Sensor selection and placement in adversarial environment

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***Abstract*—We study the problem of sensor placement, when the set of sensors are interrogated sequentially to detect a target employing evading actions. We model the sensor management problem as a two stage game between the observer and the target: in the first stage sensor locations are chosen by the observer, in the second stage the observer and the target choose open-loop control strategies to optimize error exponent for the probability of mis-detection with opposing interests. Therefore the observer chooses the sensor locations to optimize the value function of the game played in the second stage. We propose a computationally efficient gradient descent method to iteratively refine the sensor locations for maximizing the value function using the envelope theorem.**

***Index Terms*—Controlled sensing, game theory, envelope theorem, sensor placement.**

I. Introduction

In distributed sensor systems, the fusion center typically can access only a subset of sensors at any game time, to satisfy bandwidth or energy constraints while maximizing a performance metric for an inference task such as tracking or detection. In this paper we study the problem of sensor placement for this scenario. Specifically, we model the sensor placement as a two-stage game between an observer(fusion center) and adversarial target. We model the sensor management problem as a two stage game between the observer and the target: in the first stage sensor locations are chosen by the observer, in the second stage the observer and the target choose open-loop control strategies to optimize probability of detection with opposing interests

The sensor selection problem that forms the basis for the second stage interaction between observer and the target has been studied in previous work in various settings. Srivastava *et al.* [1] derived optimal randomized sensor selection strategies for M-ary hypothesis testing in the sequential setting based on Multi-hypothesis Sequential Probability Ratio Test (MSPRT). Nitinawarat *et al.* [2] analyze and solve the problem of choosing open-loop and closed-loop sensing actions that minimize the error exponent for binary and M-ary hypothesis testing in both fixed sample size and sequential setting. In an alternative direction to these previous work which assume random target actions, Ertin [3] formulated the problem of randomized sensor selection for detection of an adversarial target employing randomized strategy for selecting locations. Krause *et al.* [4] also considers the problem of sensor selection in the adversarial setting but focuses on submodular performance metrics in set of sensors selected, parametrized by target actions. We follow on our work in [3] and tie it to the sensor placement problem in Section II. The saddle point problem is parametrized as a function of the sensor configuration. Thereby, the problem of finding an optimal configuration for the sensors that maximizes the value of the saddle point problem is proposed. We provide a numerical solution to the saddle point problem in Section III along with a gradient based method with a heuristic for refinement in sensor placement. Two example scenarios for sensor selection and placement are presented in Section IV.

II. System Model and Problem Statement

A. Observation model

We model the target detection problem as a binary hypothesis test between H_0 and H_1 denoting the absence or presence of target employing randomized actions, respectively. Let K be the number of sensors, and M be the possible locations that the target can choose, where $M \gg K$. Let $X_S = \left[x_1^S, y_1^S, \cdots, x_K^S, y_K^S\right]^T$ denote co-ordinates of K sensors. Let $X_T = \left[x_1^T, y_1^T, \cdots, x_M^T, y_M^T\right]^T$ denote co-ordinates of M possible target locations. Let $\mathcal{Y}$ denote the finite observation space with cardinality J. One possible observation space is $\mathcal{Y} = \{0, 1\}$, the decision made by each sensor based on the signal measurements.
The adversarial target, and the observer employ open-loop randomized strategies in choosing their actions. The observer selects sensors using the distribution $r = [r_1, \cdots, r_K]^T$, where r_k is the probability of selecting sensor k, independently to obtain N observations $y = [y_1, \cdots, y_N]^T$, where each $y_i \in \mathcal{Y}$. The target uses the distribution $s = [s_1, \cdots, s_M]^T$ to select the location independently during each observation, where s_m is the probability that the target selects location m. Let $I = [i_1, \cdots, i_N]^T$ denote the vector of index of the selected sensor to make the N observations. The observer uses the vector $u = [y, I]^T$ to compute the decision rule. $\mathcal{U}$ is the space of all possible u. Under hypothesis H_0, we define the conditional probability of observing $j \in \mathcal{Y}$ given sensor k is chosen as p_j^k. Under hypothesis H_1, let $q_j^k(m; X_S)$ denote the conditional probability of observing $j \in \mathcal{Y}$ given observer chooses sensor k with the sensor configuration X_S, and target selects location m. The conditional probability of observing $j \in \mathcal{Y}$ given the sensor k is chosen under H_1 is $q_j^k(s; X_S) = \sum_{m=1}^{M} s_m q_j^k(m; X_S)$ when the target selects the location using the probability distribution s.

