A Large Deviations Analysis of Scheduling in Wireless Networks

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Abstract

We consider a cellular network consisting of a base station and N receivers. The channel states of the receivers are assumed to be identical and independent of each other. The goal is to compare the throughput of two different scheduling policies (a queue-length-based policy and a greedy scheduling policy) given an upper bound on the queue overflow probability or the delay violation probability. We first consider a simple channel model, where each channel is assumed to be in one of two states (ON or OFF). Given an upper bound on the delay violation probability or an upper bound on the queue overflow probability, we show that the total network throughput of the queue-length-based policy is strictly larger than the throughput of the greedy policy for all N. Further, the throughput of the queue-length-based policy is a strictly increasing function of N while the throughput of the greedy policy does not have this property. Finally, for general channel state models, we show that the relative performances of the the greedy and QLB policies have a similar behavior.

Index Terms

Multiuser wireless scheduling, Large deviations, Wireless network, Queue-length-based policy, Greedy policy.

I. Introduction

Multiuser wireless scheduling has received much attention in recent years. Consider a cellular network consisting of a base station and N users (receivers), where the base station maintains N separate queues, one corresponding to each user. Assume time is slotted and the channel states of the receivers at each time slot are known at the base station. Then, the base station can determine which queues to serve according to their channels states. In this paper, we assume that the base station operates in a TDMA fashion, i.e., the base station can serve only one queue in each time slot. Two scheduling policies have been widely studied in the literature: (i) the base station serves the user with the best (weighted) channel state (opportunistic scheduling) [14], [7], or (ii) serve the one with the best queue-length-weighted channel state (queue-length based (QLB) scheduling) [13], [5], [9], [10], [3], [1], [8]. While QLB scheduling is throughput optimal (i.e., can stabilize any set of user throughput that can be stabilized by any other algorithm), opportunistic scheduling maximizes the total network throughput if all queues are continuously backlogged. If the arrival rates to the users are identical and the channel state distribution to the receivers are identical, then these two scheduling policies have the same stability region.

While stability is the first concern of scheduling policies, quality-of-service (QoS) is important too. For example, we may require the queue overflow probability to be small or require small delays. The performance of different scheduling policies under QoS constraints has received much attention recently. For reasons of analytical tractability, much of the prior work assumes that the channels to all the receivers are independent and statistically identical. Under this assumption, and assuming identical user utilities, opportunistic scheduling policies become greedy policies in which the base station transmits to the receiver with the best channel state. In [11], the author studies a simple network consisting of two users where the channels are assumed to be independent, identically distributed ON-OFF channels. Using large-deviations techniques, it is shown that the total network throughput of the QLB policy is larger than the greedy policy under the queue overflow constraint. In [5], a wireless network with N users and ON-OFF channels is considered. It is assumed that the arrivals are identical and Poisson, and the capacity when the channel is ON is one packet per time slot. It is then shown that, when the number of users increases from N to $2N$, the expected sum of queue lengths is non-increasing under the QLB policy, while it increases linearly under the greedy policy. Further, in [4], the behavior of the greedy policy for Rayleigh fading channels is studied and it shows that under a delay constraint, the total network throughput of the greedy policy increases initially with the number of users, but eventually decreases and goes to zero when the number of the users is sufficiently large.

Motivated by these prior results, in this paper, we study the performance of the two scheduling policies (greedy and QLB) for a wireless network with ON-OFF channels or general multi-state channels. Using sample-path large deviations techniques which have been used in [2], [11] and [12], the following results are proved:

1) Under the QoS constraints described in the abstract (which will be made precise later), the total network throughput of the QLB policy is larger than the greedy policy;

2) Under the delay-violation constraint, the total network throughput using the greedy policy will decrease to zero for large number of users, but not so under the QLB policy.

The main contributions of this paper are as follows:

1) Assuming an ON-OFF channel model and a constant arrival rate in each time slot, we compute the large-deviations exponent of the probability that one queue in the network exceeds a large threshold. A key contribution here is the characterization of the queue-length trajectories that lead to queue overflow. It was conjectured that in [11] that the complexity of the calculation of the large-deviations exponent increases exponentially with increasing N, but we show here that a simple closed-form expression can be obtained.
2) Consider ON-OFF channels and QLB policy. In [5], under the assumption that the channel capacity is one packet per time slot, it is shown the expected sum of the queue lengths is nondecreasing when the number of users increases from N to $2N$. In this paper, we show that the maximum network throughput is strictly increasing in N under the delay-violation constraint or overflow constraint. Our result does not only compare performance with N users and $2N$ users, but at all intermediate values as well. Further, our result holds even when the capacity of the network is greater than one packet-per-slot. For the multi-state channel model and QLB policy, a lower bound of the maximum network throughput is obtained. We show that the lower bound is strictly increasing in N under the QoS constraints.
3) For the greedy policy, we analytically show that the throughput goes to a constant under the queue overflow constraint, and decreases to zero under the delay violation constraint. This result holds for both ON-OFF channel model and multi-state channel model, and it is consistent with the numerical results for Rayleigh fading channels in [4].
4) Under the QoS constraints, we show that the throughput of the QLB scheduling policy is larger than the throughput of the greedy policy. This conclusion has been proved true in [11] for a two users system and under queue overflow constraint. Here, we prove that it is true for an N-user system, and for both ON-OFF channel model and multi-state channel model.

The rest of the paper is organized as follows: In Section II, we describe our system model in detail. In Section III, we study the ON-OFF channel model. Then, in Section IV, a general multi-state channel model is investigated. In Section V, we compare the performance of the QLB policy and the greedy policy. In Section VI, we study the performance of the QLB policy and the greedy policy for Rayleigh fading channels and asymmetric channels using simulations. Finally, concluding remarks are provided in

Section VII.

II. BASIC MODEL

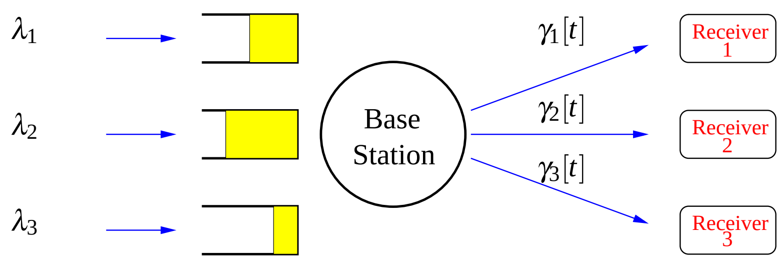


Fig. 1. Single-hop Network

Consider a wireless network shared by N users in the downlink as Fig. 1, where $N = 3$. We assume that the time is slotted and at each time slot, only one user can be chosen to transmit. Each user is associated with a channel and all channel-state processes $\gamma_i[t]$ are independent and statistically identical. In this paper, we consider multi-state channels — where each channel has L states $\{0,\ldots,L-1\}$, the probability that the channel is in state i is p_i, and we can transmit at most F_i bits/slot when a channel is in state i. Also assume that the arrival rate is constant and is equal to λ/N bits/slot for each user. In Section III, we first consider a simple multi-state channel — ON-OFF channel, where the channel has only two states. We use "0" to indicate the channel is OFF and "1" to indicate the channel is ON. When the channel is OFF, no data can be transmitted. When the channel is ON, this channel can be selected to transmit. Furthermore, we let p be the probability that the channel is in the ON state. When a channel is ON, we can transmit at most F bits to the user of that channel. The general multi-state channel model will be considered in Section IV.

For this model, we consider the total network throughput of two different scheduling policies under two different quantity-of-service (QoS) constraints. The two scheduling policies we will investigate are:

1) Queue-length based (QLB) policy: Choose user i to transmit if

$$i \in \arg\max_j \gamma_j[t]Q_j[t],$$

where $Q_i[t]$ is the queue length of user i at time t. In the ON-OFF channel model, this policy chooses the user with the largest queue length from ON channels.

2) Greedy policy: Choose user i if

$$i \in \arg\max_j \gamma_j[t].$$

If more than one user has the best channel state, we assume that the base station is equally likely to choose any one of those users.

The two QoS constraints we will consider are:

1) Queue overflow constraint:

$$\Pr(\max_i Q_i(0) > B) \leq \varepsilon,$$

where $Q_i(0)$ is the stationary queue length. So this QoS constraint requires the steady-state probability that the queue length is larger than B to be small. Instead of studying this constraint as above, we study the following approximation to the constraint:

$$\theta_B(N,\lambda) := \lim_{B\to\infty} -\frac{1}{B}\log P(\max_i Q_i(0) > B) \geq \delta, \tag{1}$$

where the large-deviation exponent θ_B is a function of the number of the users and the total arrival rate. The exponent δ can be related to ε for large B using the following approximation: $e^{-\delta B} = \varepsilon$.

2) Delay violation constraint: Define $D(t)$ to be the maximum delay experienced so far by any bit in any of the queues in slot t. Assuming that the system started at time $-\infty$, the steady-state delay violation constraint that we consider can be expressed as follows: $\Pr(D(0) > D) \leq \varepsilon$. Since the arrival rate is constant, it is easily seen that $\Pr(D(0) > D) = P\left(\max_i Q_i(0) > \frac{\lambda}{N}D\right)$. Thus, the delay violation constraint can be expressed as:

$$\Pr\left(\max_i Q_i(0) > \frac{\lambda}{N}D\right) \leq \varepsilon.$$

As before, we study the following approximation to the constraint:

$$\theta_D(N,\lambda) := \lim_{D\to\infty} -\frac{1}{D}\log P\left(\max_i Q_i(0) > \frac{\lambda}{N}D\right) \geq \delta. \tag{2}$$

From the above description of the two quantities, it is clear that if we obtain an expression for $\theta_B(N,\lambda)$, then

$$\theta_D(N,\lambda) = \frac{\lambda\,\theta_B(N,\lambda)}{N}.$$

Thus, we will primarily consider the queue overflow problem when we analyze the wireless system using large deviations.

In this paper, we use θ to denote the large deviation exponents, where the subscript indicates the QoS constraints (“B” indicates the buffer overflow constraint and “D” indicates the delay-violation constraint), and the superscript indicates the scheduling policy we use (“Greedy” is used for the greedy policy and “QLB” is used for the queue-length based policy).

III. ON-OFF CHANNEL MODEL

In this section, we focus on the simple ON-OFF channel model. Using large deviations techniques, the QLB policy is studied in Subsection III-A, and the greedy policy is studied in Subsection III-B.

A. QLB Policy

In this subsection, we study the performance of the QLB policy under the QoS constraints for a wireless network with ON-OFF channels. We compute the maximum total throughput the network can support under a fixed buffer-over flow constraint or delay-violation constraint.

