On Cooperation in Energy Efficient Wireless Networks: The Role of Altruistic Nodes

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Abstract

In wireless networks with energy limited nodes, multi-hop forwarding is usually exploited to reduce the network energy consumption. In many practical scenarios, however, nodes' selfishness raises doubts on whether each node will be willing to spend its valuable energy in forwarding packets for other users. To analyze this problem, a non-cooperative game theoretic framework is adopted in our work. Using this framework, the critical role of altruistic nodes in encouraging cooperation is established. More specifically, we show that it is sufficient to have a vanishingly small fraction of the nodes to be altruistic, i.e., relay nodes, in order to ensure full cooperation from all the nodes in the network. This result hinges on using the appropriate forwarding policies by the altruistic nodes, as detailed in the sequel. Our work also establishes the sub-optimality of traditional relaying strategies, which ignore the game-theoretic aspect of the problem. An important aspect of our work is that only reward/punishment policies that can be realized on the physical layer are used, and hence, our results establish the achievability of full cooperation without requiring additional incentive mechanisms at the application layer.

Keywords: cooperation, energy efficiency, ad-hoc networks, selfish and altruistic users, non-cooperative game

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I. Introduction

Several recent works have shown that user cooperation plays a fundamental role in wireless networks. From an information theoretic perspective, the idea of cooperative communications can be traced back to the relay channel [2]. More recent works have generalized the cooperation strategies proposed in [2] and established the utility of cooperative communications in many relevant practical scenarios [3]–[5]. In another line of work, Gupta and Kumar have shown that the simplest form of physical layer cooperation, namely multi-hop forwarding, is an indispensable element in achieving the optimal capacity scaling law in networks with asymptotically large numbers of nodes [6]. Multi-hop forwarding has also been shown to offer significant gains in the efficiency of energy limited wireless networks [7], [8]. These **physical layer** studies assume that each user is willing to expend energy in forwarding packets for other users. This assumption is reasonable in a network with a central controller with the ability to enforce the optimal cooperation strategy on the different wireless users. The popularity of ad-hoc networks and the increased programmability of wireless devices, however, raises serious doubts on the validity of this assumption, and hence, motivates investigations on the impact of **user selfishness** on the performance of wireless networks.

One important thrust in these efforts focuses on designing high-level protocols that prevent users from misbehaving and/or provide incentives for cooperation. To prevent misbehavior, several protocols based on reputation propagation have been proposed in the literature, e.g., [9], [10]. Other works have used ideas from micro-economy to construct protocols that reward cooperation. In [11], for example, a protocol based on virtual currency is proposed. Overall, these protocols are based on ideas rooted in game theory, but, in most cases, are not derived from the equilibrium perspective and are hard to analyze, due to the complicated underlying network models.

Another thrust of research analyzes the impact of user selfishness from a game theoretic perspective, e.g., [12], [13]. Since the problem is typically too involved, several simplifications to the network model are usually made to facilitate analysis and allow for extracting insights. For example, in [12], the wireless nodes are assumed to be interested in maximizing energy efficiency. At each time slot, a certain number of nodes are randomly chosen and assigned to serve as relay nodes on the source-destination route. The authors derive a Pareto optimal operating point and show that a certain variant of the well known TIT-FOR-TAT algorithm converges to this point. In [12], the authors assume that the transmission of each packet costs the same energy and each session uses the same number of relay nodes. Another example is [13], which studies the Nash equilibrium of packet forwarding in a static network by taking the network topology into consideration. More specifically, the authors assume that the transmitter/receiver pairs in the network are always fixed and derive the equilibrium conditions for both cooperative and non-cooperative strategies. Similar to [12], the cost of transmitting each packet is assumed fixed. It is worth noting that most, if not all of, the works in this thrust utilize the repeated game formulation, where cooperation among users is sustainable by credible punishment for deviating from the cooperation point.

In this paper, we adopt a game-theoretic approach to analyze the impact of user selfishness on the *energy efficiency* of wireless networks. The energy efficiency of ad hoc networks has been studied in [7], [8] under the assumption that all the nodes are interested in minimizing the overall energy consumption of the network. In the wireless setting, the transmission power must scale polynomially with the transmitter-receiver distance in order to guarantee reliable decoding of the information. In the no-cooperation scenario, where the wireless users are not willing to relay packets for each other, each user must reach its destination directly. It is then easy to see that this requires significantly higher energy levels, as compared with the globally optimal solution characterized in [7], [8]. This observation motivates our work where we consider wireless networks consisting of both selfish nodes, interested in minimizing their own energy consumption, and altruistic nodes, interested in minimizing the overall energy expenditure of the network. In practice, those altruistic nodes may correspond to wireless relay nodes endowed with more processing powers and tasked with fostering cooperation among the wireless users. Unlike the traditional approach, these relay nodes are not assumed to have any control on the behavior of the selfish nodes directly. However, the sequel establishes the critical role of these altruistic nodes in encouraging cooperation among the selfish nodes. More specifically, our work establishes the sub-optimality of traditional relaying strategies, which ignore the game theoretic aspect of the problem, and characterizes forwarding strategies that allow the relay nodes to change the cost function of the other selfish nodes such that full cooperation emerges at the equilibrium point. One of the central questions addressed in our work is: how many altruistic nodes are sufficient to guarantee full cooperation, and hence, achieve the same energy efficiency as the globally optimal solution? Interestingly, as shown in the sequel, full cooperation becomes a Nash Equilibrium in networks where only a vanishingly small fraction of the total number of nodes are altruistic. One distinguishing aspect of our work is that we confine our strategies to the physical layer and avoid introducing elements, like virtual currency, which may add significant complexity to the higher layers.

The rest of this paper is organized as follows. In Section II, we present our network model and associated notation. Section III establishes the role of one altruistic, i.e., relay, node in facilitating cooperation in a small network composed of two source-destination pairs. In section IV, we proceed to the large network scenario where a sufficient condition on the fraction of altruistic nodes needed to ensure full cooperation is derived. Finally, we offer some concluding remarks in Section V.

II. Network Model

We consider an ad-hoc wireless network with a set of half-duplex wireless nodes $\mathcal{N}$, each located at $X_i, i \in \mathcal{N}$. Our nodes are assumed to have very low rate requirements but a stringent energy budget. Among these $N = |\mathcal{N}|$ wireless nodes, there exists a set of transmitters $\mathcal{T} \subseteq \mathcal{N}$ that wish to send packets to different destinations. A time division multiple access (TDMA) approach is used where the nodes will take turns in transmitting their packets (no frequency reuse). This multiple access scheme is known to be optimal from a minimum energy per bit perspective [7], [8]. In the frame assigned to the transmitter i, it will first send a relay request to nodes in the route (we will specify the routing scheme in the sequel) to its destination. Once the relay request is received, transmitter $j \in \mathcal{T}$ will decide whether to accept or reject the relay request and send back its decision. We adopt the discrete additive white Gaussian noise (AWGN) model, i.e., when node i transmits, the signal received by node j at time k is given by

$$y_j[k] = h_{ij}x_i[k] + z_j[k],$$

where the channel gain h_{ij} from node i to node j is $h_{ij}^2 = d_{ij}^{-\gamma}$, $d_{ij} = |X_i - X_j|$ is the distance between the two nodes, $\gamma > 2$ is the channel attenuation coefficient, $z_j[k]$ is the zero-mean unit-variance Gaussian noise at node j. The noise process is assumed to be spatially and temporally white.

Our work focuses on analyzing the decision making process of accepting/rejecting relay requests generated by the different transmitters. In a fully cooperative scenario, each node **will accept** all relay requests in order to minimize the **overall** energy consumption of the network. However, the selfishness of the nodes in our network leads to a significantly more complicated process. By accepting a relay request, the individual node is committing to spend some of its valuable energy in forwarding other nodes' packets. From a selfish perspective, therefore, each node will be tempted to reject the relay requests in order to minimize its own energy consumption. This motivates our formulation which differentiates between two classes of nodes in our network, namely selfish and altruistic nodes. A selfish node is assumed to be interested in minimizing its own energy consumption whereas an altruistic node is interested in minimizing the overall energy expenditure of the network. In practice, those altruistic nodes may correspond to wireless relay nodes tasked with rewarding cooperation in the network. This correspondence justifies our assumption that the altruistic nodes are endowed with more processing power, storage, and monitoring capabilities as compared with the selfish ones. In the sequel, we will refer to the set of altruistic nodes as $\mathcal{U} \subseteq \mathcal{N}$ and let $\theta(N) = |\mathcal{U}|$ be the number of such nodes.

