Manifold Learning Methods
for Wide-angle SAR ATR

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Abstract—The automatic recognition and characterization of civilian vehicles in urban setting is motivated by an increasingly difficult class of surveillance and security challenges. These new ATR (Automatic Target Recognition) problems are motivated by new data collection capabilities, in which airborne synthetic aperture radar (SAR) systems are able to interrogate a scene, such as a city, persistently and over a large range of aspect angles. Learning and exploiting the additional information provided by wide-aspect signatures is key to developing successful algorithms. In this paper, we study manifold learning methods to learn informative projections of the feature space for ATR algorithm design, which is also amenable to performance prediction and analysis.

I. INTRODUCTION

A new class of Automatic Target Recognition (ATR) problems of detection, classification and characterization of civilian vehicles in urban environments emerged recently, prompted by advances in persistent radar sensing capabilities. Specifically, airborne synthetic aperture radar (SAR) systems are able to interrogate a scene persistently over a large range of aspect angles. Learning and exploiting the additional information provided by wide-aspect signatures is key to developing successful ATR algorithms for this application. For a persistent SAR sensor, wide-angle signature of a target is encoded in complex-baseband images obtained at different center azimuth angles. The problem of interest is to estimate the target pose and class given an image obtained at an unknown azimuth angle. Several algorithms have been proposed in the literature for this canonical problem. Image based techniques construct templates from image data and try to match images to these templates [1]. Performance of template matching methods may deteriorate in the presence of occlusions or changes in radar-target geometry. An alternate approach is to extract features from the training data and match it to features in the imagery [2]. Feature based methods rely on a computationally expensive exhaustive search step to match features from the training data to the features extracted from the test image. Theoretical analysis of feature based methods is in general intractable, because of the difficulty of modeling and learning error statistics in the feature space. In this paper, we present a data driven method to learn informative projections of the feature space for ATR algorithm design, which is also amenable to performance prediction and analysis.

Manifold learning methods for dimensionality reduction has gained a lot of interest in the past decade. The aim of dimensionality reduction is to change representation of high dimensional data sets into a low dimensional description consisting of only small number of free parameters, while preserving information about the underlying structure of the data. Consider the basic problem of estimation of target pose from SAR imagery. The radar data consists of large dimensions many of which are irrelevant to pose estimation and the remaining provide correlated features that are highly redundant. If we are only interested in pose estimation task then all the pertinent information in the data can be expressed by a single variable encoding the relative angle between the target and radar aperture. In principle, classifiers can be designed to work in the high dimensional data space directly. However, since the number of required training vectors grows exponentially with the number of dimensions for good generalization to out-of-sample data, design in the original high dimensional data space is not feasible when limited training vectors are available. Therefore to solve the association problem between different looks in high dimensional radar data requires a way to deal with curse-of-dimensionality problem.

II. REVIEW OF MANIFOLD LEARNING METHODS

Manifold learning methods [3], [4], [5], [6] provide a principled way of attacking curse-of-dimensionality problem by discovering low dimensional embeddings where classifiers can be designed robustly. Dimension reduction serves as a pre-processing step, first the data originally present at high dimensional space $\mathcal{R}^D$ is mapped to a lower dimensional space $\mathcal{R}^d$ (where $d \ll D$), while preserving information about the latent variables of interest. As a result detection, classification and learning algorithms can be designed and applied effectively using the projected data at the low dimensional space.

This information pre-processing step is made possible based on the observation that the data for a specific class of targets lies on or close to a possibly curved surface in the high dimensional space. A subset of space $\mathcal{M}$ is called a manifold if each local open neighborhood of $\mathcal{M}$ is isomorphic to a low dimensional Euclidean space of dimension $d$. Loosely speaking at each point, $\mathcal{M}$ looks locally flat like a hyperplane of dimension $d$. Manifold based dimension reduction techniques attempt to reduce the dimension of the data while
preserving relevant geometric information of the data such as distances and angles as measured on the manifold. In general, one cannot find a low dimensional embedding that preserves all distances in the high dimensional space, but the embedding problem becomes tractable when the criterion is relaxed to preserve distances only between the points in the data manifold.