To compute the maximum total throughput, we first calculate the large-deviations exponents under the QoS constraints for fixed arrival rates. Consider an N-user system, and let $\gamma_i(t)$ be the state of channel i at time t, so that $\gamma_i(t) = 1$ or 0. The state of the system depends on the state of each channel, so there are 2^N system states. Each state can be represented as an N-tuple in $\{0,1\}^N$. For example, consider a 2-user system, the channel states are: $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$, where 0 means the corresponding channel is OFF and 1 means the channel is ON. To simplify our notation, we use integers to represent the system state. So for a 2-user system, the system states are: 0, 1, 2 and 3. Thus, define the system state variable $S(t)$ as follows:

$$S(t) := \sum_{i=0}^{N-1} \gamma_i(t) 2^i, \tag{3}$$

and further define the probability vector $\mathbf{p}$, where p_j is the probability the system is at state j.

For sufficiently large T, we define $\mathbf{s}^{(B)}(t)$ on $[-T,0]$ using $S(t)$ on $[0,BT]$ as follows:

$$s_j^{(B)}(t) := \frac{1}{B} \sum_{l=0}^{B(T+t)} 1_{S(l)=j}, \text{ for } t = \frac{k}{B} - T \text{ and } k = \{0,\ldots,BT\}$$

where for values of t which are not of the form k/n, define $s_j^{(B)}(t)$ by linear interpolation. Notice that we have scaled and shifted time so that the discrete time units $0,1,\ldots,BT$ have now become the continuous time interval $[-T,0]$. For each t, the variable $s_j^{(B)}(t)$ is the amount of (scaled) time in the interval $[-T,t]$ that the system is in state j. Next, define the system channel rate processes using a 2^N-tuple — $\mathbf{u}(t)$, where $\mathbf{u}(t)$ is nonnegative, integrable, $\sum_{j=0}^{2^N-1} u_j(t) = 1$, and given $\varepsilon > 0$, for all sufficiently large B, we have for any $t_1 < t_2$

$$\left| s_j^{(B)}(t_2) - s_j^{(B)}(t_1) - \int_{t_1}^{t_2} u_j(s)\, ds \right| \leq \varepsilon.$$

Now we will consider the normalized queue length

$$q_i(t) = \frac{1}{B} Q_i(Bt).$$

Then we have

$$\Pr\left(\max_i Q_i(0) > B\right) = \Pr\left(\max_i q_i(0) > 1\right),$$

and the dynamics of the normalized queue length can be described using following differential equation:

$$\dot{q}_i(t) = \frac{\lambda}{N} - F \sum_{j \in A_i(\mathbf{q}(t))} u_j(t),$$

where $A_i(\mathbf{q}(t))$ is the set such that if $j \in A_i$, then user i will be chosen to transmit at time t when the system is at state j. Since the QLB policy is used, A_i is a function of $\mathbf{q}(t)$. The minimum cost problem that is used to find the large deviations exponents is defined in terms of the Kullback-Liebler distance from $\mathbf{p}$ to $\mathbf{u}(t)$:

$$D(\mathbf{u}(t)\|\mathbf{p}) = \sum_{j=0}^{2^N-1} u_j(t) \log \frac{u_j(t)}{p_j}.$$

Recall $\mathbf{u}(t)$ is nonnegative, integrable and $\sum_j u_j(t) = 1$. Refer $\int_{-T}^{0} D(\mathbf{u}(s)||\mathbf{p})\, ds$ as the cost function, we define following minimum cost problem:

$$\theta_B^{\text{QLB}}(N,\lambda) = \inf_{T,\mathbf{u}} \int_{-T}^{0} D(\mathbf{u}(s)||\mathbf{p})\, ds, \tag{4}$$

where $T \geq 0$, $q_i(-T) = 0$ for all i, $\max_i q_i(0) = 1$, and the QLB policy is used.

Theorem 1:

$$\theta_B^{\text{QLB}}(N,\lambda) = \lim_{B\to\infty} \frac{-1}{B} \log \Pr(\max_i q_i(0) \geq 1),$$

where $\theta_B^{\text{QLB}}(N,\lambda)$ is defined as (4), and queues are scheduled according to the QLB policy.

Proof: The proof is a straightforward extension of Theorem 6.1 in [11] for the case $N = 2$. ■

Note that the optimization problem is intuitively obvious: among all possible channel state trajectories that could lead of overflow, we pick the one that is "closest" to the mean value $\mathbf{p}$. Given $\mathbf{u}(t)$, we call $\int_{-T}^{0} D(\mathbf{u}(s)||\mathbf{p})\, ds$ the cost of the trajectory generated by $\mathbf{u}(t)$. Thus, Theorem 1 tells us that the probability of the QoS violation is related to the minimum cost problem (4).

In general, the optimal control problem (4) can be hard to solve. In this section, since we only consider the simple ON-OFF channels and assume that all channels are symmetric, the problem turns out to be tractable. In this subsection, we will show two important properties of the optimal trajectory: piecewise linearity and another property which is called the *order property*. Then, in the next subsection, we will solve the minimum cost problem. To show piecewise linearity, we segment the state space into regions such that the differential equations describing the system dynamics are unchanged in each region. For example, consider a 2-user system, there are three regions [11]:

1) If $q_1(t) > q_2(t)$, then

$$\begin{aligned}\dot{q}_1(t) &= \frac{\lambda}{2} - F(u_1(t) + u_3(t)) \\ \dot{q}_2(t) &= \left(\frac{\lambda}{2} - Fu_2(t)\right)_0^+.\end{aligned}$$

2) If $q_2(t) > q_1(t)$, then

$$\begin{aligned}\dot{q}_2(t) &= \frac{\lambda}{2} - F(u_2(t) + u_3(t)) \\ \dot{q}_1(t) &= \left(\frac{\lambda}{2} - Fu_1(t)\right)_0^+.\end{aligned}$$

3) If $q_1(t) = q_2(t)$, then

$$\dot{q}_1(t) + \dot{q}_2(t) = (\lambda - F(u_1(t) + u_2(t) + u_3(t)))_0^+.$$

Here $\dot{q}_i(t) = (a)_0^+$ is defined as follows:

$$\dot{q}_i = \begin{cases} a, & \text{if } q_i(t) > 0; \\ \max\{0, a\}, & \text{if } q_i(t) = 0. \end{cases}$$

This approach of diving the state space into regions where dynamics are invariant was first considered in [2]. We now prove following two lemmas. The first lemma is a straightforward extension of the corresponding result in [11] for the two-user case, but the second lemma is crucial to solving the problem for $N > 2$.

Lemma 2 (Piecewise Linearity): In a region of fixed system dynamics, the optimal channel rate processes of (4) are constant. Thus, the optimal trajectory is piecewise linear.

Proof: Given arbitrarily channel rate processes $\mathbf{u}(t)$, and consider a fixed system dynamics region in time interval $[t_i, t_{i+1}]$. Define

$$K_j = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u_j(s)\, ds.$$

Since $D(\mathbf{u}||\mathbf{p})$ is convex in $\mathbf{u}$, from Jensen's inequality, it follows that

$$\begin{aligned}\int_{t_1}^{t_2} D(\mathbf{u}(s)||\mathbf{p})\, ds &\geq (t_2 - t_1) D\left(\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \mathbf{u}(s)\, ds||\mathbf{p}\right) \\ &= (t_2 - t_1) D(\mathbf{K}||\mathbf{p}).\end{aligned}$$

Furthermore, the queue lengths at $t = t_2$ are unchange under $\mathbf{K}$. Thus, the optimal channel rate processes in this region are constant, and the optimal (scaled) queue length trajectory is piecewise linear. ∎

Lemma 3 (Order Property): Given any trajectory, we can find another trajectory which has the same cost and the property such that if $i \geq j$, then

$$q_i(t) \geq q_j(t).$$

Proof: This lemma exploits the fact that all channels are symmetric. First we prove following statement: given control $\bar{\mathbf{u}}(t)$ and suppose $\bar{q}_i(\bar{t}) = \bar{q}_k(\bar{t})$ at time $\bar{t}$. Then, there exists a new trajectory $\hat{\mathbf{q}}(t)$ such that $\hat{q}_i(t) = \bar{q}_k(t)$ and $\hat{q}_k(t) = \bar{q}_i(t)$ for $t \geq \bar{t}$. Furthermore, this new trajectory is identical to the original one except the indexes of the queues, and two trajectories have the same cost.

Now suppose that $\bar{q}_i(\bar{t}) = \bar{q}_k(\bar{t})$ and define new channel rate processes $\hat{\mathbf{u}}(t)$ such that

$$\hat{u}_j(t) = \begin{cases} \bar{u}_j(t), & \text{if } t < \bar{t}; \\ \bar{u}_{l_j}(t), & \text{if } t \geq \bar{t}, \end{cases}$$

where l_j is obtained from j by exchanging the i^{th} and k^{th} digits of the binary expression of j. For example, for the two-user system, we will have $\bar{u}_{(0,1)}(t) = \hat{u}_{(1,0)}(t)$, $\bar{u}_{(1,0)}(t) = \hat{u}_{(0,1)}(t)$, $\bar{u}_{(0,0)}(t) = \hat{u}_{(0,0)}(t)$, and $\bar{u}_{(1,1)}(t) = \hat{u}_{(1,1)}(t)$. Then, since $\bar{q}_i(\bar{t}) = \bar{q}_k(\bar{t})$, the dynamics of queue i and queue j are swapped after $\bar{t}$ under the channel rate processes, and we will have $\hat{q}_i(t) = \bar{q}_k(t)$ and $\hat{q}_k(t) = \bar{q}_i(t)$ for $t \geq \bar{t}$ in the new trajectory.