We model the behavior of **selfish** nodes as a static non-cooperative game $\mathcal{G} = \{\mathcal{M}, \mathcal{F}, \mathcal{C}\}$, where $\mathcal{M} = \mathcal{T} \setminus \mathcal{U}$ is the set of selfish transmitters, $\mathcal{F}_i$ is the strategy space of node i, and $\mathcal{C}_i$ is the cost function of node i. The strategy space is given by $\mathcal{F}_i = \{p_i : p_i \in [0, 1]\}$, where p_i is the probability that node i will accept a relay request. Here, we observe that our strategy space allows for mixed strategies but does not allow the selfish nodes to offer differentiated service based on the origin of the relay request. In our formulation this latter feature is reserved for the altruistic nodes. More specifically, an altruistic node will monitor the behavior of the selfish nodes and accept the relay request of selfish node i with probability $g(p_i)$. Under the assumption that the function $g(\cdot)$ is known *a-priori* by all the nodes, one should construct $g(\cdot)$ in order to encourage cooperation between all nodes (i.e., convince node i to work with a higher p_i). To simplify our task, we limit ourselves to linear functions of the form $g(p_i) = p_i + (1 - p_i)p_{nc}$, where $p_{nc} \in [0, 1]$. Intuitively, this form can be interpreted as a mixed strategy where the altruistic node **deterministically** accept the relay requests originated from **cooperative** nodes (i.e., $g(1) = 1$), but only accepts the requests of non-cooperative node with probability p_{nc} (i.e.,$g(0) = p_{nc}$). We note that $p_{nc} = 1$ corresponds to the **traditional relaying** where the altruistic nodes accept all the relay requests. For simplicity of presentation, and motivated by the results of [7], [8], our cost function corresponds only to the transmission energy[1]. Therefore, the cost function is given by $\mathcal{C}_i = E_{i,tr} + E_{i,re}$ where $E_{i,tr}$ is the energy spent on transmitting the i^{th} node own packets, and $E_{i,re}$ is the energy spent by the i^{th} node on forwarding packets for other transmitters.

A set of strategies $\mathbf{p}^* = (p_1^*, ..., p_M^*)$ is said to constitute a Nash Equilibrium (NE) if for any $i \in \mathcal{M}$

$$C_i(p_i^*, \mathbf{p}_{-i}^*) \leq C_i(p_i^{'}, \mathbf{p}_{-i}^*), \forall p_i^{'} \in [0, 1], \tag{1}$$

where $\mathbf{p}_{-i}$ is the strategies of all the players except user i. Our work seeks to characterize the Nash Equilibria of the aforementioned non-cooperative game in the two extreme cases of small and large networks. This characterization establishes rigorously the critical role of wireless altruistic nodes in stimulating cooperation in ad-hoc networks.

III. The Small Network Scenario

Aiming towards a succinct development of our ideas, we start in this section with a small network composed of five nodes as shown in Figure 1. In this setup, $\mathcal{N} = \{1, 2, 3, 4, r\}$, $\mathcal{T} = \{1, 3, r\}$, and $\mathcal{U} = \{r\}$. In other words, nodes $1, 3$ take turns to transmit information to nodes $2, 4$ at rate R and r is our altruistic relay node which does not have information to send (as illustrated in Figure 2). Due to the strict energy limitation imposed on the nodes, we operate in the asympticitally low rate regime, i.e., $R \to 0$, which is known to be the most energy efficient operating point [14].

The following lemma motivates the idea of using the relay node to stimulate cooperation.

Lemma 1: In the absence of the relay node r, no-cooperation is the only NE of our static game. Moreover, no-cooperation remains the only NE under the repeated game setup if it is known *a-priori* that the nodes will only interact for a finite number of stages and the nodes know the end of the game.

[1] As pointed out in [7], [8], the transmission energy is the dominant factor in the energy consumption in the large scale limit. We also note that our conclusions will still hold if the receiving energy is incorporated into our model but the development of the results will be more involved.

Proof: Let's consider the last stage in the repeated game scenario. In this stage, it is easy to see that the optimal strategy for any selfish node $i \in \mathcal{M}$ is to use adopt $p_i = 0$. The reason is that, for any other $p_i \neq 0$, node i can reduce $E_{i,re}$ without affecting $E_{i,tr}$, and hence, reduce its overall cost by adopting $p_i = 0$. The selfish wireless users can apply the same reasoning backward until the first stage. No cooperation at each stage is, therefore, the only Nash equilibrium. ■

At this no-cooperation equilibrium, each user will have to use sufficient transmission energy to allow the packet to reach the destination directly. For $\gamma > 2$, this implies a strictly higher energy consumption by each user, as compared to the globally optimal solution. By introducing the node r, which adopts the appropriate cooperation strategy, we will show in the following that cooperation emerges at the NE, which translates into a significant reduction in the energy per bit requirements. Note that if the relay always helps, the reasoning in Lemma 1 still applies, and hence, no-cooperation remains the only NE.

A. The Cooperation Scheme

Before proceeding further, we describe here the **underlying** cooperation scheme used in this section. Assuming that a certain node decided to cooperate with the source node, the small size of our network allows for using a more sophisticated approach, as compared with simple multi-hop forwarding. We adopt the classical Decode-Forward (DF) algorithm from [2], [15]. We illustrate in this section DF cooperation in the traditional three terminal setting (i.e., one source, one destination, and one relay which is always willing to help). This setting is sufficient to capture the main idea behind DF cooperation and, moreover, the following section outlines the generalized scenario with multiple helpers. In this setting, the relay node attempts to decode the information from the source first, then the source and the relay cooperate in transmitting the information to the destination. The basic idea is that signals from the source and the relay will add up coherently at the receiver, and hence, we can reap the beam-forming gain. Without loss of generality, we let node 1 be our source, node 2 be our destination, and node r be our relay.

In the absence of the relay node, the source-destination channel capacity and minimum energy per bit are given by [16]

$$C = \frac{1}{2}\log_2(1 + h_{12}^2 P), \quad E_1 = \lim_{C \to 0} \frac{P}{C} = \frac{2\ln 2}{h_{12}^2}, \tag{2}$$

where h_{12} is the source-destination channel gain and P is the normalized transmit power (assuming a unit variance AWGN process). Throughout the sequel, we will simplify our analysis by assuming that a capacity achieving code is used, i.e., $R = C$. This assumption will allow for using the minimum energy per bit, e.g., (2), as our figure of merit.

DF cooperation is illustrated in Figure 3. It is straightforward to see that this scheme offers a performance gain over the direct transmission benchmark if and only if $h_{1r} > h_{12}$ [2]. In this case, the relay node will be able to decode the message after receiving $\lceil \mu L \rceil$ symbols [4], where L is the length of the code word and

$$\mu = \lim_{P \to 0} \frac{\log_2(1 + h_{12}^2 P)}{\log_2(1 + h_{1r}^2 P)} = \frac{h_{12}^2}{h_{1r}^2}.$$

After successful decoding, the relay starts sending the decoded information using the same codebook as the source node with power $\alpha_r^2 P$ whereas the source node now lowers its transmit power to $\alpha_1^2 P$ ($\alpha_1 \leq 1$ and $\alpha_r \leq 1$). The signal received at the destination during this phase is

$$y_2[k] = h_{12}\alpha_1 x_1[k] + h_{r2}\alpha_r x_1[k] + z_2[k].$$

Here α_1, α_r are obtained by solving the following optimization problem

$$\begin{aligned} \min \quad & \alpha_1^2 + \alpha_r^2, \\ s.t. \quad & h_{12}\alpha_1 + h_{r2}\alpha_r = h_{12}, \alpha_1, \alpha_r \in [0,1]. \end{aligned} \tag{3}$$

