Space–Time Overlays for Convolutionally Coded Systems

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***Abstract*—In this paper, we consider the design of space–time overlays to upgrade single-antenna wireless communication systems to efficiently accommodate multiple transmit antennas. We define the overlay constraint such that the signal transmitted from the first antenna in the upgraded system is the same as that in the single-antenna system. The signals transmitted from the remaining antennas are designed according to space–time coding principles to achieve full spatial diversity in quasi-static flat fading channels. For both binary phase-shift keying (BPSK) and quaternary phase-shift keying-modulated systems, we develop an algebraic design framework that exploits the structure of existing single-dimensional convolutional codes in designing overlays that achieve full spatial diversity with minimum additional decoding complexity at the receiver. We also investigate a concatenated coding approach for BPSK overlay design in which the inner code is an orthogonal block code. This approach is shown to yield near optimal asymptotic performance for quasi-static fading channels. We conclude by offering a brief discussion outlining the extension of the proposed techniques to time-varying block fading channels.**

***Index Terms*—Convolutional codes, fading channels, multiple-input–multiple-output (MIMO), space–time coding.**

I. Introduction

ALMOST all digital wireless communication systems employ some form of channel coding to protect the raw data from channel noise and multipath fading effects. In single transmit-antenna systems, channel coding only adds temporal redundancy to the raw data aiming to exploit the temporal diversity provided by time-varying wireless fading channels. The availability of multiple transmit antennas allows for an additional degree of freedom in code design. Space–time coding was introduced in [1] as a two-dimensional (2-D) coding paradigm that exploits the spatial diversity provided by multiple transmit antennas in quasi-static flat fading channels. This principle was later generalized in [2] to construct 2-D space–time codes that exploit both temporal and spatial diversity available in multiantenna systems operating in multiple-input–multiple-output (MIMO) block fading channels.

Thus far, research on space–time code design has not fully considered the limitations and capabilities of existing single-antenna wireless systems [1], [3], [4]. On one hand, the optimized physical layer parameters obtained from those designs may not satisfy certain practical constraints imposed on the system, for example, certain designs require larger constellation sizes to achieve the same throughput [5]. On the other hand, those designs did not exploit the single-dimensional channel coding already employed in almost all practical single-antenna systems. In this paper, we consider the design of space–time overlays for upgrading convolutionally coded single-antenna wireless systems to efficiently accommodate multiple transmit antennas. We develop an algebraic design approach that utilizes the structure of single-dimensional convolutional codes to construct space–time overlays that achieve full spatial transmit diversity while satisfying a certain overlay constraint. This constraint ensures that the signal transmitted from the first antenna in the upgraded system is the same as that in the single antenna system. This allows for a smooth upgrade when moving from a single transmit-antenna system to a multiple transmit-antenna system. The constraint also ensures that the modulation will not be changed (the orthogonal designs, for example, in some cases require larger constellations to support the same rate [5]).

For binary phase-shift keying (BPSK)-modulated systems with rate k/n binary convolutional codes, we construct space–time overlays that preserve the same trellis complexity of the single-dimensional code. On the contrary, for quaternary phase-shift keying (QPSK), the general design approach entails the use of systematic inner codes that achieve full diversity. This approach suffers from the additional complexity required to decode the inner space–time code. However, for the special case of QPSK systems using rate $1/n$ binary convolutional codes with Gray mapping, we present a new space–time overlay construction with the same trellis complexity as the single-dimensional code. Therefore, in most cases, our designs allow for a space–time maximum-likelihood (ML) decoder with the same trellis complexity as the original single-dimensional decoder. The only difference is that the branch labeling needs to be modified at the receiver. This can be stored on a look-up table and does not require changing the chip.

We also investigate the use of the orthogonal block designs proposed by Tarokh *et al.* as inner space–time codes with full spatial transmit diversity [5]. We argue in favor of the optimality of this approach, in terms of the product distance metric [1], for BPSK-modulated systems in quasi-static fading channels. In QPSK systems, however, the inner orthogonal coding

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approach only satisfies the overlay constraint for systems with two transmit antennas [5], which limits its practical utility.

While the development in this paper focuses primarily on quasi-static fading channels, the extension to time-varying block fading channels is straightforward. This extension is possible through the framework introduced in [2]. One important result in this regard, outlined in Section V, pertains to the inner orthogonal coding approach and its inability to achieve the maximum possible diversity advantage in such channels.

We note that throughout the paper only a very limited number of antennas is assumed at the receiver. This scenario represents the down-link of most wireless systems where the number of receive antennas at the terminal is limited by the weight, size, and battery consumption requirements. As argued in [2], the space–time code design problem in such systems is more challenging than that in systems with a large number of receive antennas. In the latter scenario, efficient signal processing algorithms can be exploited to separate the signals transmitted from different antennas at the receiver. This reduces the code design problem to a single-dimensional code design in time-varying block fading channels, whereas in the scenario, underhand 2-D code design is required to account for the mutually interfering transmitted signals.

The rest of the paper is organized as follows. The system model is presented in Section II. In Section III, we consider the design of space–time overlays for BPSK- and QPSK-modulated systems. The use of orthogonal block designs as inner space–time codes in existing convolutionally coded wireless systems is discussed in Section IV. In Section V, we offer a brief discussion outlining the extension of the proposed techniques to time-varying block fading channels. In Section VI, we exhibit some design examples with simulation results that demonstrate the efficacy of the proposed approach. Finally, we present some concluding remarks in Section VII.