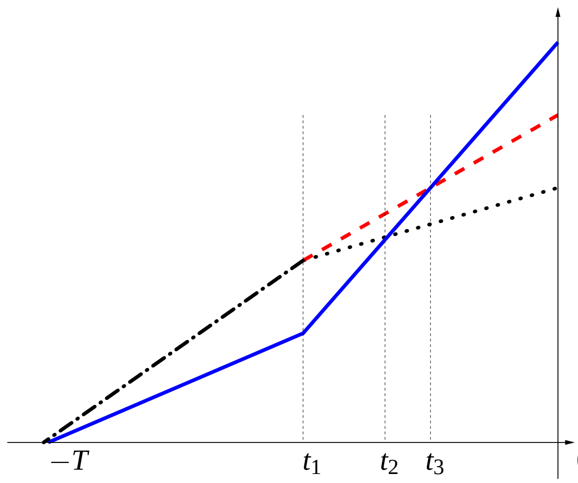
Furthermore, the channels are symmetric, it is easy to see $p_j = p_{l_j}$ because the binary expressions of j and l_j have the same number of "0"s and "1"s. So we can conclude that the new trajectory have the same cost as the original one because

$$\int_{\bar{t}}^{0} D(\hat{\mathbf{u}}(s)\|\mathbf{p})\,ds = \int_{\bar{t}}^{0} \sum_j \hat{u}_j(s) \log \frac{\hat{u}_j(s)}{p_j}\,ds = \int_{\bar{t}}^{0} \sum_j \bar{u}_{l_j}(s) \log \frac{\bar{u}_{l_j}(s)}{p_j}\,ds = \int_{\bar{t}}^{0} D(\bar{\mathbf{u}}(s)\|\mathbf{p})\,ds.$$

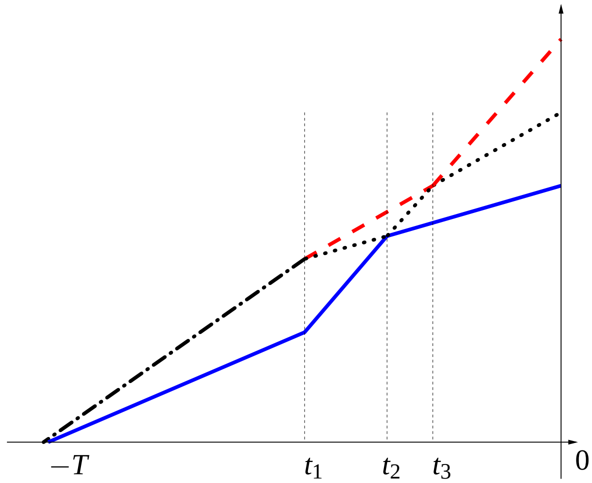
Now, we have proved that if two queues have the same queue length at time $\bar{t}$, there exists a new trajectory with the same cost such that the lengths of the two queues are swapped after $\bar{t}$. It is also easy to see that if we have more than two queues with the same length at time $\bar{t}$, then we can swap any two of them after $\bar{t}$ to get a new trajectory with the same cost. Thus, give any trajectory, we can get a new trajectory with the same cost such that $q_i(t) \geq q_j(t)$ if $i \geq j$.

For example, consider a trajectory in Fig. 2(a). There are three queues, we use the solid line for q_1, the dotted line for q_2, the dashed line for q_3, and dashed-dotted line for $q_2 + q_3$. Then by swapping q_1 and q_2 after t_2 and swapping q_2 and q_3 after t_3, we get Fig. 2(b) which satisfies $q_3(t) \geq q_2(t) \geq q_1(t)$ for any t.

■



(a) An Arbitrary Sample Path



(b) Ordered Sample Path with the Same Cost as the Other Sample Path

Fig. 2. Order Property

We have proved two important properties of the optimal trajectory: piecewise linearity and order property. Next, we use these two properties to prove that the linearity of the optimal trajectory. Also, we provide the closed-form expressions of $\theta_B^{\mathrm{QLB}}(N,\lambda)$ and $\theta_D^{\mathrm{QLB}}(N,\lambda)$.

Before we solve the minimum cost problem (4), we first consider a simpler optimization problem $\mathrm{OP}(M,N)$ as follows:

$$\mathrm{OP}(M,N): \qquad C_M^N(h) = \inf_{\mathbf{u},T} TD(\mathbf{u}||\mathbf{p}) \tag{5}$$

$$\text{Subject to}: \quad T\left(\frac{M}{N}\lambda - F\sum_{j=2^{N-M}}^{2^N-1} u_j\right) = Mh \tag{6}$$

$$\sum_{j=0}^{2^N-1} u_j = 1 \tag{7}$$

$$u_j \geq 0 \quad \forall j. \tag{8}$$

This problem is related to the (4) as follows: (5) uses the same cost function as (4), but it is assumed that the channel rate processes are constant., the lengths of the queues $N-M$ through $N-1$ are all equal and larger than the remaining queue lengths and that

$$q_{N-M}(0) = \ldots = q_{N-1}(0) = h.$$

We do not impose any restrictions on q_1 through q_{N-M-1}, other than the fact that these are smaller than $q_{N-M}(t)$ for all $t \in [-T,0]$. However, in problem (5), we do not verify that the trajectory is a feasible trajectory under the QLB policy, i.e., we give priority to the users indexed $N-M$ through $N-1$ without verifying that their queue lengths are larger than the queue lengths of the remaining users (we simply assume this in stating the optimization problem, but do not verify it).

Notice that

$$T = \frac{Mh}{\frac{M}{N}\lambda - F\sum_{j=2^{N-M}}^{2^N-1} u_j},$$

so

$$C_M^N(h) = \inf_{\mathbf{u}} \frac{Mh}{\frac{M}{N}\lambda - F\sum_{j=2^{N-M}}^{2^N-1} u_j} D(\mathbf{u}||\mathbf{p}) = hC_M^N(1).$$

Now we focus on $C_M^N(1)$ and obtain a closed-form expression for it in Lemma 4. Then in Theorem 5, we prove that the linearity of the optimal trajectory, and that $\theta_B^{\text{QLB}}(N,\lambda) = \min_M C_M^N(1)$.

Lemma 4: Consider the optimization problem (5), we have

$$C_M^N(1) = \inf_{0 \le x < \frac{\lambda M}{FN}} \frac{N}{\lambda - xF\frac{N}{M}} D(\mathbf{u}^x(M)||\mathbf{p}), \tag{9}$$

where

$$D(\mathbf{u}^x(M)||\mathbf{p}) = x\log\frac{x}{1-(1-p)^M} + (1-x)\log\frac{1-x}{(1-p)^M}.$$

Proof: To solve (5), we first fix T and solve the following optimization problem:

$$\inf_{\mathbf{u}} \sum_{j=0}^{2^N-1} u_j \log\frac{u_j}{p_j}$$

$$\text{Subject to:} \quad \sum_{j=0}^{2^N-1} u_j = 1;$$

$$\sum_{j=2^{N-M}}^{2^N-1} u_j = x;$$

$$\frac{M}{F}\left(\frac{\lambda}{N} - \frac{1}{T}\right) = x.$$

$D(\mathbf{u}||\mathbf{p})$ is a strictly convex function, so the optimal solution is unique. We use Lagrange multipliers to solve this problem. Define the Lagrangian

$$L = \sum_{j=0}^{2^N-1} u_j \log\frac{u_j}{p_j} + \lambda_1(\sum_{j=0}^{2^N-1} u_j - 1) + \lambda_2(\sum_{j=1}^{2^N-1} u_j - x).$$

The first-order optimality conditions obtained by differentiating the Lagrangian and setting it equal to zero yield

$$\log\left(\frac{u_j}{p_j}\right)+1 \quad = \lambda_1+\lambda_2 \quad \text{for } j=2^{N-M},\dots,2^N-1; \tag{10}$$

$$\log\left(\frac{u_j}{p_j}\right)+1 \quad = \lambda_1 \quad \text{for } j=0,\dots,2^{N-M}-1; \tag{11}$$

$$\sum_{j=0}^{2^N-1} u_j \quad = 1; \tag{12}$$

$$\sum_{j=2^{N-M}}^{2^N-1} u_j \quad = x, \tag{13}$$

where $x=\frac{M}{F}\left(\frac{\lambda}{N}-\frac{1}{T}\right)$. Since this optimization problem is solved for a fixed T (fixed x), we denote the optimal solution by $\mathbf{u}^x(M)$. Solving for $u_j^x(M)$ from (10)-(13), we get

$$u_j^x(M)=\begin{cases} \frac{x}{1-(1-p)^M}p_j & \text{for } j=2^{N-M},\dots,2^N-1; \\ \frac{1-x}{(1-p)^M}p_j, & \text{for } j=0,\dots,2^{N-M}-1. \end{cases} \tag{14}$$

So

$$D(\mathbf{u}^x(M)\|\mathbf{p})=x\log\frac{x}{1-(1-p)^M}+(1-x)\log\frac{1-x}{(1-p)^M}$$

and

$$C_M^N(1)=\inf_{0\le x<\frac{\lambda M}{FN}}\frac{1}{\frac{\lambda}{N}-\frac{xF}{M}}D(\mathbf{u}^x(M)\|\mathbf{p}),$$

where we require $0\le x<\frac{\lambda M}{FN}$ because $T>0$ and $u_j\ge 0$ for all j. ■

In the following theorem, we prove that $\theta_B^{\text{QLB}}(N,\lambda)=\min_M C_M^N(1)$ and $\theta_D^{\text{QLB}}(N,\lambda)=\frac{\lambda}{N}\min_M C_M^N(1)$.

Theorem 5: For an N-user network, the optimal channel rate processes that solves (4) are constant, and hence the optimal queue length trajectories are linear, and further

$$\theta_B^{\text{QLB}}(N,\lambda)=\min_M C_M^N(1) \qquad \text{and} \qquad \theta_D^{\text{QLB}}(N,\lambda)=\frac{\lambda}{N}\min_M C_M^N(1).$$

Proof: Since $\theta_B^{\text{QLB}}(N,\lambda)=\frac{\lambda}{N}\theta_D^{\text{QLB}}(N,\lambda)$, we only need to prove $\theta_B^{\text{QLB}}(N,\lambda)=\min_M C_M^N(1)$. From Lemma 2 and Lemma 3, we only need to consider the trajectories which are piece-wise linear and $q_i(t)\ge q_k(t)$ for $i\ge k$. Pick any piece of this trajectory which is in a fixed dynamic region, i.e., the channel rate processes are constant in this region. Suppose this piece of trajectory is in the time interval $[t_1,t_2]$, $q_{N-1}(t_2)-q_{N-1}(t_1)=h$, and $q_{N-1}(t)=\ldots=q_{N-M}(t)>q_{N-M-1}(t)$ for any $t\in(t_1,t_2)$. Thus, in this region, the dynamics of queue $N-1,\ldots,N-M$ are as follows:

$$\sum_{i=N-M}^{N-1} \dot{q}_i(t) = \frac{M}{N}\lambda - F\left(\sum_{j=2^{N-M}}^{2^N-1} u_j\right).$$

Then, it is easy to see that

$$(t_2-t_1)\left(\frac{M}{N}\lambda - F\sum_{j=2^{N-M}}^{2^N-1} u_j\right) = Mh. \tag{15}$$