This constrained optimization minimizes the total transmit power while maintaining the same effective capacity, from the destination perspective. Straightforward calculations yield the following optimal values for α_1, α_2, the energy per bit expended by node 1, and the minimum **total** energy per bit expended by the network (i.e., nodes 1 and r), under DF cooperation

$$\begin{aligned} \alpha_1 &= \frac{h_{12}^2}{h_{12}^2 + h_{r2}^2}, \alpha_r = \frac{h_{12}h_{r2}}{h_{12}^2 + h_{r2}^2}. \\ E_{1,s} &= \frac{2\ln 2}{h_{1r}^2} + \left(1 - \frac{h_{12}^2}{h_{1r}^2}\right)\frac{2\ln 2 h_{12}^2}{(h_{12}^2 + h_{r2}^2)^2}, E_t = \frac{2\ln 2}{h_{1r}^2} + \left(1 - \frac{h_{12}^2}{h_{1r}^2}\right)\frac{2\ln 2}{h_{12}^2 + h_{r2}^2}. \end{aligned} \tag{4}$$

Comparing (2) and (4) shows that, as expected, the relay node offers gains in the energy consumption for $h_{1r} > h_{12}$ (i.e., setting $h_{1r} = h_{12}$ yields $E_t = E_1$).

B. Equilibrium Analysis

To obtain the equilibrium point of our non-cooperative game, it is sufficient to calculate the cost functions under the pure strategies, i.e., cooperation and no-cooperation, as shown in the Table I.

1)(no-cooperation, no-cooperation)

In this case, the relay will help the two nodes with probability p_{nc} ($g(0) = p_{nc}$). Using the results developed in the previous section on DF cooperation, we can see that the average energy per bit required for the two nodes are

$$C_{1,nn} = (1 - p_{nc})E_1 + p_{nc}E_{1,s}, \quad C_{3,nn} = (1 - p_{nc})E_3 + p_{nc}E_{3,s},$$

where $E_3, E_{3,s}$ can be obtained from (2) and (4) by replacing subscripts 1 and 2 with 3 and 4, respectively.

2)(cooperation, no-cooperation)

Let's consider first a packet transmitted by node 1. Since the relay node always cooperates ($g(1) = 1$) but node 3 doesn't, this scenario reduces to the classical three terminal DF cooperation described in section III-A, and hence, the energy consumption for node 1 on its packet is $E_{1,tr} = E_{1,s}$ and the energy consumption by node 3 on relaying is $E_{3,re} = 0$.

On the other hand, when a packet is generated by node 3, we have two distinct scenarios. When the relay doesn't cooperate, which happens with probability $1 - p_{nc}$, the energy consumption of the nodes can be written as φ_1, $E_3 - \varphi_3$ respectively, where

$$\varphi_1 = \Big(1 - \frac{h_{34}^2}{h_{31}^2}\Big)\frac{h_{14}^2 h_{34}^2}{(h_{34}^2 + h_{14}^2)^2}E_3, \quad \varphi_3 = \Big(1 - \frac{h_{34}^2}{h_{31}^2}\Big)\frac{2h_{34}^2 h_{14}^2 + h_{14}^4}{(h_{34}^2 + h_{14}^2)^2}E_3, \tag{5}$$

based on the DF cooperation scheme with node 1 now playing the role of the helper node. If the relay node also cooperates, which happens with probability p_{nc}, the energy consumption of the nodes depends on the decoding order of node 1 and the relay nodes. Here, we assume that both h_{31}, h_{3r} are larger than h_{34} since otherwise this scenario is reduced to the three terminal case considered earlier. If $h_{31} > h_{3r}$, the operation sequence is shown in Figure 4. After a

$$\mu_{3,1}^{'} = \lim_{P_3 \to 0} \frac{\frac{1}{2}\log_2(1 + h_{34}^2 P_3)}{\frac{1}{2}\log_2(1 + h_{31}^2 P_3)} = \frac{h_{34}^2}{h_{31}^2}$$

fraction of the frame, node 1 will decode the information and start to help. In this period, the relay node will still keep on listening, and

$$y_4[k] = h_{14}\alpha_1^{'}x_3[k] + h_{34}\alpha_3^{'}x_3[k] + z_4[k], \quad y_r[k] = h_{1r}\alpha_1^{'}x_3[k] + h_{3r}\alpha_3^{'}x_3[k] + z_r[k],$$

where $\alpha_1^{'} = \frac{h_{14}h_{34}}{h_{14}^2+h_{34}^2}, \alpha_3^{'} = \frac{h_{34}^2}{h_{14}^2+h_{34}^2}$ are the solution to an optimization problem similar with (3). Then, after an additional

$$\mu_{3,2}^{'} = \lim_{P_3 \to 0} \frac{\frac{1}{2}\log_2(1+h_{34}^2 P_3) - \frac{\mu_{3,1}^{'}}{2}\log_2(1+h_{3r}^2 P_3)}{\frac{1}{2}\log_2\left(1+(h_{1r}^2\alpha_1^{'2} + h_{3r}^2\alpha_3^{'2})P_3\right)} = \frac{(h_{31}^2 - h_{3r}^2)(h_{14}^2 + h_{34}^2)^2}{(h_{1r}^2 h_{14}^2 + h_{34}^2 h_{3r}^2)^2 h_{31}^2} \tag{6}$$

fraction of the frame, the relay will be able to decode the message [4], [15], and then join in transmitting. In this period, the signal that node 4 receives is given by

$$y_4[k] = h_{14}\beta_1 x_3[k] + h_{r4}\beta_2 x_3[k] + h_{34}\beta_3 x_3[k] + z_4[k]. \tag{7}$$

Similar to (3), the value of β_is are obtained as $\beta_1 = \frac{h_{14}h_{34}}{h_{14}^2+h_{r4}^2+h_{34}^2}, \beta_2 = \frac{h_{r4}h_{34}}{h_{14}^2+h_{r4}^2+h_{34}^2}, \beta_3 = \frac{h_{34}^2}{h_{14}^2+h_{r4}^2+h_{34}^2}$ which correspond to the solution to the following optimization problem

$$\begin{aligned} \min \quad & \sum_{i=1}^{3} \beta_i^2, \\ s.t. \quad & h_{14}\beta_1 + h_{r4}\beta_2 + h_{34}\beta_3 = h_{34}, \beta_i \in [0,1]. \end{aligned} \tag{8}$$

Hence if $h_{31} > h_{3r}$, the energy consumptions of the two nodes are

$$\begin{aligned} \varphi_1^{'} &= \mu_{3,2}^{'}\alpha_1^{'2}E_3 + (1-\mu_{3,1}^{'} - \mu_{3,2}^{'})\beta_1^2 E_3, \\ \varphi_3^{'} &= \mu_{3,1}^{'}E_3 + \mu_{3,2}^{'}\alpha_3^{'2}E_3 + (1-\mu_{3,1}^{'} - \mu_{3,2}^{'})\beta_3^2 E_3. \end{aligned} \tag{9}$$

The case where $h_{31} \leq h_{3r}$ is similar, with the decoding order of node 1 and the relay node interchanged. The energy consumption of the nodes can, therefore, be written as

$$\varphi_1^{''} = (1-\mu_{3,1}^{''} - \mu_{3,2}^{''})\beta_1^2 E_3, \quad \varphi_3^{''} = \mu_{3,1}^{''}E_3 + \mu_{3,2}^{''}\alpha_3^{''2}E_3 + (1-\mu_{3,1}^{''} - \mu_{3,2}^{''})\beta_3^2 E_3.$$

Combining these two cases together, we obtain the following relationship for the energy consumption of the two nodes

$$\varphi_1^{'''} = \varphi_1^{'} I(h_{31} > h_{3r}) + \varphi_1^{''} I(h_{31} \leq h_{3r}), \quad \varphi_3^{'''} = \varphi_3^{'} I(h_{31} > h_{3r}) + \varphi_3^{''} I(h_{31} \leq h_{3r}),$$

where $I(\cdot)$ is the indicator function. Finally, the cost functions of the two nodes under the (cooperation, no-cooperation) pure strategy is

$$C_{1,cn} = E_{1,s} + (1-p_{nc})\varphi_1 + p_{nc}\varphi_1^{'''}, \quad C_{3,cn} = (1-p_{nc})(E_3 - \varphi_3) + p_{nc}\varphi_3^{'''}.$$

3)(no-cooperation, cooperation)

For this case, we can follow the same steps as above, and get the cost functions of the nodes

$$C_{1,nc} = (1-p_{nc})(E_1 - \psi_1) + p_{nc}\psi_1^{'''}, \quad C_{3,nc} = E_{3,s} + (1-p_{nc})\psi_3 + p_{nc}\psi_3^{'''}.$$

Here the expressions for $\psi_i, \psi_i^{'''}$ are similar with $\varphi_i, \varphi_i^{'''}$ with the roles of node 1 and node 3 interchanged.