Manifold learning techniques such as Locally linear Embedding (LLE), Isomap, Laplacian Eigenmaps all rely on capturing data geometry by constructing the proximity graph of the data and then finding a projection to low dimensional space to preserve the distances in the proximity graph. Here we focus on the Laplacian Eigenmap, since it is a computationally efficient global linear mapping that can embed out-of-samples directly. The vertices of the neighborhood graph are given by the \( N \) data points \( \{x_1, x_2, \ldots, x_N\} \). The \( N \times N \) weight matrix \( W \) provides the edge weights between each pair of data points encoding neighborhood information. Possible choices for \( W \) include: Binary adjacency matrix where \( W_{ij} = 1 \) if point \( x_j \) is among the \( K \)-nearest neighbors of point \( x_i \), and \( 0 \) otherwise, and Gaussian heat kernel \( W_{ij} = \exp(-d(x_i, x_j)/2\sigma^2) \) providing a soft measure of nearliness between data points.

Given the proximity graph, Laplacian Eigenmaps try to minimize the norm of Laplace-Beltrami operator on the manifold \( \int_M \|\nabla \Phi(x)\|^2dx \) based on the intuition that the linear embedding function \( \Phi(x) = w^T x \) should have a small gradient on the manifold.

The discrete equivalent optimization problem is given by

\[
\min_{\phi} \sum_{ij} W_{ij} \|\phi(x_i) - \phi(x_j)\|.
\]

In other words the embedding function should map points that are close on the manifold to close points in the low dimensional embedding space. The embedding solution is given by the eigenvectors of the generalized eigenvalue problem

\[
Lw = \lambda Dw,
\]

where the embedding function is given by the eigenvectors corresponding to the smallest \( d \) non-zero eigenvalues. Here the graph \( D \) is the diagonal matrix of the row sum of the weight matrix and \( L \) is the graph Laplacian defined as

\[
L = D - W.
\]

It is interesting to note that classical algorithms for dimensionality reduction such as principal component analysis (PCA) and linear discriminant analysis (LDA) can be obtained as special cases of the Laplacian method through proper choice of the weight matrix. In particular, PCA corresponds to the fully populated weight matrix of ones, where every pair of points are considered neighbors. Similarly, LDA corresponds to the case where two data points are neighbors if and only if they are in the same class.

III. DISCOVERING MANIFOLDS FOR SAR IMAGERY

Manifold based dimensionality reduction techniques have been proposed previously with varying success on radar data. Direct application to the complex valued imagery is problematic, since the phase information in the imagery encodes several nuisance parameters such as materials, imaging platform height and local EM interactions. Therefore we propose to apply first a transformation that is invariant to the nuisance parameters and produces a representation that is independent of nuisance parameters, revealing target geometry. One possibility to use an attributed scattering center abstraction: First detect dominant scattering mechanisms and then encode their location and other properties such as polarization signature as a variable length vector. Figure 1 shows this representation of a Honda Civic data obtained with an X-band radar simulation of 640 MHz bandwidth. Intuitively, we expect a low dimensional structure to be present in this target DNA that manifold methods can key on.

To apply manifold learning methods, we will have to first define appropriate distance metric in this data space. Each data point is the complex baseband SAR image obtained by backprojection over 15 degrees azimuth window with 128x128 pixels at 15 cm resolution. Next, a feature extraction algorithm is utilized for extracting local peaks in the magnitude images. To define the distance between two sets of scattering centers, we employ a computationally simple distance metric that relies on the \( L_2 \) distance between the Parzen type density estimate of the scattering center representation.

\[
d(x_i, x_j) = \|f_{x_i} - f_{x_j}\|^2,
\]

where \( f_{x_i} \) is the normalized density estimate of the set of scattering centers \( \{z_k(x_i)\}_k \) extracted from the image \( x_i \).

\[
f_{x_i}(z) = C \sum_k \exp\left(z - z_k(x_i)\right)/2\sigma^2,
\]

Once distances for the scattering center data representation defined, the neighborhood graph and weight matrix can be calculated using the k-nearest or the exponential method. For various target classes we constructed embedding of the scattering center representation of the SAR imagery on three basis vectors. Figure 2 shows the results of the Laplacian Eigenmap based embedding of the data. The results show clearly the nonlinear single dimensional manifold structure that encodes the pose. The manifold representation also indicates poses that are likely to be confused since these poses become close neighbors in the representation, indicating close distance in the original data space. Figure 3 shows the embedding obtained by the traditional principal component analysis (PCA), which does not capture the relevant geometry with respect to target pose.