Define $T = t_2 - t_1$, then (15) is similar to (6). Let $D(\mathbf{u}||\mathbf{p})$ be the cost of this piece of the trajectory, we have

$$(t_2-t_1)D(\mathbf{u}||\mathbf{p}) \geq C_M^N(h).$$

Define $M^* \in \arg\min_M C_M^N(1)$, then we have

$$(t_2-t_1)D(\mathbf{u}||\mathbf{p}) \geq hC_M^N(1) \geq hC_{M^*}^N(1).$$

Now consider a piecewise linear and ordered trajectory, and divide $[-T,0]$ into subintervals $[t_i,t_{i+1}]$ such that the trajectory in each subinterval is in a fixed system dynamic region. So the channel rate processes in $[t_i,t_{i+1}]$ are constant — we call it $\mathbf{u}_i$. Also define $h_i = q_{N-1}(t_{i+1}) - q_{N-1}(t_i)$. Then, we have

$$\begin{aligned}\int_{-T}^{0} D(\mathbf{u}(s)||\mathbf{p})\,ds &= \sum_i (t_{i+1}-t_i)D(\mathbf{u}_i||\mathbf{p}) \geq \sum_i h_i C_{M^*}^N(1) \\ &= C_{M^*}^N(1)\end{aligned}$$

This is true for each piece-wise linear and ordered trajectory, and thus,

$$\theta_B^{\text{QLB}}(N,\lambda) \geq C_{M^*}^N(1).$$

We have shown that $\min_M C_M^N(1)$ is a lower bound on $\theta_B^{\text{QLB}}(N,\lambda)$. If there exists a trajectory that has the cost $\min_M C_M^N(1)$, then we can conclude that $\theta_B^{\text{QLB}}(N,\lambda) = \min_M C_M^N(1)$. Define $\mathbf{u}(M)$ to be the optimal channel rate processes corresponding to $C_M^N(1)$, $T(M)$ to be the optimal overflow time corresponding to $C_M^N(1)$, and $U := \{M : C_M^N(1) = \min_M C_M^N(1)\}$. First, it is obvious that if $N \in U$, $\mathbf{u}(N)$ generates a feasible trajectory under the QLB policy because all queues are identical under $\mathbf{u}(N)$. So we only need to consider $N \notin U$. Note that $M^* \in U$ and suppose that

$$\frac{1}{M^*}\sum_{j=2^{N-M^*}}^{2^N-1} u_j(M^*) < \frac{1}{N-M^*}\sum_{j=1}^{2^{N-M^*}-1} u_j(M^*). \tag{16}$$

Then, we argue that the queue length dynamics will satisfy

$$\sum_{i=N-M^*}^{N-1} \dot{q}_i = \frac{M^*}{N}\lambda - F\sum_{j=2^{N-M^*}}^{2^N-1} u_j(M^*); \tag{17}$$

$$\sum_{i=0}^{N-M^*-1} \dot{q}_i = \frac{N-M^*}{N}\lambda - F\sum_{j=1}^{2^{N-M^*}-1} u_j(M^*). \tag{18}$$

To see this, note that from the structure of the optimal solution $\mathbf{u}^x(M^*)$ defined by (14), queue i and queue k have the same channel state processes if $i,k < M^*$ or $i,k \geq M^*$. Thus, under dynamics (17) and (18), we will have $q_i(t) = q_k(t)$ if $i,k < M^*$ or $i,k \geq M^*$. If condition (16) holds, then

$$q_{N-1}(t) = \ldots = q_{N-M}(t) > q_{N-M-1}(t) = \ldots = q_0(t).$$

Then, this trajectory generated by (17) and (18) is a feasible trajectory under the QLB policy (i.e., when a user in $\{N-M,\ldots,N-1\}$ is ON and a user in $\{0,\ldots,N-M-1\}$ is ON, then a user in $\{N-M,\ldots,N-1\}$ is scheduled as the QLB policy dictates), and the cost of the trajectory is $C^N_{M^*}(1)$.

Otherwise, if condition (16) doesn't hold, then we have

$$\frac{1}{M^*}\sum_{j=2^{N-M^*}}^{2^N-1} u_j(M^*) \geq \frac{1}{N}\sum_{j=1}^{2^N-1} u_j(M^*). \tag{19}$$

Choose $\hat{T}$ such that

$$\hat{T}\left(\lambda - \sum_{j=1}^{2^N-1} u_j(M^*)\right) = N. \tag{20}$$

Since

$$T(M^*)\left(\frac{M^*}{N}\lambda - \sum_{j=2^{N-M^*}}^{2^N-1} u_j(M^*)\right) = M^*,$$

from inequality (19), we can conclude that $\hat{T} \leq T(M^*)$. Thus,

$$\hat{T}D(\mathbf{u}(M^*)\|\mathbf{p}) \leq T(M^*)D(\mathbf{u}(M^*)\|\mathbf{p}) = C^N_{M^*}(1).$$

Furthermore, from (20) and definition (5), we know that $(\hat{T},\mathbf{u}(M^*))$ is a feasible solution of $OP(N,N)$. Thus,

$$C^N_N(1) \leq \hat{T}D(\mathbf{u}(M^*)\|\mathbf{p}) \leq C^N_{M^*}(1),$$

which contradicts $N \notin U$.

From all of the above, we can conclude that there exists a trajectory under the QLB policy which has the minimum cost $C^N_{M^*}(1)$. Thus, $\theta_B^{\text{QLB}}(N,\lambda) = \min_M C^N_M(1)$. ■

When the total arrival rate is λ, from Theorem 5, we know that the buffer-overflow constraint (the delay-violation constraint) θ is feasible if $\theta \leq \theta_B^{\text{QLB}}(N,\lambda)$ ($\theta \leq \theta_B^{\text{QLB}}(N,\lambda)$). We would also like to compute the maximum total throughput given a QoS constraint θ. Before computing the maximum throughput of our system, we first study a single user network, where the arrival rate is λ_s and the service rate $\{\mu_s[t]\}$ are nonnegative, identical, and independent random variables. From [6], given the buffer overflow constraint θ, the arrival rate λ_s is supportable (supportable means that the large-deviations exponent is less than or equal to θ when the arrival rate is λ_s) if and only if

$$\lambda_s \leq -\frac{\Lambda(-\theta)}{\theta}, \tag{21}$$

where $\Lambda(\theta) = \log E\left[e^{\theta\mu_s[t]}\right]$. Further, give the delay violation constraints θ, it is easy to see that the arrival rate λ_s is supportable if and only if [4]

$$1 \leq -\frac{\Lambda(-\frac{\theta}{\lambda_s})}{\theta}. \tag{22}$$

Consider an ON-OFF channel such that $\mu_s[t] = F_s$ with probability p_s, and $\mu_s[t] = 0$ with probability $1 - p_s$. Then, from equation (21), we conclude that λ_s is supportable if and only if

$$\lambda_s \leq -\frac{\log\left(1 - p_s + p_s e^{-\theta F_s}\right)}{\theta}. \tag{23}$$

Fix the arrival rate λ_s, then specializing Theorem 5 to the case $N = 1$ shows that the buffer-overflow constraint θ is feasible if and only if

$$\theta \leq \inf_{0 \leq x < \frac{\lambda_s}{F_s}} \frac{1}{\lambda_s - xF_s}\left(x\log\frac{x}{p_s} + (1-x)\log\frac{1-x}{1-p_s}\right). \tag{24}$$

Thus, we can conclude $\{\lambda_s, \theta\}$ satisfies (23) if and only if it satisfies (24).

Similarly, given the delay violation constraint θ, the arrival rate λ_s is supportable if and only if

$$\lambda_s \leq -\frac{\theta}{\log\left(p_s - 1 + e^{-\theta F_s}\right) - \log p_s}. \tag{25}$$

Given the arrival rate λ_s, the delay violation constraint θ is feasible if and only if

$$\theta \leq \inf_{0 \leq x < \frac{\lambda_s}{F}} \frac{\lambda_s}{\lambda_s - xF_s}\left(x\log\frac{x}{p_s} + (1-x)\log\frac{1-x}{1-p_s}\right). \tag{26}$$

So, under the delay-violation constraint $\{\lambda_s, \theta\}$ satisfies (25) if and only if it satisfies (26). In the next theorem, we will use inequalities (23)-(26) to obtain the maximum throughput of an N-user system under the QLB policy.

Use $\lambda^{\text{QLB}}(N, \theta)$ to denote the maximum throughput of an N-user system under the QoS constraints, we have following theorem. Note that the maximum throughput is equal to the maximum arrival rate the network can support under the QoS constraints.

Theorem 6: Suppose the QLB policy is used. Given the queue-overflow constraint $\theta_B^{\text{QLB}}(N,\lambda)=\theta$, the maximum total throughput of the network is

$$\lambda_B^{\text{QLB}}(N,\theta) = -\max_{1\le M\le N} \frac{N}{\theta}\log\left((1-p)^M + \left(1-(1-p)^M\right)e^{-\frac{F\theta}{M}}\right). \tag{27}$$

Given the delay-violation constraint $\theta_D^{\text{QLB}}(N,\lambda)=\theta$, the maximum total throughput of the network is

$$\lambda_D^{\text{QLB}}(N,\theta) = -\max_{1\le M\le N} \frac{\theta NF}{M\left(\log\left(e^{-\theta}-(1-p)^M\right)-\log\left(1-(1-p)^M\right)\right)}. \tag{28}$$

Proof: Given the buffer-overflow constraint $\theta_B^{\text{QLB}}(N,\lambda)=\theta$, from Theorem 5, we have that λ is supportable if and only if

$$\theta \le \inf_{0\le x<\frac{\lambda M}{FN}} \frac{1}{\frac{\lambda}{N}-x\frac{F}{M}}\left(x\log\frac{x}{1-(1-p)^M}+(1-x)\log\frac{1-x}{(1-p)^M}\right)$$

holds for each M. Let $\lambda_s=\lambda/N$, $F_s=F/M$ and $p_s=1-(1-p)^M$. From (23) and (24), λ is supportable if and only if

$$\frac{\lambda}{N} \le -\frac{1}{\theta}\log\left((1-p)^M + \left(1-(1-p)^M\right)e^{-\frac{F\theta}{M}}\right)$$

holds for each M, which implies that the maximum throughput of the network is

$$\lambda_B^{\text{QLB}}(N,\theta) = -\max_{1\le M\le N} \frac{N}{\theta}\log\left((1-p)^M + \left(1-(1-p)^M\right)e^{-\frac{F\theta}{M}}\right).$$

Similarly, given the delay violation constraint $\theta_D^{\text{QLB}}(N)=\theta$, the maximum throughput is

$$\lambda_D^{\text{QLB}}(N,\theta) = -\max_{1\le M\le N} \frac{\theta NF}{M\left(\log\left(e^{-\theta}-(1-p)^M\right)-\log\left(1-(1-p)^M\right)\right)}.$$

■

In the next theorem, we characterize the behavior of $\lambda_B^{\text{QLB}}(N,\theta)$ and $\lambda_D^{\text{QLB}}(N,\theta)$ as a function of N.