4)(cooperation, cooperation)

It's easy to see that when node 3 transmits, the transmission energy of the nodes are $\varphi_1^{'''}, \varphi_3^{'''}$ respectively. When node 1 transmits, the transmission energy of the nodes are $\psi_1^{'''}, \psi_3^{'''}$ respectively. Hence the cost functions are

$$C_{1,cc} = \psi_1^{'''} + \varphi_1^{'''}, \quad C_{3,cc} = \psi_3^{'''} + \varphi_3^{'''}.$$

Theorem 2: If $C_{1,cn} < C_{1,nn}$ or $C_{3,nc} < C_{3,nn}$, then the relay node will stimulate cooperation (i.e., no-cooperation is not a NE anymore). More strongly, If $C_{1,cc} < C_{1,nc}, C_{3,cc} < C_{3,cn}, C_{1,cn} < C_{1,nn}, C_{3,nc} < C_{3,nn}$, then full cooperation is the only NE.

Proof: If $C_{1,cn} < C_{1,nn}$, node 1 can reduce its cost by deviating from no-cooperation to cooperation. Similarly, if $C_{3,cn} < C_{3,nn}$, node 3 can reduce its cost by deviating from no-cooperation to cooperation. If $C_{1,cc} < C_{1,nc}, C_{3,cc} < C_{3,cn}, C_{1,cn} < C_{1,nn}, C_{3,nc} < C_{3,nn}$, no-cooperation is a dominated strategy, and hence, full-cooperation is the only equilibrium with $\mathbf{p}^* = \mathbf{1}$. When pure strategies do not yield a NE, the users will adopt mixed strategies to arrive at an equilibrium point. For our two users matrix form game, we can readily compute those mixed strategies. At the NE, user 1 will choose p_1^* such that user 3 will have the same cost under either cooperation or no-cooperation:

$$C_3^* = (1-p_1^*)C_{3,nn} + p_1^* C_{3,cn} = (1-p_1^*)C_{3,nc} + p_1^* C_{3,cc}.$$

User 3 will choose p_3^* in a similar way, hence

$$p_1^* = \frac{C_{3,nn} - C_{3,nc}}{C_{3,cc} - C_{3,cn} + C_{3,nn} - C_{3,nc}}, \quad p_3^* = \frac{C_{1,nn} - C_{1,cn}}{C_{1,cc} - C_{1,nc} + C_{1,nn} - C_{1,cn}}.. \tag{10}$$

and $C_1^* = (1 - p_3^*)C_{1,nn} + p_3^* C_{1,cn}$. ■

Under the equilibrium strategy (p_1^*, p_3^*), the average **total** energy per bit is $E_t = \frac{1}{2}\big(C_1^* + C_3^* + E_r^*\big)$, where E_r^* is the energy consumption of the relay node. Therefore, the altruistic node determines the optimal strategy which corresponds to finding $p_{nc}^* = \operatorname*{argmin}_{p_{nc}\in[0,1]} E_t$. In summary, the main insight from Theorem 2 is that, by adopting the appropriate cooperation strategy, the altruistic node can stimulate cooperation among nodes 1 and 3 under certain conditions on the *topology* of the networks, which determines the channel gain between every pair of nodes.

C. Numerical Results

Armed with Theorem 2, we now obtain numerical results for the network shown in Figure 5. In this example, we place nodes 1 and 3 (the transmitters) at coordinates $(0, 0.1), (0, -0.1)$, respectively, and nodes 2 and 4 (the receivers) at $(1, 0.2), (1, -0.2)$, respectively. The relay node is placed at position (x, y). In the simulation, we let $\gamma = 3$. Figure 6 shows the region corresponding to the optimal location(s) for the relay node, i.e., if the relay node is located in this region full-cooperation becomes the only equilibrium of our non-cooperative game. Figure 7 further illustrates the gain offered by the altruistic node as a function of its position. As benchmarks, we use 1) the average transmission energy per bit, E_{co}, in the idealistic cooperative network where node 1 and 3 are not selfish and 2) the dumb relaying strategy where the altruistic node **always** helps the sender (hence, based on Lemma 1, no-cooperation is the only equilibrium among the transmitters.). We also define

$$\zeta_t = 10\log_{10}\left\{\frac{E_t(p_{nc}^*)}{(E_1 + E_3)/2}\right\}, \zeta_{co} = 10\log_{10}\left\{\frac{E_{co}}{(E_1 + E_3)/2}\right\}$$

to quantify the gain resulting from cooperation in our non-cooperative game (ζ_t) and the idealistic case (ζ_{co}). In Figure 7, we show the gain by letting this node move at the x-axis, that is the position of the relay is $(x, 0)$. We can see that the introduction of the relay node reduces the energy consumption of the network at the equilibrium significantly (e.g., a gain as large as 7 dB). It is also shown that full-cooperation is the NE when the relay node is between -0.2 and 0.7, since the curve of ζ_t coincides with that of ζ_{co}.

IV. LARGE RANDOM NETWORK

Now we proceed to ad-hoc networks with large numbers of nodes N. We consider the case where these N nodes are randomly distributed on the surface of a sphere of area $A(N)$ according to a uniform distribution as shown in Figure 8. We keep the density ρ of the node as constant, and hence, as the number of nodes in the network N increases, the area of the network $A(N) = N/\rho$ increases accordingly. This corresponds to the extended network model considered in the literature [7], [8]. Without loss of generality, we let $\rho = 1$ resulting in $A(N) = N$. As before, we let X_i be the position of node i and $d_{ij} = |X_i - X_j|$ be the distance between nodes i and j. In our model, we allow all the nodes in the set $\mathcal{N}$ to be transmitters, that is $\mathcal{T} = \mathcal{N}$.

It is easy to see that the design and analysis of the DF cooperation scheme, adopted in the small network scenario, will become intractable in this large network. Therefore, in the large network, we will limit ourselves to the simplest form of cooperation, namely packet forwarding. More specifically, the packets propagate in the network in a hop-by-hop fashion. We assume that, at each frame, each node randomly picks another node in the network as its destination and sends a packet to it. Also at the beginning of each transmission, the source node identifies a route to the destination. Let Γ_{ij} be the set of nodes in the route from the source i to the destination j. We use the routing scheme described in [6]: divide the whole area into small Voronoi cells[2], whose size is properly chosen to guarantee that there is at least one node at each cell, and the packet hops from the source to the destination through the cells that have intersection with a line connecting the source and the destination. If a particular cell has multiple nodes, the relay request is assigned randomly to any of the nodes in this cell. Before sending a message to its destination j, source i first broadcasts a relay request to the nodes on the route Γ_{ij}, then node k on this route will decide whether to accept this request or not and sends back a response to the source node. We assume that the nodes always respond[3] to the relay requests and are not allowed

[2]Given a set of points $a_1, ..., a_n$ in the surface of sphere, the Voronoi cell $V(a_i)$ is the set of all points which are closer to a_i than to any of the other a_j's [6].