B. Detection rule

Given the observation vector u, the observer's decision rule partitions the space $\mathcal{U}$ into decision regions $\mathcal{U}_1, \mathcal{U}_0$ such that if

$u \in \mathcal{U}_i$, then $\delta(u) = H_i$. The probability of mis-detection P_M^N and false alarm P_{FA}^N, given N observations, can be computed as follows:

$$P_M^N(r,s;X_S) = \sum_{[y,I]\in\mathcal{U}_0} \prod_{n=1}^{N} r_{i_n} q_{y_n}^{i_n}(s;X_S) \tag{1}$$

$$P_{FA}^N(r,s) = \sum_{[y,I]\in\mathcal{U}_0} \prod_{n=1}^{N} r_{i_n} p_{y_n}^{i_n} \tag{2}$$

The decision rule employed by the observer is restricted to the family of rules such that $P_{FA}^N(r,s) \leq \lambda$, $\lambda \in [0,1]$. The maximum achievable negative error exponent associated with H_1 as the number of samples $N \to \infty$ is as follows:

$$\mathcal{J}(r,s;X_S) = -\lim_{N\to\infty} \inf_{\mathcal{U}_0 : P_{FA}^N(r,s)\leq\lambda} P_M^N(r,s;X_S) \tag{3}$$

It was shown in [3] that for open-loop randomized strategies, the negative error exponent (3) can be computed as follows:

$$\mathcal{J}(r,s;X_S) = \sum_{k=1}^{K} r_k D\left(p^k \mid\mid q^k(s;X_S)\right), \tag{4}$$

where $D\left(p^k \mid\mid q^k(s;X_S)\right) = \sum_{j\in\mathcal{Y}} p_j^k \log \frac{p_j^k}{q_j^k(s;X_S)}$.

C. Sensor selection game

The sensor selection problem is formulated as a surveillance game between the observer and the target with the payoff function as the largest achievable error exponent (4) as shown

$$V(X_S) = \max_{r\in\mathcal{P}^K} \min_{s\in\mathcal{P}^M} \mathcal{J}(r,s;X_S), \tag{5}$$

where $\mathcal{P}^K, \mathcal{P}^M$ are the K and M dimensional probability simplexes, respectively, $V(X_S)$ corresponds to the value of the surveillance game given sensor configuration X_S, and $(r^*(X_S), s^*(X_S))$ corresponds to the saddle point solution. It can be seen that (4) is convex in s and linear or concave in r, and the saddle point solution to (5) exists as shown in [3]. The saddle point solution $(r^*(X_S), s^*(X_S))$ corresponds to the Nash equilibrium strategy for the two opponents, where the sensor selection distribution $r^*(X_S)$ safeguards against an adversarial target that minimizes (4). This can be stated as

$$\begin{aligned}\mathcal{J}(r, s^*(X_S); X_S) &\leq \mathcal{J}(r^*(X_S), s^*(X_S); X_S) \\ &\leq \mathcal{J}(r^*(X_S), s; X_S),\end{aligned} \tag{6}$$

which is true for any $r \in \mathcal{P}^K, s \in \mathcal{P}^M$.

D. Sensor placement problem

Instead of randomly deploying sensors in the field, we aim at placing sensors to guarantee maximal coverage in terms of probability of detection even in the presence of an adversarial target. The payoff function in (4) is non-convex in sensor co-ordinates as it is invariant to interchange between the sensor locations in the case of homogeneous sensors. Therefore, we parametrize the surveillance game in terms of sensor configuration X_S. The sensor placement problem is formulated as

$$\max_{X_S} V(X_S), \tag{7}$$

where $V : K \to \mathbb{R}$, K is a compact set in $\mathbb{R}^2$. The next section presents the methods to solve the problems (5) and (7).