IV. PERFORMANCE PREDICTION AND NOISE STATISTICS IN THE EMBEDDING SPACE

For SAR imagery making probabilistic estimates for the class and pose of vehicles in clutter is particularly challenging problem. This is due to the unknown noise statistics in the high dimensional radar data space. While performing scattering center extraction provides certain noise immunity, clutter can lead to false peaks extracted from the data and occlusion of
some of the target peaks. The resulting perturbations in the high dimensional data are extremely hard to parameterize and learn since they are akin to the shot noise commonly observed in optical noise with non-uniform spatial correlation unlike the optical system. As an example, characterization of the Hausdorff distance between two sets in high dimensions in the presence of spatial point process is an open problem. Without a statistical model for clutter perturbations, accurate decisions on posterior probabilities cannot be performed without relying on heuristics.

Manifold learned based dimensionality reduction methods provides a promising direction in modeling and learning the clutter perturbations in the low dimensional embedding space. In this space correlated perturbations at various dimensions are accumulated to small number of real variables. Therefore, one can invoke central limit theorem to argue that in the low dimensional space the perturbations can be modeled as multivariate Gaussian noise model, which is easily parametrized and learned by the covariance matrix of the perturbations. To validate this promising direction we performed preliminary simulation experiments. We consider a single aspect of the Honda Civic class (10 degrees azimuth) and introduced 5% false peaks into the imagery randomly in the bounding box of the target. Next, we use Laplacian Eigenmaps learned from the original dataset to perform dimensionality reduction to three dimensions. The results are given in Figure 4, we observe that the perturbations are centered around the 10 degree pose and show a regular structure akin to Gaussian cloud. To explore goodness of fit of the Gaussian density we consider the Mahalonobis distances between the points in the perturbation cloud. If the cloud density follows multivariate Gaussian then the Mahalonobis distances should follow Chi-square density with three degrees of freedom. Figure 5 gives the quantile-quantile plot of the sample density versus chi-square density which indicates a good fit. We also computed Mardias kurtosis test statistics as 0.56 indicating no rejection of the Gaussian distribution hypothesis.

V. EXAMPLES

A. MANIFOLD LEARNING METHODS FOR CLASSIFICATION

To evaluate the effectiveness of manifold learning methods in ATR class design algorithm, we performed a simulation experiment with two target classes: Honda Civic and Toyota Camry. These two targets are both compact civilian cars with similar features, providing a challenging discrimination task. We have used X-band radar simulation results of 640 MHz bandwidth to get signatures for each target class observed from 360 degree aperture. We formed complex baseband SAR image obtained by backprojection over 15 degrees azimuth window with 128x128 pixels at 15cm resolution. We divided the data set into 90% training and 10% test data. Then we applied PCA and Laplacian Maps (with neighborhood size of K=6 and K=10) to the training data to learn projections onto subspaces of varying dimension. Next, we used a simple nearest neighbor classifier for each embedding to estimate the target class. Figure 6 shows the probability of misclassification
VI. MANIFOLD LEARNING METHODS FOR PERFORMANCE PREDICTION