[3]For the sake of simplicity, we ignore the protocol overhead, which can be easily incorporated into our model without changing our main conclusions.

to misbehave later. Furthermore, we assume that if a selfish node accepts a relay request, it will use a sufficient power level to ensure successful decoding **only** at the next hop on the route (this assumption is consistent with the node selfishness). Therefore, if any node on the route rejects the relay request, the route is interrupted. In this case, the source node will transmit the packet to the destination in one hop (i.e., direct transmission). Clearly, one can envision more sophisticated routing strategies where the source attempts to establish another route when the first route is interrupted. Here, we limit ourselves to the aforementioned routing strategy for the sake of simplicity, and analytical tractability. It also appears that the potential gains from more sophisticated routing strategies will be rather marginal due to the selfishness of the majority of nodes in our network (i.e., every selfish nodes will not be willing to **waste** energy in forwarding a packet for a long hop). Finally, the source node is assumed to have no prior information about the identity of the nodes in the routes (whether a certain node is selfish or altruistic). On the other hand, every node knows the fraction of altruistic nodes present in the network and the functional form of the strategy employed by those nodes (i.e., $g(.)$).

To ensure successful decoding of the transmitted signal at a receiver d_{ij} away from the transmitter, the energy spent on sending one packet (in the low rate regime) is give by $E_{tr}(d_{ij}) = c_1/h_{ij}^2 = c_1 d_{ij}^{\gamma}$, where c_1 is a constant. Therefore, the required energy per hop scales polynomially, in the hop length, with order γ. It follows that, if all the nodes in the network are cooperative and accept all the relay requests, the optimal energy per packet scales as $\Theta\left(\sqrt{N}\log^{(\gamma-1)/2}(N)\right)$ as shown in [7], [8][4]. This optimal scaling is achieved by the routing scheme used here. On the other hand, if all the nodes are selfish then the result in Lemma 1 will hold, implying that no-cooperation will be the unique NE. At this equilibrium point, each source node must send the packet to the destination in one hop resulting in an average energy per packet which scales as $\Theta(N^{\gamma/2})$. In the following, we show how altruistic nodes can be utilized to close the huge gap between those two scenarios.

We now recall that there are $\theta(N)$ altruistic nodes among these N nodes, that is $|\mathcal{U}| = \theta(N)$, as

[4]In this paper, we use the following Knuth's asymptotic notations: 1)$f(N) = o(g(N))$ means $\forall c > 0, \exists N_0, \forall N > N_0, f(N) < cg(N)$, 2) $f(N) = \omega(g(N))$ means $\forall c > 0, \exists N_0, \forall N > N_0, g(N) < cf(N)$, 3) $f(n) = \Theta(g(N))$ means $\exists c_1, c_2 > 0, N_0, \forall N > N_0, c_1 g(N) \leq f(N) \leq c_2 g(N)$.

shown in Figure 8. For simplicity, in the function $g(p_i) = p_i + (1 - p_i)p_{nc}$, we set the parameter p_{nc} to be 0, that is $g(p_i) = p_i$. Let $\hat{\Psi}_i = \{j : j \neq i, j \in \mathcal{N}, \text{ s.t. } \exists k \in \Gamma_{ij}, k \in \mathcal{U}\}$, which means that $\hat{\Psi}_i$ is the set of possible destinations for node i such that the route between it and node i includes at least one altruistic node.

The probability that node i can transmit its packet to the destination j through the relays is $\prod_{k \in \Gamma_{ij}} p_k$ (note that, if $k \in \mathcal{U}, p_k = p_i$). Let $i^{'}(j)$ be the first node on the route Γ_{ij}, then with probability $\prod_{k \in \Gamma_{ij}} p_k$, the energy that node i spends on sending this packet is $c_1 d^{\gamma}_{ii'(j)}$. If the relay request is rejected, which happens with probability $1 - \prod_{k \in \Gamma_{ij}} p_k$, node i has to transmit the packet directly to its destination j with an energy expenditure of $c_1 d^{\gamma}_{ij}$. Therefore, the expected energy that node i spends on sending its own packet is

$$E_{i,tr} = \frac{1}{N-1}\Big\{\sum_{\substack{j=1\\ j\neq i}}^{N}\Big[\prod_{k\in\Gamma_{ij}} p_k c_1 d^{\gamma}_{ii'(j)} + \Big(1 - \prod_{k\in\Gamma_{ij}} p_k\Big) c_1 d^{\gamma}_{ij}\Big]\Big\},$$

since node i will choose its destination among other $N-1$ nodes with equal probability. Besides spending energy for sending its own packets, node i also needs to relay packets for the other nodes. Let $\Lambda_i = \{\Gamma_{kj} : i \in \Gamma_{kj}, k, j \in \mathcal{N}, k \neq i, j \neq i\}$ be the set of routes that include node i. Λ_i is a random set, but at each TDMA super-frame which includes one slot assigned to each source node, $|\Lambda_i|$ is upper-bounded by N, since there are at most N relay requests asking for node i to help. Let $\Gamma \in \Lambda_i$ be one of the routes and $\Gamma(i)$ be node i's direct next hop in the route Γ. We have $E_{i,re} = \mathbb{E}\Big\{\sum_{\Gamma\in\Lambda_i} p_i \prod_{\substack{k\in\Gamma\\ k\neq i}} p_k c_1 d^{\gamma}_{i\Gamma(i)}\Big\}$, here $\mathbb{E}\{\cdot\}$ means the expectation.

Hence the total cost for node i is

$$C_i = \frac{1}{N-1}\Big\{\sum_{\substack{j=1\\ j\neq i}}^{N}\Big[\prod_{k\in\Gamma_{ij}} p_k c_1 d^{\gamma}_{ii'(j)} + \Big(1 - \prod_{k\in\Gamma_{ij}} p_k\Big) c_1 d^{\gamma}_{ij}\Big]\Big\} + \mathbb{E}\Big\{\sum_{\Gamma\in\Lambda_i} p_i \prod_{\substack{k\in\Gamma\\ k\neq i}} p_k c_1 d^{\gamma}_{i\Gamma(i)}\Big\}. \qquad (11)$$

Now, we proceed to establish a sufficient condition on $\theta(N)$ that ensures full cooperation in the network. Since we consider random networks, our results hold in a probabilistic sense, i.e., we use the notion *w.h.p* to mean that a certain result is true for any realization of the random network with probability which goes to 1 as the number of nodes in the network increases. First, we need to recall some preliminary results on the topology of random network.

Lemma 3 (see [6]): Let $r(N)$ be the radius of a disk with area $100\log N$ in the surface of a sphere. If we divide the total area $A(N) = N$ into Voronoi cells, each contains a disk of area $100\log N$ and is contained in a disk with radius $2r(N)$, then there exists a sequence $\delta(N) \to 0$, such that Prob(every cell contains a node) $\geq 1 - \delta(N)$.

Based on the described routing scheme, the packet will hop from cell to the adjacent cell. Since the radius of each cell, needed to ensure the presence of at least one node, is at most $2r(N)$, the maximum distance of any hop is $8r(N)$. Hence, this lemma shows the existence of a constant $c_2 > 0$ such that the distance of any hop in the route is at most $c_2\sqrt{\log N}$. That is, for any route Γ_{ij}, we have

$$d_{ii'(j)} \leq c_2\sqrt{\log N}, \quad d_{i\Gamma(i)} \leq c_2\sqrt{\log N}. \tag{12}$$

Next, we focus on the set of destination nodes which are far away from the source i, since the transmission to these destinations requires large energy expenditure, especially if the node has to transmit the packet directly. Let $\mathcal{D}_i = \{j : j \in \mathcal{N}, d_{ij} \geq \sqrt{N}/4\}$, which is the set of nodes that are more than $\sqrt{N}/4$ away from node i. Let S be area of a disc in the sphere with radius $\sqrt{N}/4$, and $c_4 = (N-S)/N$. Then it is easy to prove that $|\mathcal{D}_i| = c_4 N + o(N)$ *w.h.p.* This says that the number of nodes that are at least $\sqrt{N}/4$ far from the source i is $c_4 N$, i.e., has the same order as the number of nodes in the network. Let η_{ij} be the number of hops that a packet has to travel from its source node i to a destination $j \in \mathcal{D}_i$ via the preferred route assuming that all the relay requests are accepted. Then *w.h.p*, we have

$$\eta_{ij} \geq \frac{d_{ij}}{c_2\sqrt{\log N}} \geq \frac{\sqrt{N}}{4c_2\sqrt{\log N}}, \tag{13}$$

since the length of each hop is at most $c_2\sqrt{\log N}$ and $d_{ij} \geq \sqrt{N}/4$.