III. Optimal strategies for sensor selection and placement

A. Surveillance game solution

For a given sensor placement X_S, solution for (5) can be obtained by solving simultaneously the following optimization problems

$$\max_{r\in\mathcal{P}^K} \mathcal{J}(r, s^*(X_S); X_S), \text{ and} \tag{8}$$

$$\min_{s\in\mathcal{P}^M} \mathcal{J}(r^*(X_S), s; X_S), \tag{9}$$

where $(r^*(X_S), s^*(X_S))$ is the saddle point solution for the convex-concave game. This problem can be numerically solved by using the interior point method [5]. The joint Lagrangian of the optimization problems (8) and (9) can be written as

$$\mathcal{L}(r,s,\lambda_1,\lambda_2;t) = \sum_{k=1}^{K} r_k \sum_{j=1}^{J} p_j^k \log\left(\frac{p_j^k}{\sum_{m=1}^{M} q_j^k(m) s_m}\right) + \frac{1}{t}\sum_{k=1}^{K}\log(r_k) - \frac{1}{t}\sum_{m=1}^{M}\log(s_m) + \lambda_1(\mathbf{1}^T r - 1) + \lambda_2(\mathbf{1}^T s - 1), \tag{10}$$

where t is the parameter associated with the log barrier method, and $\lambda_1.\lambda_2$ are the Lagrange multiplier associated with the simplex equality constraint. The solution is obtained by iteratively solving the following system of non linear equations as $t \to \infty$:

$$\lim_{t\to\infty} \nabla\mathcal{L}\left(r^*(t), s^*(t), \lambda_1^*(t), \lambda_2^*(t); t\right) = \mathbf{0}, \tag{11}$$

where $\lim_{t\to\infty}(r^*(t), s^*(t))$ is the primal solution, and $\lim_{t\to\infty}(\lambda_1^*(t), \lambda_2^*(t))$ is the dual solution to the optimization problem (8) and (9). However, in practice t is increased until the following convergence condition is satisfied

$$\left|\max_{r\in\mathcal{P}^K} \mathcal{J}(r, s^*(t); X_S) - \min_{s\in\mathcal{P}^M} \mathcal{J}(r^*(t), s; X_S)\right| \leq \epsilon, \tag{12}$$

where ϵ is an appropriately chosen threshold. This guarantees the convergence to a saddle point solution.

The solution $(r^*(X_S), s^*(X_S))$ depends on the particular sensor configuration X_S. In the next section we look at how the sensor configuration can be modified, which leads to increase in the value of the surveillance game given in (5).

B. Sensor placement solution

We solve the sensor placement problem in (7) by making use of the gradient based method. If the value function is differentiable or directionally differentiable in X_S, then a gradient ascent or subgradient method can be used to iteratively solve (7) by updating the sensor configuration as follows

$$X_S^{(i)} = X_S^{(i-1)} + \eta \partial V(X_S), \tag{13}$$

where $X_S^{(i)}$ is the updated sensor congiguration, η is the step size calculated by line search or is set constant, and $\partial V(X_S)$ is the subdifferential of V with respect to X_S. In order to compute $\partial V(X_S)$, we make use of the envelope theorem for saddle point problems given as Theorem 4 in [6]. The envelope theorem provides a computationally efficient method of computing the differential of the value functions $\partial V(X_S)$ at any saddle point $(r^*(X_S), s^*(X_S))$ as:

$$\nabla_{X_S} V(X_S) = \mathcal{J}_{X_S}\left(r^*(X_S), s^*(X_S); X_S\right). \tag{14}$$

provide that the following conditions are met:

1) $\mathcal{J}(r, s, .)$ is absolutely continuous in X_S for all $(r, s) \in \mathcal{P}^K \times \mathcal{P}^M$
2) The saddle set $\{r^*(X_S) \times s^*(X_S)\} \neq \emptyset$ for almost all $X_S \in K \subset \mathbb{R}^2$
3) there exists an integrable function $b : K \to \mathbb{R}_+$ such that $|\mathcal{J}_{X_S}(r, s; X_S) \leq b(X_S)|$ for all $(r, s) \in \mathcal{P}^K \times \mathcal{P}^M$ and almost every $X_S \in K$.

