, \text{ and } \varepsilon$, it turns out this is minimized when $b/a \approx 3.6$. The resulting cable impedance is then $Z_0 = \frac{1}{2\pi} \sqrt{\mu / \varepsilon}$.
- For air filled cables, this results in $Z_0 = 77$ Ohms, while for polyethylene filled cables $\varepsilon_r = 2.25$, $Z_0 = 51.2$ Ohms.
- Commercial coaxial cables typically have $Z_0 = 50$ ohms or 75 Ohms because of this.
- See the link on the course web page for a good section on practical coaxial cables.

Parallel plate line

The parallel plate line has two plates of width $w$ separated by distance $d$. A material may fill the region between the plates.
Parallel plate equations

- The relationship between parallel plate properties and the transmission line parameters is

\[ R' = \frac{2R_s}{w}, \quad G' = \frac{\sigma w}{d}, \quad R_s = \sqrt{\frac{\pi f \mu_0}{\sigma_c}} \]

\[ L' = \frac{\mu d}{w}, \quad C' = \frac{\varepsilon w}{d} \]

- Again to make \( R' \) small, we should make \( R_s \) small, or \( \sigma_c \) large. This means using high conductivity metals.
- To make \( G' \) small, we should make \( \sigma \) as small as possible; use good insulators between conductors.
- Smaller \( w \) means larger \( R' \). Also, \( R_s \) increases as \( \sqrt{d} \)
- All parameters other than \( R' \) depend on \( w/d \); absolute size of line doesn’t matter for these
- For a lossless line, \( Z_0 = \sqrt{\mu/\varepsilon} \frac{d}{w} \)

Propagation velocities again

- Ulaby also talks about the “two-wire” line: see eqns. in book
- Note that the phase velocity for both lossless line types is

\[ u_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu \varepsilon}} \]

- Thus only the insulator material is important in determining propagation velocity on a lossless cable. For air filled cables, \( 1/\sqrt{\mu \varepsilon_0} = 3 \times 10^8 \) m/sec!
- Our voltage and current “waves” are really electromagnetic waves propagating in the material of the cable. Larger dielectric constants slow the wave down.
- The wavelength of voltage and current waves on a lossless line is

\[ \lambda = \frac{2\pi}{\beta} = \frac{1}{f \sqrt{\mu \varepsilon}} \]

The wavelength gets shorter for increasing \( \varepsilon \)
- Other general line relationships: \( L'C' = \mu \varepsilon \) and \( \frac{G'}{\varepsilon} = \frac{\pi}{4} \)

Practical parallel plate lines

- Although a true parallel plate line is unusual, a “microstrip line” is the default “wire” on printed circuit boards
- Although not exactly a parallel plate line, it is an ok approximation to model microstrip as parallel plate with the width of the plate that of the top trace
- We can then fabricate varying impedances easily by changing the trace width