Let $\hat{\mathcal{D}}_i = \{j : j \in \mathcal{D}_i,$ s.t. $\exists k \in \Gamma_{ij}, k \in \mathcal{U}\}$ be the set of node destinations (for source node i), which are at least $\sqrt{N}/4$ away from node i and have at least one altruistic node in the routes between them and node i. The following result lower bounds the cardinality of the set $\hat{\mathcal{D}}_i$.

Lemma 4: $|\hat{\mathcal{D}}_i| \geq c_4\Big(N - N(1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}\Big) + o\Big(N(1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}\Big), \forall i,$ *w.h.p.* In particular, if $\theta(N) = \omega(\sqrt{N\log N})$, then $|\hat{\mathcal{D}}_i| \geq c_4 N + o(N), \forall i,$ *w.h.p.*

Proof: Please refer to Appendix I. ■

Roughly speaking, Lemma 4 argues that if the number of altruistic nodes in the network scales as $\omega(\sqrt{N\log N})$, then almost all the routes from source i to its destinations that are more than $\sqrt{N}/4$

away from it, will contain at least one altruistic node as one of the hops. This fact allows the altruistic nodes to efficiently enforce the reward/punishment strategy by accepting the relay request from only the cooperative nodes. Based on this observation, the following result establishes an upper bound on the fraction of altruistic nodes needed to ensure full cooperation.

Theorem 5: If $\theta(N) = \omega(\sqrt{N \log N})$, then full cooperation $\mathbf{p} = \mathbf{1}$ is a Nash equilibrium. At this equilibrium, the average energy per packet approaches the optimal scaling law of $\Theta\left(\sqrt{N}\log^{(\gamma-1)/2}(N)\right)$.

Proof: Please refer to Appendix II. ∎

Finally, we observe that

$$\lim_{N\to\infty} \theta(N)/N = 0,$$

implying the need for **only** a vanishingly small fraction of the nodes to be altruistic in order to converge to a full cooperation equilibrium in our energy limited wireless network.

V. Conclusions

In this work, we adopted a game theoretic approach for analyzing the impact of user selfishness on the performance of energy limited ad-hoc network. Our results have established the critical role of altruistic, i.e., relay, nodes in stimulating full cooperation between all nodes. In the small network setting, our numerical results show that the introduction of one relay node, which employs the appropriate cooperation strategy, yields significant energy savings. In the large network scenario, we have derived an upper bound on the number of altruistic nodes required to ensure full cooperation. This upper bound shows that full cooperation is possible in networks where only a vanishingly small fraction nodes is altruistic. Our results also shed light on the structure of optimal **physical layer** reward policies and established the strict sub-optimality of the traditional relaying approach, where the relay node offers the same forwarding service to all the nodes in the network.

Appendix I
Proof of Lemma 4

For any destination $j \in \mathcal{D}_i$, define Y_{ij} be the Bernoulli random variables, such that $Y_{ij} = 1$ if $\forall k \in \Gamma_{ij}, k \notin \mathcal{U}$, and $Y_{ij} = 0$ otherwise. This means that if the route Γ_{ij} only consists of selfish nodes, then $Y_{ij} = 1$, otherwise, if there exists at least one altruistic nodes in the route Γ_{ij}, $Y_{ij} = 0$. Since these $\theta(N)$ altruistic nodes are distributed in the network randomly, we have

$$P\{Y_{ij} = 1\} = \prod_{k=0}^{\eta_{ij}-1} \left(1 - \frac{\theta(N)}{N-k}\right) < (1-\theta(N)/N)^{\eta_{ij}} \leq (1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})} \tag{14}$$

due to (13) and the fact that $1 - \theta(N)/N < 1$.

We want to bound the number of routes that only consists of selfish nodes. Let $Y_i = \sum_{j \in \mathcal{D}_i} Y_{ij}$, then

$$\mathbb{E}\{Y_i\} = \mathbb{E}\{\sum_{j \in \mathcal{D}_i} Y_{ij}\} = \sum_{j \in \mathcal{D}_i} \mathbb{E}\{Y_{ij}\} \leq c_4 N (1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}. \tag{15}$$

For any given i, Y_{ij}s are not independent. For example, let $j_1, j_2 \in \mathcal{D}_i$ be such that Γ_{ij_2} includes all the nodes in Γ_{ij_1} and some other nodes $k \in \mathcal{N}$, that is $\Gamma_{ij_1} \subset \Gamma_{ij_2}$, then if $y_{ij_1} = 0$, y_{ij_2} also equals to 0. The reason is that $y_{ij_1} = 0$ means that $\exists k \in \mathcal{U},\ s.t.\ k \in \Gamma_{ij_1}$. Now, since $\Gamma_{ij_1} \subset \Gamma_{ij_2}$, we have $k \in \Gamma_{ij_2}$, hence, $y_{ij_2} = 0$. Due to this dependence, we couldn't use the Chernoff bound for i.i.d. Bernoulli random variables directly. Instead, we use similar technique as [17] and define $\hat{Y}_i = \sum_{j \in \mathcal{D}_i} \hat{Y}_{ij}$, where $\hat{Y}_{ij}$ be the i.i.d. Bernoulli random variables with

$$\Pr(\hat{Y}_{ij} = 1) = (1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}. \tag{16}$$

We know that if $\forall y_{ij_1} = 0$, and $\Gamma_{ij_1} \subset \Gamma_{ij_2}$, we will have $y_{ij_2} = 0$, thus, it is easy to check that

$$\mathbb{E}\{Y_i^m\} \leq \mathbb{E}\{\hat{Y}_i^m\} \tag{17}$$

for any $m > 0$. Hence $\mathbb{E}\{\exp(\phi Y_i)\} \leq \mathbb{E}\{\exp(\phi \hat{Y}_i)\}$ for any $\phi > 0$.

Let $P(\hat{Y}_i, \delta) = \Pr\{\hat{Y}_i \geq (1+\delta)E\{\hat{Y}_i\}\}, P(Y_i, \delta) = \Pr\{Y_i \geq (1+\delta)E\{Y_i\}\},$ by the Chernoff bound for i.i.d. Bernoulli random variables, we have for any $\delta > 0$,

$$P(\hat{Y}_i, \delta) \leq \exp(-\delta^2 \mathbb{E}\{\hat{Y}_i\}/2).$$

From (17), and follows [17], we have

$$P(Y_i, \delta) \leq P(\hat{Y}_i, \delta) \leq \exp(-\delta^2 \mathbb{E}\{\hat{Y}_i\}/2). \tag{18}$$

Let $\delta = 2\sqrt{\log N/\mathbb{E}\{\hat{Y}_i\}}$, we have

$$\Pr\Big(\hat{Y}_i \geq \mathbb{E}\{\hat{Y}_i\} + 2\sqrt{\log N \mathbb{E}\{\hat{Y}_i\}}\Big) \leq 1/N^2. \tag{19}$$

We know that $|\hat{\mathcal{D}}_i| = c_4 N - Y_i$, hence combining (16)(18)(19), we have that with probability larger than $1 - 1/N^2$, there are

$$c_4\Big(N - N(1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}\Big) + o\Big(N(1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}\Big)$$

destinations in $\mathcal{D}_i$, whose routes from node i includes at least one altruistic node.

Using union bound, we have

$$|\hat{\mathcal{D}}_i| > c_4\Big(N - N(1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}\Big) + o\Big(N(1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})}\Big), \forall i \in \mathcal{N} \tag{20}$$

with probability larger than $1 - 1/N$.

In particular, if $\theta(N) = \omega(\sqrt{N \log N})$,

$$0 \leq \lim_{N\to\infty} \frac{\mathbb{E}\{Y_i\}}{N} \leq \lim_{N\to\infty} \Big[(1-\theta(N)/N)^{N/\theta(N)}\Big]^{\theta(N)/(4c_2\sqrt{N\log N})} = 0, \tag{21}$$

since $\lim_{N\to\infty}(1-\theta(N)/N)^{N/\theta(N)} = 1/e$ and $\theta(N) = \omega(\sqrt{N \log N})$. Hence $N(1-\theta(N)/N)^{\sqrt{N}/(4c_2\sqrt{\log N})} = o(N)$, the claim is proved.