The conditions of absolute continuity of $\mathcal{J}(r, s; .)$ in X_S $\forall (r, s) \in \mathcal{P}^K \times \mathcal{P}^M$ can be verified for physical sensing problems. For all $X_S \in K \subset \mathbb{R}^2$, the set of saddle points $\{r^*(X_S), s^*(X_S)\} \neq \emptyset$ because (4) is concave in r, and convex in s. Finally, bounding $\nabla_{X_S} |\mathcal{J}(r, s; X_S)| \leq b(X_S)$ with an integrable function $b(X_S)$ can be shown as follows:

$$\frac{\partial \mathcal{J}(r, s; X_S)}{\partial x_i^S} = r_i \sum_{j \in \mathcal{Y}} \frac{p_j^i}{q_j^i(s); X_S} \sum_{m=1}^{M} s_m \frac{\partial q_j^i(m; X_S)}{\partial x_i^S} \tag{15}$$

$$\frac{\partial \mathcal{J}(r, s; X_S)}{\partial y_i^S} = r_i \sum_{j \in \mathcal{Y}} \frac{p_j^i}{q_j^i(s; X_S)} \sum_{m=1}^{M} s_m \frac{\partial q_j^i(m; X_S)}{\partial y_i^S} \tag{16}$$

Using (15), (16), we get

$$\nabla_{X_S} |\mathcal{J}(r, s; X_S)| \leq \sum_{i=1}^{K} \left(\max_{j \in \mathcal{Y}, m} \frac{1}{q_j^i(s; X_S)} \left| \frac{\partial q_j^i(m; X_S)}{\partial x_i^S} \right| + \max_{j \in \mathcal{Y}, m} \frac{1}{q_j^i(s; X_S)} \left| \frac{\partial q_j^i(m; X_S)}{\partial y_i^S} \right| \right). \tag{17}$$

If the sensing model is such that the conditional probability $q_j^k(m; X_S)$ is continuous and differentiable in X_S, and $q_j^k(s; X_S) \neq 0, \forall j, k, m$, then the partial derivative is bounded by an integrable function.

Equipped with this result, we follow a simple approach for optimizing sensor locations iteratively. At each iteration, the surveillance game is solved with the current sensor coordinates $X_S^{(i)}$, and then compute the gradient of the value function with respect to sensor coordinates using (15) and (16). Then using (14) and (13) $X_S^{(i+1)}$ is computed. The sensor coordinates are updated until convergence in value function $V(X_S^{(i)})$.

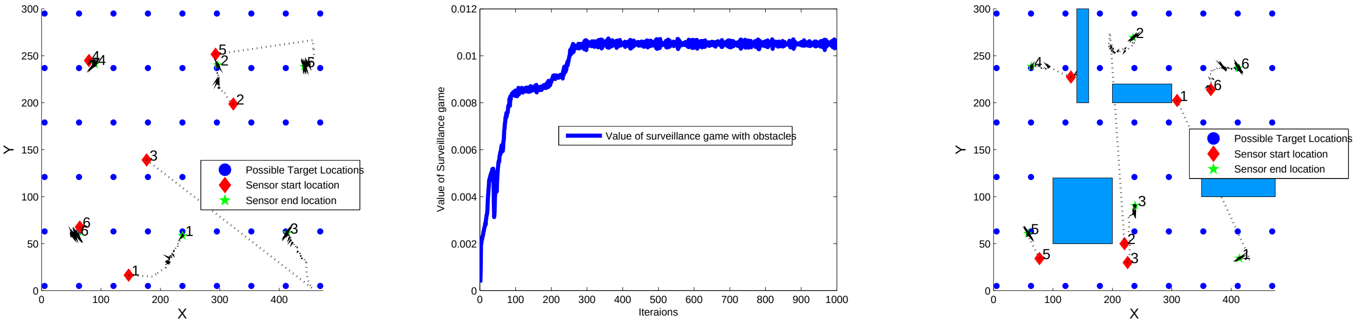
As discussed earlier, the value of the game is non-convex in X_S. In order to find a global maxima to the problem (7), we can employ random restarts. An alternate heuristic is to take advantage of the saddle point solution. It is evident from (15) and (16) that sensors that have very low probability of being selected from distribution $r^*\left(X_S^{(i-1)}\right)$ do not change the position in subsequent iterations. It is also clear that the target places more probability mass in locations that have high probability ot mis-detection due to the payoff function of the surveillance game. In order to increase the coverage in terms of probability of detection, the observer could move the unselected sensor to the location, where the target places maximum probability mass. This might guide the gradient based optimization process to converge towards global optima. In the next section, we present some examples where the proposed method was used to compute the sensor selection, and placement strategies.