Theorem 7: Under the QLB policy, the maximum support rates under the QoS constraints satisfy:

$$\lambda_B^{\text{QLB}}(N,\theta) < \lambda_B^{\text{QLB}}(N+1,\theta) \quad \text{and} \quad \lambda_D^{\text{QLB}}(N,\theta) < \lambda_D^{\text{QLB}}(N+1,\theta).$$

Proof: From the inequalities (23)-(26), we can see that the theorem holds if the large-deviations exponents behave as follows:

$$\theta_B^{\text{QLB}}(N,\lambda) < \theta_B^{\text{QLB}}(N+1,\lambda) \quad \text{and} \quad \theta_D^{\text{QLB}}(N,\lambda) < \theta_D^{\text{QLB}}(N+1,\lambda).$$

Also, we know that $\theta_B^{\text{QLB}}(N,\lambda)=\frac{N}{\lambda}\theta_D^{\text{QLB}}(N,\lambda)$. If we can prove $\theta_D^{\text{QLB}}(N,\lambda)<\theta_D^{\text{QLB}}(N+1,\lambda)$, then $\theta_B^{\text{QLB}}(N,\lambda)<\theta_B^{\text{QLB}}(N+1,\lambda)$ holds automatically.

To prove $\theta_D^{\text{QLB}}(N,\lambda) < \theta_D^{\text{QLB}}(N+1,\lambda)$, we will show the following two inequalities:

$$\frac{1}{N}C_M^N(1) < \frac{1}{N+1}C_M^{N+1}(1) \text{ and } \frac{1}{N}C_N^N(1) < \frac{1}{N+1}C_{N+1}^{N+1}(1),$$

where

$$\frac{1}{N}C_M^N(1) = \inf_{0\le x<\frac{\lambda M}{FN}} \frac{1}{\lambda - xF\frac{N}{M}} D(\mathbf{u}^x(M)\|\mathbf{p}).$$

We know that for an N-user system $x \in [0, M\lambda/NF)$; and for a $N+1$-user system, $x \in [0, M\lambda/(N+1)F)$. Now fix $x \in (0, M\lambda/(N+1)F) \subset (0, M\lambda/NF)$,

$$\frac{1}{\lambda - \frac{xFN}{M}} < \frac{1}{\lambda - \frac{xF(N+1)}{M}}.$$

Define

$$G(x) = \frac{1}{\lambda - xF\frac{N}{M}}\left(x\log\frac{x}{1-(1-p)^M} + (1-x)\log\frac{1-x}{(1-p)^M}\right),$$

it is easy to show $G(\lambda M/FN) = \infty$ and $\frac{dG(x)}{dx}|_{x=0} = -\infty$. Thus,

$$x^* = \arg\inf_{0\le x<\frac{\lambda M}{FN}} \frac{1}{\lambda - xF\frac{N}{M}} D(\mathbf{u}^x(M)\|\mathbf{p}) \in (0, \lambda M/FN),$$

which yields

$$\frac{1}{N}C_M^N(1) < \frac{1}{N+1}C_M^{N+1}(1).$$

Next consider

$$\begin{aligned}\frac{C_N^N(1)}{N} &= \inf_{0\le x<\frac{\lambda}{F}} \frac{1}{\lambda - xF} D(\mathbf{u}^x(N)\|\mathbf{p}) \\ &= \inf_{0\le x<\frac{\lambda}{F}} \frac{1}{\lambda - xF}\left(h(x) + g(N)\right),\end{aligned}$$

where $h(x) = x\log x + (1-x)\log(1-x)$, and

$$g(N) = N\log\frac{1}{1-p} - x\log\left(\frac{1}{(1-p)^N} - 1\right).$$

Since $h(x)$ is independent on N, if $g(N)$ is non-decreasing in N for fixed $x \in [0, \lambda/F)$, then we can conclude that $\frac{C_N^N(1)}{N}$ is also non-decreasing in N. Taking the derivative of $g(N)$, we get

$$\begin{aligned}g'(N) &= \log\frac{1}{1-p} - \frac{x}{\left(\frac{1}{(1-p)^N} - 1\right)}\frac{1}{(1-p)^N}\log\frac{1}{1-p} \\ &= \left(1 - \frac{x}{1-(1-p)^N}\right)\log\frac{1}{1-p}.\end{aligned}$$

When we compare the N-user system with the $(N+1)$-user system for fixed λ, we must have $\lambda \le F\left(1-(1-p)^N\right)$ to guarantee the stability of the system. Thus,

$$x < \frac{\lambda}{F} \le 1 - (1-p)^N$$

and

$$1-(1-p)^N - x > 1-(1-p)^N - 1 + (1-p)^N = 0.$$

So $g(N)$ is an increasing function in N, and

$$\frac{C_N^N(1)}{N} < \frac{C_{N+1}^{N+1}(1)}{N+1}.$$

From all above, we can conclude that

$$\theta_B^{\text{QLB}}(N,\lambda) < \theta_B^{\text{QLB}}(N+1,\lambda) \quad \text{and} \quad \theta_D^{\text{QLB}}(N,\lambda) < \theta_D^{\text{QLB}}(N+1,\lambda),$$

which implies the theorem. ■

From the theorem above, we see that under the QLB policy, the total throughput is strictly increasing with the number of the users. In the next subsection, it will be shown that under the greedy policy, the maximum throughput with the QoS constraint does not have this property.

B. Greedy Policy

In this section, we will consider the greedy policy. We obtain the expressions for $\lambda_B^{\text{Greedy}}(N,\lambda)$ and $\lambda_D^{\text{Greedy}}(N,\lambda)$. From the symmetric of the channel states and the greedy scheduling scheme, we know that

$$\Pr(q_i(0) \geq 1) \leq \Pr(\max_i q_i(0) > 1) \leq N\Pr(q_i(0) \geq 1),$$

which implies

$$-\lim_{B\to\infty} \frac{1}{B} \log \Pr(q_i(0) \geq 1) = -\lim_{B\to\infty} \frac{1}{B} \Pr(\max_i q_i(0) > 1).$$

Thus, we can obtain $\theta_B^{\text{Greedy}}(N,\lambda)$ by calculating $\lim_{B\to\infty} \frac{-1}{B} \Pr(q_i(0) \geq 1)$. It is easy to see that, under the greedy policy, the probability of one user is scheduled to transmit is $\frac{1}{N}(1-(1-p)^N)$, then we have following theorem.

Theorem 8: For an N-user system,

$$\lambda_B^{\text{Greedy}}(N,\theta) = -\frac{N \log\left(1-\hat{p}+\hat{p}e^{-\theta F}\right)}{\theta},$$

and

$$\lambda_D^{\text{Greedy}}(N,\theta) = -\frac{N\theta F}{\log\left(\hat{p}-1+e^{-\theta}\right) - \log \hat{p}},$$

where

$$\hat{p} = \frac{1}{N}(1-(1-p)^N).$$

Proof: Since $\hat{p}$ is the probability of one user is scheduled to transmit, we consider a single user network, where $\lambda_s = \lambda/N,\ p_s = \hat{p}$, and $F_s = F$. Use (23) and (25), we obtain the lemma. ■

In the next theorem, we will show that as the number of users increases, the throughput under the queue overflow constraint will converge to a constant and the throughput under the delay constraint decreases.

Theorem 9: Fix the QoS constraints, the maximum throughput under the greedy policy satisfy

$$\lim_{N\to\infty} \lambda_B^{\text{Greedy}}(N,\theta) = \frac{1-e^{-F\theta}}{\theta}, \tag{29}$$

and

$$\lambda_D^{\text{Greedy}}(N,\theta) = 0$$

for $N \geq a$, where $a > 0$ and $\frac{1}{a}(1-(1-p)^a) = 1-e^{-\theta}$.

Proof: First we know that

$$\begin{aligned}\lambda_B^{\text{Greedy}}(N,\theta) &= -\frac{N\log\left(1-\hat{p}+\hat{p}e^{-\theta F}\right)}{\theta} \\ &= -\frac{1}{\theta}\log\left(1-(1-(1-p)^N)\frac{1-e^{-\theta F}}{N}\right)^N.\end{aligned}$$

Since $\lim_N 1-(1-p)^N = 1$, we have

$$\lambda_B^{\text{Greedy}}(N,\theta) = \frac{1-e^{-\theta F}}{\theta}.$$

Next, consider

$$\lambda_D^{\text{Greedy}}(N,\theta) = -\frac{N\theta F}{\log\left(\hat{p}-1+e^{-\theta}\right)-\log\hat{p}}.$$

We need $\left(\hat{p}-1+e^{-\theta}\right) > 0$ to guarantee a positive $\lambda_D^{\text{Greedy}}(N,\theta)$. ■

This completes the analysis of the ON-OFF channel model. We have obtained the formulas for the maximum network throughput under the QoS constraints. We have also shown that under the QLB policy, the maximum throughput is strictly increasing in N, but this is not true for the system under the greedy policy. Further, from the formulas of the maximum throughput, we have following observations:

1) From (28), we can see that, under the QLB policy and the delay-violation constraint, the maximum throughput is a linear function of F.
2) Also from (28), we have that under the QLB policy, the maximum delay-violation constraint the network can support is $\log\frac{1}{1-p}$, which is a function of p and does not depend on F.
3) From (29), we have that under the greedy policy and the buffer overflow constraint, the maximum throughput converges to a constant which is less than $1/\theta$. So increasing the channel capacity F will not significantly increase the throughput of the network with large number of users. Further,

this throughput is far less than the throughput without QoS constraint which equals F when N goes to infinity.

IV. Multi-State Channel

In this section, we study a more general channel model — multi-state-channel model, where each channel has L states. Let p_i be the probability that the channel is in state i, and F_i be the number of bits can be transmitted per time slot when the channel is in state i. Assume $F_0 = 0$, and $F_i > F_j$ for $i > j$. Also, define $\gamma_i(t)$ to be the state of channel i at time t, so $\gamma_i(t) \in \{0, \ldots, L-1\}$.