Sample based methods of performance estimation use cross-validation (e.g. leave-one-out estimate) techniques to construct performance bounds. For the wide-angle SAR problem, the high dimensionality of the signatures causes sample-based estimates of performance to be highly biased. Dimensionality reduction techniques employed in this paper can mitigate this bias. In particular, manifold learning methods reveal geometrical of the high dimensional data and can be useful in predicting performance of inference algorithms that attempt to extract information from samples obtained from the underlying data model. As discussed in section IV, for each of the civilian vehicle datasets we analyze in this work, their high dimensional SAR signatures live close to a one dimensional manifold articulated by the target pose angle $\theta$ and perturbations from the manifold in the embedding space are well approximated by correlated Gaussian random vectors:

\[ \phi(x_i) = \mathcal{M}(\theta_i) + n_i, \]

where $\phi(x_i)$ is the projection of the $i$’th SAR signature vector obtained at azimuth angle $\theta_i$ to the Laplacian eigenspace, $n_i$ is a random vector with zero mean and $d \times d$ covariance matrix $\Sigma_n$. For data collected with azimuth labels, we can fit a parametric model to the recovered manifold structure in the $d$ dimensional embedding space such as a truncated Fourier series, or spline model and estimate the covariance matrix of the fit-errors. This concise parametric model learned from the data can be used in traditional methods of performance estimation such as Cramér-Rao Bound (CRB) analysis. In general, for the parametric manifold model with intrinsic dimension $m$, the entries of the $m \times m$ Fisher information matrix [7] for the estimation of the $d$ dimensional articulating vector $\theta$ can be computed as

\[ J_{kl} = \frac{\partial M^T(\theta)}{\partial \theta_k} \Sigma_n^{-1} \frac{\partial M(\theta)}{\partial \theta_l}. \]

In turn, the Fisher information matrix can be used to bound error covariance of any unbiased estimator $\hat{\theta}(x) : \mathcal{R}^D \rightarrow \mathcal{R}^m$ using

\[ \text{Cov} \left( \hat{\theta}(x) \right) = J^{-1} \text{ is p.s.d.}. \]
For the case of the civilian vehicle manifolds, manifold is parameterized with a single dimensional parameter, and the bound in 3 simplifies to

$$\text{var}\left(\hat{\theta}(x)\right) \geq \frac{1}{J(\theta)}.$$  \hspace{1cm} (4)

To illustrate this manifold learning based method of performance estimation, we used Laplacian Eigenmaps to embed the wide-angle SAR signatures for Honda Civic using an embedding dimension of \(d = 3\). As before, the original signatures were backprojection images computed over 15 degrees azimuth window with 128x128 pixels at 15cm resolution using X-band radar simulation results of 640 MHz bandwidth. Next we used a two term Fourier-Series model to provide a parametric fit to the resulting one-dimensional closed curve and sample covariance estimate for learning the covariance matrix \(\Sigma_x\). Next, we computed the Fisher information and the associated CRB for this parametric model. The results are given in Figure 7. We observe that we predict the performance of the pose estimation method vary periodically according to the true pose angle. The predicted performance is best at cardinal angles of \(0^\circ, 90^\circ, 180^\circ, 270^\circ\) degrees. This is in agreement with empirical results reported in [8], and intuitively appealing since a higher number of scatterers are present at these angles to provide discriminability.

\[ \begin{array}{cc}
\text{Fig. 7. Cramér Rao bound computed from the parametrization of the target manifold.} \\
\end{array} \]

### VII. Conclusions and Future Work

In this work we have considered each vehicle in a separate class when designing the manifold embedding function. The only degree of freedom in this corresponds to pose in azimuth and elevation. An interesting extension is to construct meta-classes of civilian vehicles, SUVs, compact vehicles and learn manifolds for these meta-classes. In this case the manifold dimension is expected to increase since targets within a class will exhibit variations in geometry (width, height, length) as well as ratio of macro features such as trunk, hood, roof, doors. The promise of the Laplacian Eigenmaps is its ability to provide generalized templates (eigenvectors corresponding to different dimensions) to focus on these factors as well as the pose.

The performance prediction method for pose estimation presented in this paper requires the pose angle to be labeled for each data sample. In practice, this labeling could be hard or costly to obtain. Typically, one just have data for a vehicle class from wide set of angles that are uniformly sampled. Then one cannot utilize a parametric model. An interesting research direction is to consider non-parametric methods that estimate the curvature of the data space locally from when the signature space is uniformly sampled. These methods [9] rely on the convergence of the eigenset structure of the graph Laplacian to the Laplacian of the underlying data [10]. Then, as shown in this paper, the local curvature can be used to compute the Fisher information and associated performance metrics.

### References