Appendix II

Proof of Theorem 5

If $\mathbf{p} = \mathbf{1}$, then

$$E_{i,tr} = \frac{1}{N-1}\sum_{j\neq i} c_1 d_{ii'(j)}^{\gamma}, E_{i,re} = \mathbb{E}\Big\{\sum_{\Gamma\in\Lambda_i} c_1 d_{i\Gamma(i)}^{\gamma}\Big\},$$

since all its relay request will be accepted.

Hence the cost under this point is

$$C_{i,co} = E_{i,tr} + E_{i,re} \leq c_1 c_2^{\gamma} \log^{\gamma/2} N + c_1 N c_2^{\gamma} \log^{\gamma/2} N = \Theta(N \log^{\gamma/2} N), \tag{22}$$

due to the bound in (12), and at each frame, there are at most N relay requests for node i at each frame.

On the other hand if node i changes its strategy to $p_i < 1$, then

$$E_{i,tr}(p_i) = \frac{1}{N-1}\Bigg(\sum_{j\in\Psi_i\setminus\hat{\Psi}_i} c_1 d_{ii'(j)}^{\gamma} + \sum_{j\in\hat{\Psi}_i} p_i^{f(\Gamma_{ij})} c_1 d_{ii'(j)}^{\gamma} + \sum_{j\in\hat{\Psi}_i}\big(1 - p_i^{f(\Gamma_{ij})}\big) c_1 d_{ij}^{\gamma}\Bigg), \tag{23}$$

where $f(\Gamma_{ij})$ is the number of altruistic nodes in the route Γ_{ij}, and

$$E_{i,re}(p_i) = \mathbb{E}\Big\{p_i \sum_{\Gamma\in\Lambda_i} c_1 d_{i\Gamma(i)}^{\gamma}\Big\}. \tag{24}$$

Hence for $\forall \epsilon > 0$, $p_i < 1 - \epsilon$, we have

$$
\begin{aligned}
C_i(p_i) &= E_{i,tr}(p_i) + E_{i,re}(p_i) \\
&\geq \frac{1}{N-1} \sum_{j \in \hat{\Psi}_i} \left(1 - p_i^{f(\Gamma_{ij})}\right) c_1 d_{ij}^{\gamma} && (25) \\
&\geq \frac{1}{N-1} \sum_{j \in \hat{\mathcal{D}}_i} \left(1 - p_i^{f(\Gamma_{ij})}\right) c_1 d_{ij}^{\gamma} && (26) \\
&\geq \epsilon c_1 \frac{c_4 N}{N-1} N^{\gamma/2} / 4^{\gamma} && (27) \\
&= \Theta(N^{\gamma/2}) > C_{i,co}. && (28)
\end{aligned}
$$

(25) is true, since we only consider the energy spent on direct transmission and ignore the energy spends on helping. (26) is true, since $\hat{\mathcal{D}}_i \subseteq \hat{\Psi}_i$. (27) is true, because: 1)$|\hat{\mathcal{D}}_i| \geq c_4 N$ (see lemma 4), 2) $\forall j \in \hat{\mathcal{D}}_i$, $d_{ij} \geq \frac{\sqrt{N}}{4}$ (definition), and 3) $1 - p_i^{f(\Gamma_{ij})} \geq 1 - p_i \geq \epsilon$, since $f(\Gamma_{ij}) \geq 1, \forall j \in \hat{\mathcal{D}}_i$. (28) is true when N is large, since $\gamma > 2$.

Also

$$
\begin{aligned}
\frac{\partial C_i}{\partial p_i}|_{p_i=1} &= -\frac{1}{N-1} \sum_{j \in \hat{\Psi}_i} f(\Gamma_{ij}) c_1 \Big(d_{ij}^{\gamma} - d_{ii'(j)}^{\gamma}\Big) + \mathbb{E}\Big\{ \sum_{\Gamma \in \Lambda_i} c_1 d_{i\Gamma(i)}^{\gamma} \Big\} \\
&\leq -\frac{1}{N-1} \sum_{j \in \hat{\mathcal{D}}_i} c_1 \Big(d_{ij}^{\gamma} - d_{ii'(j)}^{\gamma}\Big) + c_1 N c_2^{\gamma} \log^{\gamma/2} N && (29) \\
&\leq -\frac{c_1 c_4 N}{N-1} (N^{\gamma/2}/4^{\gamma} - c_2^{\gamma} \log^{\gamma/2} N) + c_1 N c_2^{\gamma} \log^{\gamma/2} N && (30) \\
&< 0, && (31)
\end{aligned}
$$

for sufficiently large N.

(29) is true since: 1) $\hat{\mathcal{D}}_i \subseteq \hat{\Psi}_i$, 2)$\forall j \in \hat{\Psi}_i$, $f(\Gamma_{ij}) \geq 1$, and 3) at each frame, there are at most N relay requests for each node. (30) is true, because of lemma 4, and $\forall j \in \hat{\mathcal{D}}_i$, $d_{ij} \geq \sqrt{N}/4$. (31) is true since $\gamma > 2$. (31) is true for all the nodes in the network *w.h.p.*

Hence no user will deviates from $p_i = 1$, full-cooperation is a nash-equilibrium. At this equilibrium, the average energy per packet scales the same as the cooperative network, which is $\Theta\Big(\sqrt{N} \log^{(\gamma-1)/2}(N)\Big)$ given by [7], [8].

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TABLE I

THE GAME PAYOFF MATRIX WITH ONE RELAY NODE.

	cooperation	no-cooperation
cooperation	$(C_{1,cc}, C_{3,cc})$	$(C_{1,cn}, C_{3,cn})$
no-cooperation	$(C_{1,nc}, C_{3,nc})$	$(C_{1,nn}, C_{3,nn})$

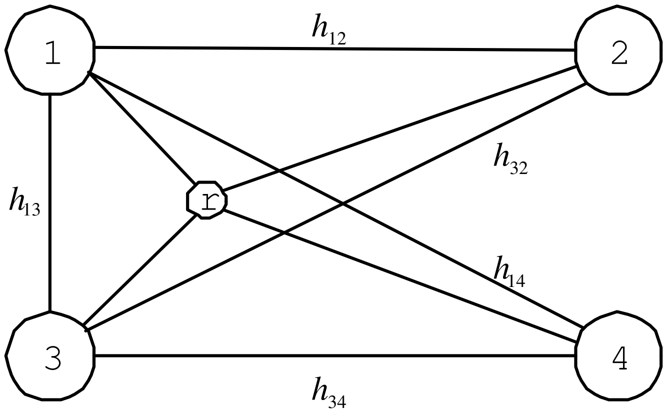

Fig. 1. A small wireless network with two selfish senders, one altruistic node and two receivers.

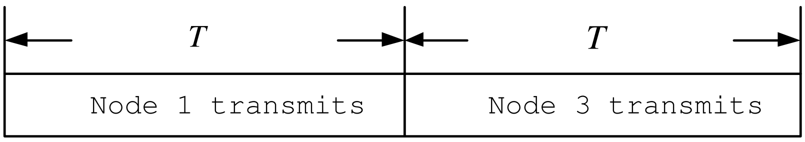

Fig. 2. The TDMA data frame.