A. QLB Policy

Consider a multi-state-channel system under the QLB policy. Define the state of the system using the composite state of all the channels, so there are L^N system states. Each system state can be represented as an N-tuple in $\{0, \ldots, L-1\}^N$. To simplify the notation, we use integer $j \in \{0, \ldots, L^N - 1\}$ to represent the system state, and define the system state variable $S(t)$ as follows:

$$S(t) := \sum_{i=0}^{N-1} \gamma_i(t) L^i. \tag{30}$$

Sometimes, we will also use the N-tuple representation of the system state. We define an N-tuple S^j in $\{0, \ldots, L-1\}^N$ to denote the state j. Let S_i^j be the i^{th} entry of S_i^j, it is also the state of channel i when the system is in system state j. Further, define the probability vector $\mathbf{p}$, where p_j is the probability the system is in state j. As in the case of the ON-OFF channel model, for sufficiently large T, we define $\mathbf{s}^{(B)}(t)$ on $[-T, 0]$ using $\mathbf{S}(t)$ on $[0, BT]$ as follows:

$$s_j^{(B)}(t) := \frac{1}{B} \sum_{l=0}^{B(T+t)} 1_{S(l)=j}, \qquad \text{for} \qquad t = \frac{k}{B} - T, \qquad \text{and} \qquad k = \{0, \ldots, BT\}$$

where for values of t which are not of the form k/n, define $s_j^B(t)$ by linear interpolation. Next, define the system channel rate processes using a L^N-tuple — $\mathbf{u}(t)$, where $\mathbf{u}(t)$ is nonnegative, integrable, $\sum_{j=0}^{L^N-1} u_j(t) = 1$, and for large B and small ε, we have for any $t_1 < t_2$

$$\left| s_j^{(B)}(t_2) - s_j^{(B)}(t_1) - \int_{t_1}^{t_2} u_j(s)\, ds \right| \leq \varepsilon.$$

Consider the normalized queue length

$$q_i(t) = \frac{1}{B} Q_i(Bt),$$

the dynamics of the normalized queue length can be described as follows:

$$\dot{q}_i(t) = \frac{\lambda}{N} - \sum_{j \in A_i(\mathbf{q}(t))} F_{S_i^j} u_j(t),$$

where $A_i(\mathbf{q}(t))$ is the set such that if $j \in A_i(\mathbf{q}(t))$, then user i will be chosen to transmit when the system is in state j at time t, and the channel state of user i is S_i^j, so the user i can at most transmit $F_{S_i^j}$ bits/sec. Since the QLB policy is used, A_i depends on the $\mathbf{q}(t)$. Next, define the Kullback-Liebler distance

$$D(\mathbf{u}(t)\|\mathbf{p}) = \sum_{j=0}^{L^N-1} u_j(t) \log \frac{u_j(t)}{p_j},$$

and optimization problem:

$$\theta_B^{\text{QLB}}(N,\theta) = \inf_{T,\mathbf{u}} \int_{-T}^{0} D\left(\mathbf{u}(s)||\mathbf{p}\right) ds, \tag{31}$$

where $T \geq 0$, $q_i(-T) = 0$ for all i, $\max_i q_i(0) = 1$, and the QLB policy is used.

Theorem 10:

$$\theta_B^{\text{QLB}}(N,\lambda) = \lim_{B\to\infty} \frac{-1}{B} \log \Pr(\max_i q_i(0) \geq 1),$$

where $\theta_B^{\text{QLB}}(N)$ is defined as (31), and queues are scheduled according to the QLB policy.

Proof: It is a straightforward extension of ON-OFF channel model. ■

While the minimum cost problem was proved to be tractable for the ON-OFF channel system, it seems to be hard to solve for a general multi-state-channel system. One difficulty is that the piecewise linearity property seems to be hard to prove for the multi-state channel model. But we can still obtain a lower bound of the minimum cost, because the order property still holds.

Lemma 11 (Order Property): Given any trajectory, we can find another trajectory which has the same cost and the property such that if $i \geq j$, then

$$q_i(t) \geq q_j(t).$$

Proof: Since the channels are still symmetric in multi-state channel system, this lemma is a straightforward extension of ON-OFF channel system. ■

Since the order property still holds, we only need to consider ordered trajectories. We first define following sets:

1) $\hat{A}^M$: If $j \in \hat{A}^M$, then at least one of channels $\{N-M,\ldots,N-1\}$ is in state $L-1$ when the system in state j.
2) $A^M(\mathbf{q}(t))$: If $j \in A^M(\mathbf{q}(t))$, then one of users $\{N-M,\ldots,N-1\}$ will be scheduled to transmit when the system is in state j at time t and users' queue lengths are $\mathbf{q}(t)$.

Then, consider following optimization problem:

$$\text{OP}^m(M,N): \qquad \hat{C}_M^N(1) = \inf_{\mathbf{u},T} TD(\mathbf{u}||\mathbf{p}) \tag{32}$$

$$\text{Subject to}: \quad T\left(\tfrac{M}{N}\lambda - F_{L-1}\textstyle\sum_{j\in\hat{A}^M} u_j\right) = Mh \tag{33}$$

$$\textstyle\sum_{j=0}^{L^N-1} u_j = 1 \tag{34}$$

$$u_j \geq 0 \quad \forall j. \tag{35}$$

From Lemma 4, it is easy to obtain

$$\hat{C}_M^N(1) = \inf_{0\leq x<\frac{\lambda M}{F_{L-1}N}} \frac{1}{\frac{\lambda}{N} - \frac{x}{M}F_{L-1}} \left(x\log\frac{x}{1-(1-p_{L-1})^M} + (1-x)\log\frac{1-x}{(1-p_{L-1})^M} \right). \tag{36}$$

In the following theorem, we show that $\min_M \hat{C}_M^N(1)$ is a lower bound of the minimum cost of (31).

Theorem 12: For an N-user system with multi-state channels, the minimum cost of optimal control problem (31) is lower bounded by $\min_M \hat{C}_M^N(1)$, thus

$$\theta_B^{\text{Greedy}}(N,\lambda) \geq \min_M \hat{C}_M^N(1).$$

Proof: Consider an ordered trajectory generated by control $\mathbf{u}(t)$. Define a L^N-tuple $\mathbf{K}$ such that

$$K_j = \frac{1}{T}\int_{-T}^{0} u_j(s)\,ds.$$

Since $D(\mathbf{u}(t)||\mathbf{p})$ is convex in $\mathbf{u}(t)$, from Jensen's inequality, we have

$$\int_{-T}^{0} D(\mathbf{u}(s)||\mathbf{p})\,ds \geq TD\left(\frac{1}{T}\int_{-T}^{0}\mathbf{u}(s)\,ds||\mathbf{p}\right) = D(\mathbf{K}||\mathbf{p}).$$

Define $F(\mathbf{q}(t),j)$ to be the capacity of the user which will scheduled if the system is in state j and the users' queue lengths are $\mathbf{q}(t)$. Then, if $q_{N-1}(0) = \ldots = q_{N-M}(0) = 1 > q_{N-M-1}(0) \geq \ldots \geq q_0(0)$, we have

$$M = T\frac{M\lambda}{N} - \int_{-T}^{0} \sum_{j\in A^M(\mathbf{q}(t))} F(\mathbf{q}(t),j)u_j(t)\,dt.$$

Now, suppose the system is in state j at time t, where $j\in\hat{A}^M$. From the definition of $\hat{A}^M$, there is at least one of channels $\{N-M,\ldots,N-1\}$ in state $L-1$. Since the trajectory is ordered, then one of users $\{N-M,\ldots,N-1\}$ will be scheduled. So $\hat{A}^M \subseteq A^M(\mathbf{q}(t))$ and

$$M \leq T\frac{M\lambda}{N} - \int_{-T}^{0} F_{L-1}\sum_{j\in\hat{A}^M} u_j(t)\,dt = T\left(\frac{M\lambda}{N} - F_{L-1}\sum_{j\in\hat{A}^M} K_j\right).$$

Now choose $\hat{T}$ such that

$$M = \hat{T}\left(\frac{M\lambda}{N} - F_{L-1}\sum_{j\in\hat{A}^M} K_j\right),$$

then $\hat{T} \leq T$ and $\{\hat{T}, \mathbf{K}\}$ is a feasible solution of $\mathrm{OP}^m(M,N)$. So we can conclude that

$$\int_{-T}^{0} D(\mathbf{u}(s)||\mathbf{p})\, ds \geq TD(\mathbf{K}||\mathbf{p}) \geq \hat{T}D(\mathbf{K}||\mathbf{p}) \geq \hat{C}_M^N(1) \geq \min_M \hat{C}_M^N(1),$$

which implies that $\theta_B^{\mathrm{QLB}}(N) \geq \min_M \hat{C}_M^N(1)$. ■

Theorem 13: Suppose the QLB policy is used. Given the buffer overflow constraint $\theta_B^{\mathrm{QLB}}(N,\lambda) = \theta$, the maximum throughput of the network satisfies

$$\lambda_B^{\mathrm{QLB}}(N,\theta) \geq \underline{\lambda}_B^{\mathrm{QLB}}(N,\theta) := -\max_{1\leq M\leq N} \frac{N}{\theta} \log\left((1-p_{L-1})^M + \left(1-(1-p_{L-1})^M\right) e^{-\frac{F_{L-1}\theta}{M}} \right).$$

Given the delay-violation constraint $\theta_D^{\mathrm{QLB}}(N,\lambda) = \theta$, the maximum throughput of the network satisfies

$$\lambda_D^{\mathrm{QLB}}(N,\theta) \geq \underline{\lambda}_D^{\mathrm{QLB}}(N,\theta) := -\max_{1\leq M\leq N} \frac{\theta N F_{L-1}}{M\left(\log\left(e^{-\theta}-(1-p_{L-1})^M\right) - \log\left(1-(1-p_{L-1})^M\right)\right)}.$$

Furthermore,

$$\underline{\lambda}_B^{\mathrm{QLB}}(N,\theta) < \underline{\lambda}_B^{\mathrm{QLB}}(N+1,\theta) \qquad \text{and} \qquad \underline{\lambda}_D^{\mathrm{QLB}}(N,\theta) < \underline{\lambda}_D^{\mathrm{QLB}}(N+1,\theta).$$

Proof: The lower bounds of the maximum throughput are obtained from (23)-(26). The proof of the behavior of the lower bounds is same as in Theorem 7. ■

B. Greedy Policy

In this subsection, the greedy policy is considered. Under the greedy policy, the probability $\hat{p}_l$ for a user at state l and picked to transmit is

$$\hat{p}_l = \frac{1}{N}\left(\left(1 - \sum_{j=l+1}^{L-1} p_j\right)^N - \left(1 - \sum_{j=l}^{L-1} p_j\right)^N \right). \tag{37}$$

We have argued that, under the greedy policy, the large-deviations exponents of the system are same as the large-deviations exponents of one user at the beginning of Subsection III-B. So we can compute the large-deviations exponents under the greedy policy by considering a single user system with arrival rate λ/N and probability vector $\hat{\mathbf{p}}$, where $\hat{p}_l$ is defined by (37).