IV. Simulation results

We consider two examples to demonstrate the solution strategies obtained using the proposed method for sensor selection and placement. The first example considered is the case when K=6 sensors, and the target selects from M = 54 possible locations in a field without obstacles. The next example considered has 4 rectangular obstacles with K=6 sensors and M=47 locations. In both these cases the space of observations from each sensor is the decision made by each sensor $\mathcal{Y} = \{0, 1\}$ after making a signal measurement Z. The signal model for sensor k conditioned on absence of target or presence of target at location m is

$$H_0 : Z^k = N$$
$$H_1(m) : Z^k = \frac{A \exp -B(r_{k,m})}{1 + r_{k,m}^{\alpha}} + N,$$

where Z^k is the signal observed by sensor k, α is the signal attenuation factor with distance, A is a constant associated with signal strength, $B(r_{k,m})$ is the signal attenuation factor due to obstacles, which is computed as shown in [7], $r_{k,m} = \sqrt{(x_k^S - x_m^T)^2 + (y_k^S - y_m^T)^2}$, and N is a zero mean Gaussian noise with standard deviation σ_N. We model the effect of medium on the signal strength as a scalar field of attenuation factor. The attenuation factor is 0 for air, and a large value for obstacles proportional to the distance the signal propagates in the medium. In the case of multiple obstacles,



(a) Sensor placement with no obstacles (b) Value of surveillance game with obstacles (c) Sensor placement with obstacles

Fig. 1. Sensor placement solution for the two examples

the attenuation scalar field $B(r_{k,m})$ is

$$B(r_{k,m}) = r_{k,m} \sum_{i=1}^{T} \beta^{(i)} \int_{\lambda=0}^{\lambda=1} \exp \Bigg(- \left[\frac{\lambda(x_k^S - x_m^T) - \bar{x}_i + x_m^T}{\sigma_{x_i}} \right]^6 - \left[\frac{\lambda(y_k^S - y_m^T) - \bar{y}_i + y_m^T}{\sigma_{y_i}} \right]^6 \Bigg), \quad (18)$$

where $\beta^{(i)} \gg 1$ is the attenuation factor of obstacle i, $(\bar{x}_i, \bar{y}_i)$ is the centroid of the i$^{\text{th}}$ obstacle, and σ_{x_i}, σ_{y_i} are parameters calculated based on the width of the obstacle.

Each sensor uses the Neyman-Pearson decision rule for a fixed false alarm rate at $p_1^k = \gamma$ for sensor k, the decision threshold τ is obtained as

$$\tau = \Phi_{H_0}^{-1}(1-\gamma), \quad (19)$$

where $\Phi_{H_0}^{-1}$ is the inverse normal cdf under H_0. The probability of mis-detection under H_1 is

$$q_0^k(m) = Pr(Z_m^K \leq \tau \mid H_1) = \Phi_{H_1}(\tau) \quad (20)$$

Using the probability of mis-detection and false alarm, the surveillance game is solved for the current sensor configuration X_S. The gradient is calculated using (15),(16). We employ a constant step size rule for updating the sensor coordinates. The change in value of game as the sensors are moved in the direction of gradient when no obstacles are present is shown in Fig 2. We note that the solution obtained in Fig 1(a) is not the global optimal, and a better placement configuration can be obtained by further decreasing the x coordinates of sensors indexed as 4,2, and 5. For the example with obstacles, we note that in Fig 1(b) there is a decrease in value function in iteration 40, which is due to the constant step size rule we employ. We make use of the heuristic discussed earlier to re-locate the sensors with low probability of being selected to locations that the mode of the least favorable distribution that the adversarial target uses to select locations, which aids in the convergence to a more favorable local maxima.

V. Conclusion

We presented a method for computing an open loop randomized policy for sensor selection in the presence of an

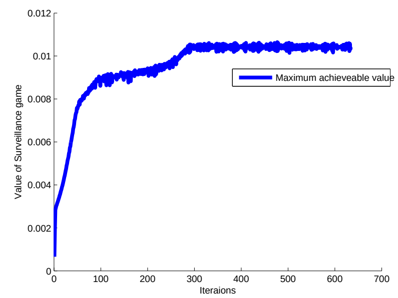


Fig. 2. Value of surveillance game with no obstacles

adversarial target that independently chooses control actions at each time step. We also presented a method to refine the sensor placement that leads to larger worst case negative error exponent. We assumed that both target and the observer employ randomized open loop control policies. An interesting future work would be to consider the effect of adapting the control policies for sensor selection for hypothesis testing in an adversarial setting.

Acknowledgment

This research was partially supported by Army Research Office grant W911NF-11-1-0391.

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