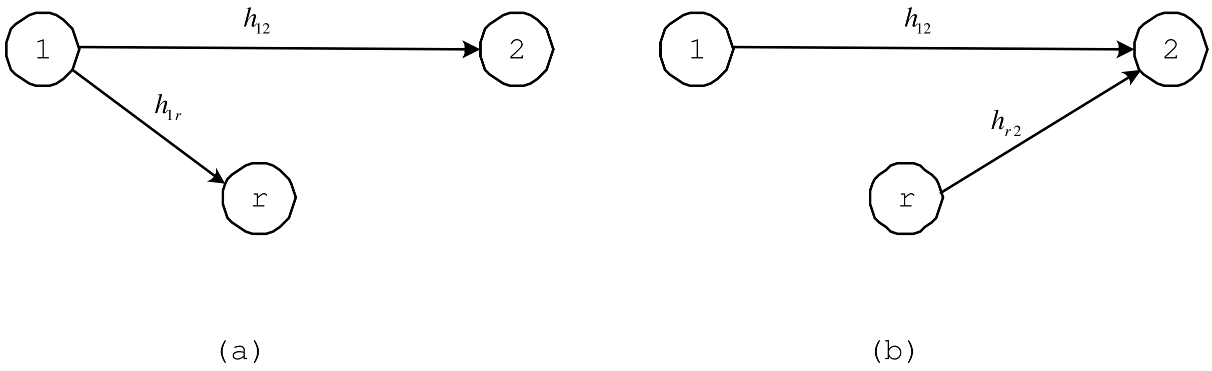

Fig. 3. The cooperation scheme in the small network: a) both the relay and node 2 keep receiving, lasting μT b) both node 1 and the relay transmit, lasting $(1-\mu)T$.

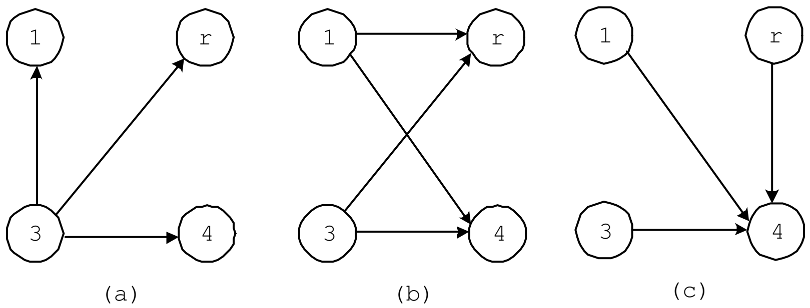

Fig. 4. The operation sequence when node 1 helps node 3 and $h_{31} > h_{3r}$, (a) node 1 and the relay keep listening , lasting $\mu^{'}_{3,1}T$ (b) node 1 helps transmitting, lasting $\mu^{'}_{3,2}T$ part of the time (c) both node 1 and the relay help, lasting $(1-\mu^{'}_{3,1}-\mu^{'}_{3,2})T$.

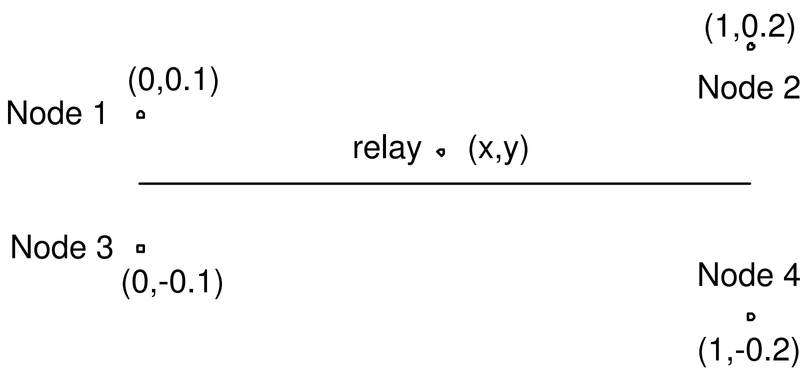

Fig. 5. The network example.

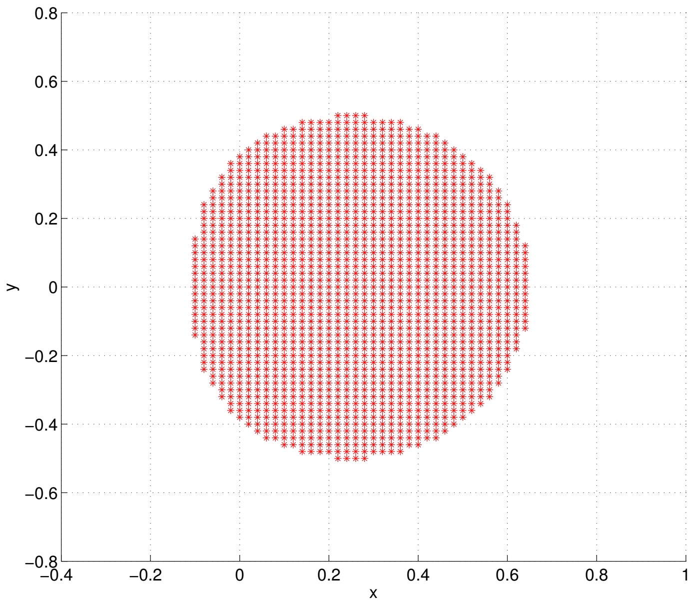

Fig. 6. The region corresponding to the relay node positions where full-cooperation is the only equilibrium for the selfish nodes.

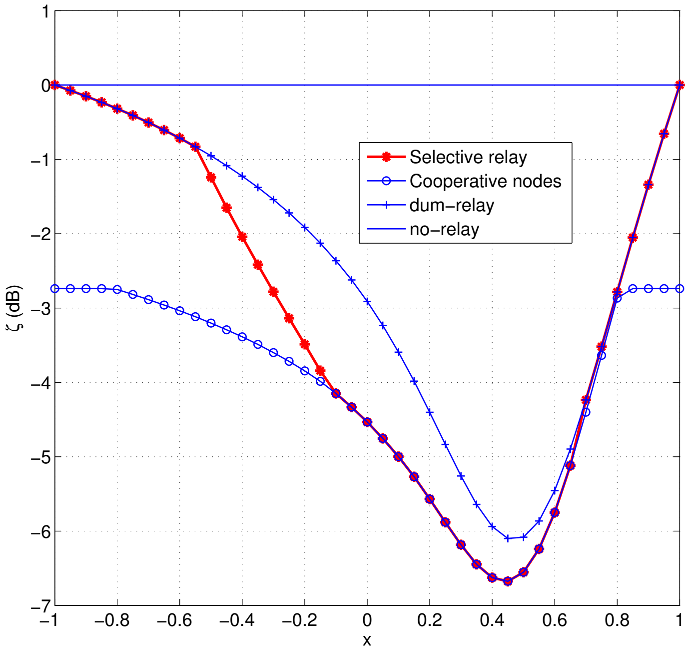

Fig. 7. Comparison between relay strategies (dumb relaying refers to the case where the relay always helps).

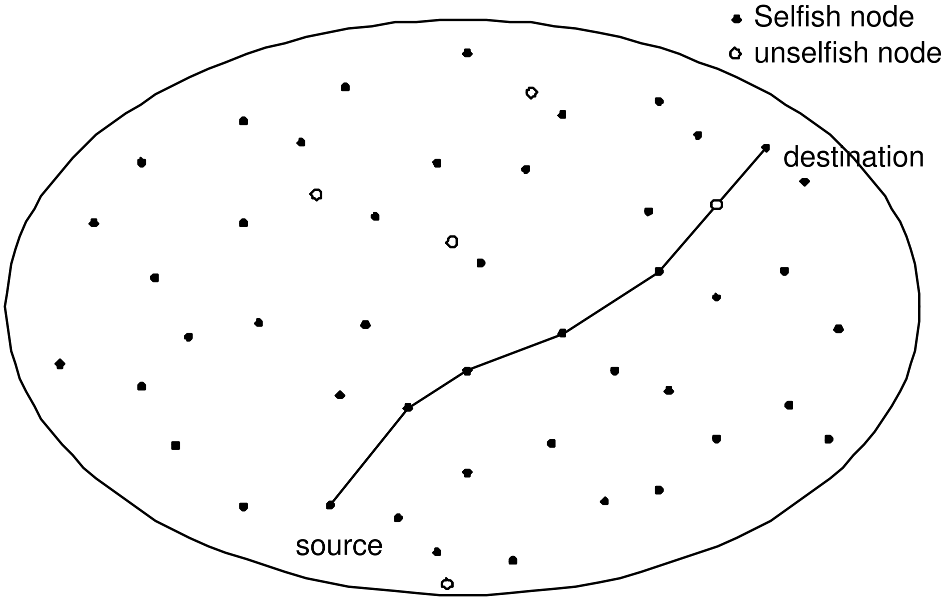

Fig. 8. Our large random network with altruistic nodes.