Theorem 14: Suppose the greedy policy is used. Given the buffer-overflow constraint $\theta_B^{\mathrm{Greedy}}(N,\lambda) = \theta$, the maximum throughput the network can support is

$$\lambda_B^{\mathrm{Greedy}}(N,\theta) = -\frac{N \log\left(\sum_{l=0}^{L-1} e^{-F_l\theta}\hat{p}_l\right)}{\theta}.$$

Given the delay-violation constraint $\theta_D^{\mathrm{Greedy}}(N,\lambda) = \theta$, the maximum throughput $\lambda_D^{\mathrm{Greedy}}(N,\theta)$ the network can support satisfies:

$$1 = -\frac{\log\left(\sum_{l=0}^{L-1} \hat{p}_l e^{-\frac{NF_l\theta}{\lambda_D^{\mathrm{Greedy}}(N,\theta)}}\right)}{\theta}$$

Proof: Use (21) and (22). ■

Theorem 15: When N goes to infinity,

$$\lim_N \lambda_B^{\text{Greedy}}(N) \leq \frac{1-e^{-\theta F_{L-1}}}{\theta}.$$

There exits $a > 0$ such that if $N \geq a$,

$$\lambda_D^{\text{Greedy}}(N) = 0.$$

Proof: First we know that

$$-\frac{N\log\left(\sum_{l=0}^{L-1} e^{-F_l\theta}\hat{p}_l\right)}{\theta} \leq -\frac{N\log\left(\hat{p}_0+(1-\hat{p}_0)e^{-F_{L-1}\theta}\right)}{\theta} = -\frac{N\log\left((1-\hat{p}_0)(e^{-F_{L-1}\theta}-1)+1\right)}{\theta}$$

where $1-\hat{p}_0 = \frac{1}{N}\left(1-p_0^N\right)$. Since $\lim_N \left(1-p_0^N\right) = 1$, we can conclude that

$$\lim_N -\frac{N\log\left(\hat{p}_0+(1-\hat{p}_0)e^{-F_{L-1}\theta}\right)}{\theta} = \frac{1-e^{-\theta F_{L-1}}}{\theta}.$$

Next consider the delay-violation constraint $\theta_D^{\text{Greedy}}(N,\lambda) = \theta$, from Theorem 14, we know that λ is supportable if

$$1 \leq -\frac{\log\left(\sum_{l=0}^{L-1} \hat{p}_l e^{-\frac{NF_l\theta}{\lambda}}\right)}{\theta},$$

which requires

$$e^{-\theta} - \hat{p}_0 \geq \sum_{l=1}^{L-1} \hat{p}_l e^{-\frac{NF_l\theta}{\lambda}} \geq 0.$$

Since $\lim_N \hat{p}_0 = 1$, there exists $a > 0$ such that for $N \geq a$, we have $e^{-\theta} < \hat{p}_0$ and $\lambda_D^{\text{Greedy}}(N,\theta) = 0$. ■

For the multi-state channel model, from Theorem 13 and Theorem 15, we can conclude that under the delay-violation constraint, the throughput of the QLB policy is strictly larger than the throughput of the greed policy for large N. Actually, since the maximum throughput of the greedy policy decreases to zero, but the lower bound of the QLB is strictly increasing, the performances of these two scheduling schemes are dramatically different when N is large.

V. QLB Policy vs Greedy Policy

For the ON-OFF channel model and the delay-violation constraint, we have shown that the total throughput of the QLB policy is increasing in N; while the throughput of the greedy policy decreases to 0. For a general multi-state channel model, we have shown that the lower bound of the throughput of the QLB policy is increasing in N; while the throughput of the greedy policy decreases to zero. Since $\theta_B(N) = \frac{N}{\lambda}\theta_D(N)$, we can conclude that, for both the ON-OFF and the multi-state channel model, the maximum total throughput of the QLB policy is larger than the greedy policy under the QoS constraints for large N. In this section, we show it is actually true for all N. Note that since the ON-OFF channel model is a simple multi-state channel model, we only prove the fact for the general multi-state channel model.

We first define the system state and the channel rate processes for an N-user system under the greedy policy. We use $\{0,(j,i)\}$ to indicate the state of the system, where 0 means all channels are OFF, $j = 1,\dots L^N - 1$ represents the composite state of all the channels, and i indicates the channel which is chosen to transmit. Let l_j be the best channel state when the system is in state j, and m_j be the number of channels in state l_j when the system in state j. Then, we have

$$p_{j,i} = \begin{cases} \frac{1}{m_j} p_j, & \text{if } S_i^j = l_j; \\ 0, & \text{otherwise} . \end{cases}$$

Use $\tilde{\mathbf{p}}$ to denote the probability vector. Further, similar to the ON-OFF channel model, we define the channel rate processes $\{u_0, u_{j,i}\}$.

Theorem 16: For an N users system, the total maximum throughput under the QLB policy is larger than the throughput under the greedy policy:

$$\lambda_B^{\text{QLB}}(N,\theta) \geq \lambda_B^{\text{Greedy}}(N,\theta) \qquad \text{and} \qquad \lambda_D^{\text{QLB}}(N,\theta) \geq \lambda_D^{\text{Greedy}}(N,\theta).$$

Proof: Theorem holds if

$$\theta_B^{\text{QLB}}(N,\lambda) \geq \theta_B^{\text{Greedy}}(N,\lambda) \qquad \text{and} \qquad \theta_D^{\text{QLB}}(N,\lambda) \geq \theta_D^{\text{Greedy}}(N,\lambda).$$

Since $\theta_B(N,\lambda) = \frac{N}{\lambda}\theta_D(N,\lambda)$, we only need to show one of the inequalities holds. We consider $\theta_B(N,,\lambda)$ and show $\theta_B^{\text{QLB}}(N,\lambda) \geq \theta_B^{\text{Greedy}}(N,\lambda)$.

Consider an ordered trajectory under the QLB policy with channel rate processes $\mathbf{u}(t)$. Suppose $q_i(-T) = 0$ for all i and $q_{N-1}(0) = 1$, so

$$\int_{-T}^{0} \left(\frac{1}{N}\lambda - \sum_{j \in A(\mathbf{q}(t))} F(\mathbf{q}(t), j) u_j(t) \right) = 1,$$

where $A_{N-1}(\mathbf{q}(t))$ is the set such that if $j \in A_{N-1}(\mathbf{q}(t))$, then user $N-1$ will be selected to transmit at time t if the system is in state j at time t.

Now define a new control $\tilde{\mathbf{u}}$ for the system under the greedy policy such that for $j > 0$,

$$\tilde{u}_{j,i}(t) = \begin{cases} \frac{1}{m_j} u_j(t), & \text{if } S_i^j = l_j; \\ 0, & \text{otherwise.} \end{cases} \tag{38}$$

Define a set $\hat{A}_{N-1}$ such that if $j \in \hat{A}_{N-1}$, then $S_{N-1}^j \geq S_i^j$ for all i. Thus, if $j \in \hat{A}_{N-1}$, then user $N-1$ has the best channel state when the system is in state j. Also, user $N-1$ has the largest buffer size, so under the QLB policy, user $N-1$ will be selected to transmit when the system is in state j. So we have $\hat{A}_{N-1} \subseteq A_{N-1}$. Now consider the system under the greedy policy and channel rate processes $\tilde{\mathbf{u}}(t)$. Suppose $q_i(-T)=0$ for all i, then

$$\begin{aligned} q_{N-1}(0) &= \int_{-T}^{0} \left(\frac{\lambda}{N} - \sum_{j \in \hat{A}_{N-1}} F_{S^j_{N-1}} \tilde{u}_{j,N-1}(t) \right) dt \geq \int_{-T}^{0} \left(\frac{\lambda}{N} - \sum_{j \in \hat{A}_{N-1}} F_{S^j_{N-1}} u_j(t) \right) dt \\ &\geq \int_{-T}^{0} \left(\frac{\lambda}{N} - \sum_{j \in A_{N-1}} F_{l_j} u_j(t) \right) dt = 1. \end{aligned}$$

Thus, the overflow time $\tilde{T}$ under the greedy policy and control $\tilde{\mathbf{u}}(t)$ is less than T. On the other hand,

$$D(\tilde{\mathbf{u}}(t)||\tilde{\mathbf{p}}) = D(\mathbf{u}(t)||\mathbf{p}),$$

from which we can conclude that $\theta_B^{\text{QLB}}(N) \geq \theta_B^{\text{Greedy}}(N)$, and the theorem holds. ■

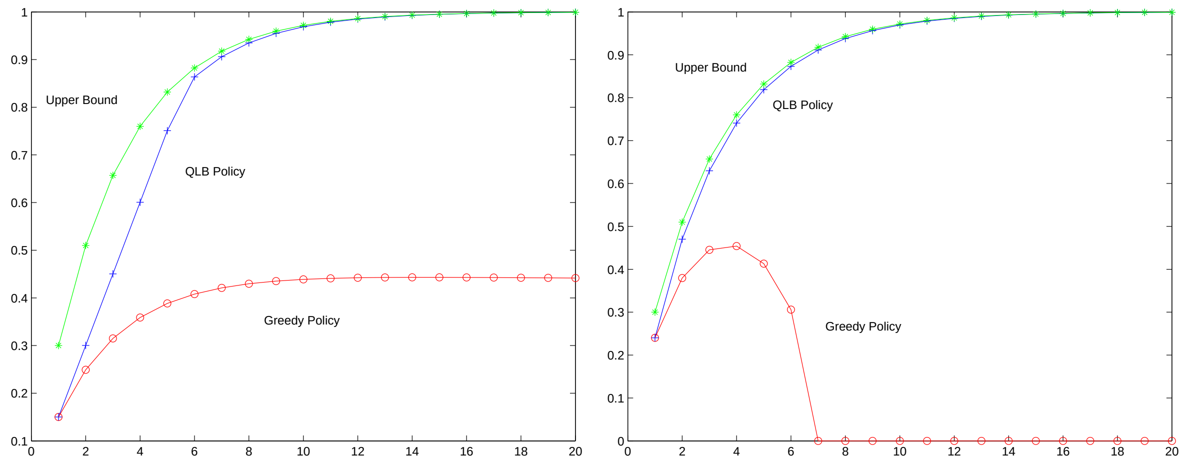
Remark: Consider the ON-OFF channel model, we have shown the optimal channel rate processes are constant and $u_j(M^*) \neq 0$ for all j (since $x(M^*) \neq 0$). Let $\tilde{\mathbf{u}}(M^*)$ be the channel rate processes generated from $\mathbf{u}(M^*)$ according to (38), then it is easy to verify that

$$\int_{-T}^{0} \left(\frac{\lambda}{N} - \sum_{j \in \hat{A}_{N-1}} F_{S^j_{N-1}} \tilde{u}_{j,N-1}(M^*) \right) dt > \int_{-T}^{0} \left(\frac{\lambda}{N} - \sum_{j \in \hat{A}_{N-1}} F_{S^j_{N-1}} u_j(M^*) \right) dt,$$

because there exists j such that $j \in \hat{A}_{N-1}$ and $\tilde{u}_{j,N-1}(M^*) < u_j(M^*)$. Thus, we can conclude that under the greedy policy, there exists a trajectory whose cost is strictly less than the minimum cost of the trajectories under the QLB policy. So for the ON-OFF channel model, the maximum throughput under the QLB policy is strictly larger than the throughput under the greedy policy.

Example: Consider an ON-OFF channel system with $p=0.3$ and $F=1$. For this system, we calculate the maximum throughput of the system under the QoS constraints for $N=1,\ldots,20$. We also calculate the maximum throughput without QoS constraint, which is a upper bound of the throughput under QoS constraints.

1) Fixing the buffer overflow constraint to be $\theta=2$, in Fig. 3(a), we plot the maximum throughput $\lambda_B(N,\theta)$. We can see that the throughput under the QLB policy is almost equal to the throughput without QoS constraint for $N \geq 6$, while the throughput under the greedy policy is less than 0.5.
2) Fixing the delay-violation constraint to be $\theta=0.15$, in Fig. 3(b), we plot the maximum throughput $\lambda_D(N,\theta)$. We can see that the throughput under the QLB policy is almost equal to the throughput without QoS constraint for all N, while the throughput under the greedy policy decreases to 0 for $N \geq 7$.



(a) The Maximum Throughput under the Queue Overflow Constraint

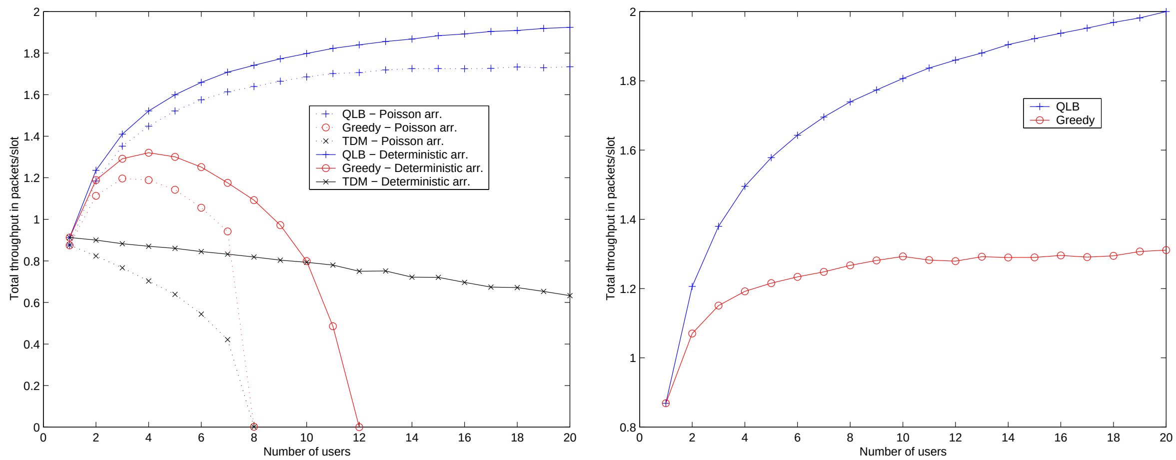
(b) The Maximum Throughput under the Delay Constraint

Fig. 3. The Maximum Throughput for QLB Policy and Greedy Policy

VI. SIMULATIONS

In this section we study the performance of the Greedy and the QLB policies for a more realistic channel model than the ON-OFF model. Moreover, we examine the performance of the schedulers under asymmetric channel conditions. We aim to compare the behavior of the schedulers with the analytical results, and investigate whether similar behaviors are observed under the realistic conditions. For this purpose, we consider a Rayleigh fading channel that is independently distributed across the N users and across the time slots. It is assumed that the time slots are of duration $T_c = 1$ msecs, and the available bandwidth for transmission is $W = 1.25$ MHz. The traffic entering the buffers is measured in *packets/slot* for convenience. Here, a *packet* is defined to be of size $W \times T_c = 1.25$ Knats.

EXPERIMENT 1: We assume symmetric channel conditions, where the Signal-to-Noise Ratio (*SNR*) between the base station and each of the users is equal to $3\,dB$. In addition to the QLB and Greedy Schedulers, we also include the performance of a Time-Division-Multiplexing (TDM) Scheduler, which provides service to the users in a periodic fashion regardless of the channel conditions of that user in the slot that it is served. It has been observed in [4] that the TDM scheduler can outperform the Greedy Scheduler if N is large enough. In Figure 4(a) we study the maximum throughput levels attainable by each of the schedulers when the delay constraints are $D = 100$ and $\varepsilon = 0.001$. The dotted lines correspond to the case when the arrivals to each queue is Poisson distributed with mean equal to λ packets/slot. The solid lines, on the other hand, corresponds to deterministic arrivals at rate λ packets/slot.



(a) The Maximum Throughput under the Delay Constraint (b) The Maximum Throughput under the Queue Overflow Constraint

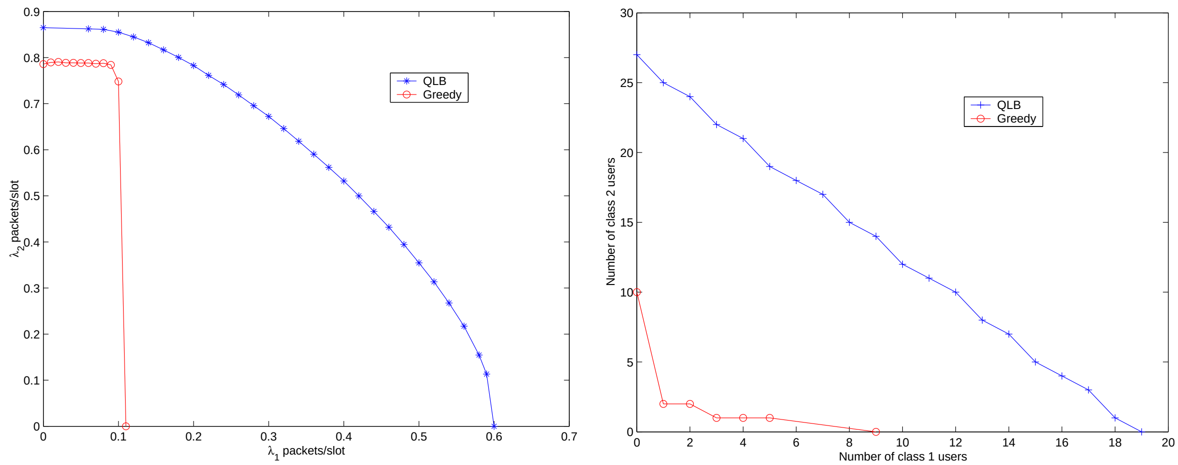
Fig. 4. The Maximum Throughput for Rayleigh Fading Channel

It is not surprising that the randomness in the arrivals cause a decrease in the total throughput achievable for each of the schedulers. However, each scheduler is affected to a different extent. For example, we observe that the TDM policy, contrary to the deterministic arrival case, no longer possesses an advantage over the Greedy policy for any N under the random arrival scenario. We also observe that for both the deterministic and Poisson distributed arrival processes, the behavior of the QLB and Greedy policies are very similar to those predicted in our analysis, and those presented in Figure 3(b). In the remainder of the simulations, we will consider the deterministic arrival case since the relative behaviors of the schedulers are not different with random arrivals.

In the second part of the experiment, we study the maximum achievable total throughput levels of the QLB and Greedy policies under the buffer constraint. Figure 4(b) depicts the results when $B = 20$ and $\varepsilon = 0.001$. Again, we observe that the behaviors of the two policies are the same as in Figure 3(a).

Both of these simulations support the predicted behavior of the policies for channel processes other than the ON-OFF model, and even under the case of random arrivals. Next, we compare the QLB and Greedy policies under asymmetric channel conditions.

EXPERIMENT 2: We assume two classes of users. Class-1 users have an average SNR of $3\,dB$, whereas Class-2 users have an average SNR of $7\,dB$. We let N_i and λ_i respectively denote the number of users in Class$-i$ and the arrival rate to each user that belongs to Class-i. We study the performance of delay constrained traffic with $D = 100$ and $\varepsilon = 0.001$ for all the users.



(a) The Maximum Supportable Throughput levels

(b) The Maximum Number of Supportable Users

Fig. 5.

In the first part of the experiment, we fix (N_1, N_2) to $(2, 2)$. That is, there are two users to be served in each class. In Figure 5(a), we plot the region of maximum rates, (λ_1, λ_2), that can be supported by the QLB and Greedy policies. It is observed that the QLB policy can support any rate that is supportable by the Greedy policy. Moreover, QLB Scheduler exhibits a gradually changing boundary, whereas the Greedy Scheduler has abrupt changes. This is because the Greedy policy is not able to adjust its service rate allocation as a function of the arrival rates, whereas the QLB policy can transfer the service rate of the less demanding class to the class with high arrival rates. We also observe in Figure 5(a) that for both of the schedulers, the supportable rate region is asymmetric in favor of Class-2 users which is to be expected since the channel conditions are better for them. However, we note that the QLB Scheduler is much more capable of adapting its services to support both class users to a fairly good level. On the other hand, the Greedy Scheduler is unable to support any rate if λ_1 exceeds a threshold level of 0.11 packets/slot.

In the second part of the experiment, we fix the arrival rates of both class of users to $\lambda_1 = \lambda_2 = 0.1$ packets/slot. Then we investigate the number of users that can be supported in each class for the same delay constraints as in the first part. Figure 5(b) depicts the region of supportable users in each class for the QLB and Greedy policies. Again, we observe that QLB is far superior to the Greedy policy. It can maintain a much larger number of users in each class. On the other hand, the region for the Greedy policy exhibits convexity properties. The main reason for the poor performance of the Greedy Scheduler is its inability to respond to asymmetric conditions. For this scheduler, by increasing N_1 from 0 to 1, we need to decrease N_2 sharply from 10 to 2. On the other hand, the QLB Scheduler can successfully adapt itself to maintain both class of users due to the feedback it receives concerning the buffer occupancy levels.

The simulations presented in this section suggest that the large deviations analysis performed for the simple channel model, is able to provide performance characteristics for the QLB and Greedy policies that hold for more realistic and complex models.

VII. Conclusion

In this paper, we use a large deviations analysis to investigate the performance of different scheduling policies for the downlink of a cellular network under QoS constraints. For a simple ON-OFF channel model and a multi-state channel model, we prove that the throughput of queue-length based policy is larger than that of the greedy policy. Furthermore, when the number of users increases, the throughput of the greedy policy decreases while the QLB policy increases. Also for the ON-OFF channel model, closed-form expressions for the maximum throughput have been provided.

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