II. System Model

In the single-antenna system, the source generates k information symbols from the discrete alphabet $\mathcal{X}$, which are encoded by the error control code $C^{(s)}$ to produce code words $[c_1^{(s)}, c_2^{(s)}, \ldots, c_{n-1}^{(s)}, c_n^{(s)}]$ of length n over the symbol alphabet $\mathcal{Y}$. The modulator mapping function $f : \mathcal{Y} \rightarrow \Omega$ then maps the encoded symbols into constellation points from the discrete complex-valued signaling constellation Ω for transmission across the channel. In the multi-antenna system, the k information symbols are encoded by the composite error control code C to produce code words of length $N = nL_t$ over the symbol alphabet $\mathcal{Y}$. The encoded symbols are parsed among L_t transmit antennas and, as part of the overlay constraint, mapped by the same modulator f into constellation points. The modulated streams for all antennas are transmitted simultaneously. At the receiver, there are L_r receive antennas to collect the incoming transmissions. The received baseband signals are subsequently decoded by the space–time decoder. Each spatial channel (the link between one transmit antenna and one receive antenna) is assumed to experience statistically independent flat Rayleigh fading.

We formally defined a space–time code to consist of an underlying error control code together with the spatial parsing formatter.

Definition 1: An $L_t \times n$ space–time code $\mathcal{C}$ of size M consists of an $(L_t n, M)$ error control code C and a spatial parser σ that maps each code word vector $\bar{c} \in C$ to an $L_t \times n$ matrix $\mathbf{c}$ whose entries are rearrangement of those of $\bar{c}$. The space–time code $\mathcal{C}$ is said to be linear if both C and σ are linear.

We will assume that the standard parser maps

$$\bar{c} = \left(c_1^{(1)}, c_1^{(2)}, \ldots, c_1^{(L_t)}, c_2^{(1)}, c_2^{(2)}, \ldots, c_2^{(L_t)}, \ldots, c_n^{(1)}, c_n^{(2)}, \ldots, c_n^{(L_t)}\right) \in C$$

to the matrix

$$\mathbf{c} = \begin{bmatrix} c_1^{(1)} & c_2^{(1)} & \cdots & c_n^{(1)} \\ c_1^{(2)} & c_2^{(2)} & \cdots & c_n^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{(L_t)} & c_2^{(L_t)} & \cdots & c_n^{(L_t)} \end{bmatrix}.$$

In this notation, it is understood that $c_t^{(i)}$ is the code symbol assigned to transmit antenna i at time t. Therefore, the overlay requirement translates to the following constraint:

$$\left[c_1^{(s)}, c_2^{(s)}, \ldots, c_{n-1}^{(s)}, c_n^{(s)}\right] = \left[c_1^{(1)}, c_2^{(1)}, \ldots, c_{n-1}^{(1)}, c_n^{(1)}\right]. \quad (1)$$

Let $\mathbf{s} = f(\mathbf{c})$ be the baseband version of the code word as transmitted across the channel, then we have the following baseband model of the received signal for the overlay system:

$$y_t^j = \sqrt{E_s} \sum_{i=1}^{L_t} \alpha_t^{ij} s_t^{(i)} + n_t^j \quad (2)$$

where $\sqrt{E_s}$ is the energy per transmitted symbol; α_t^{ij} is the complex path gain from transmit antenna i to receive antenna j at time t; $s_t^{(i)} = f(c_t^{(i)})$ is the transmitted constellation point from antenna i at time t; n_t^j is the additive white Gaussian noise (AWGN) sample for receive antenna j at time t. The noise samples are independent samples of zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. The different path gains α_t^{ij} are assumed to be statistically independent. The fading model of primary interest is that of a quasi-static flat Rayleigh fading process in which the complex fading gains are constant over the same code word and are independent from one code word to the next. Channel state information is assumed to be available *a priori* only at the receiver.

The diversity advantage of a space–time code is defined as the minimum of the absolute value of the asymptotic slope of the pairwise probability of error versus signal-to-noise ratio (SNR) curve on a log-log scale. The following rank criterion was proposed in [1] and [3] to maximize the spatial diversity advantage provided by the multiple transmit antenna.

- *Baseband Rank Criterion*: Maximize $d = \text{rank}(f(\mathbf{c}) - f(\mathbf{e}))$ over all pairs of distinct code words $\mathbf{c}, \mathbf{e} \in \mathcal{C}$.

Full spatial transmit diversity is achieved if and only if $\text{rank}(f(\mathbf{c}) - f(\mathbf{e})) = L_t$ for all pairs of distinct code words $\mathbf{c}$,

$\mathbf{e} \in \mathcal{C}$. It can be shown that [1], in the presence of L_r receive antennas, the total diversity advantage achieved by this code in quasi-static fading channels is $L_t L_r$.

III. Space–Time Overlay Design

In this section, we present a design framework for full-diversity space–time codes that satisfy the overlay constraint (1) and require minimal additional decoding complexity over the single-dimensional Viterbi decoder used in the single-antenna system.

A. BPSK Modulation

For BPSK modulation, the natural discrete symbol alphabet $\mathcal{Y}$ is the field $\mathbb{F} = \{0, 1\}$ of integers modulo 2. Modulation is performed by mapping the symbol $x \in \mathbb{F}$ to the constellation point $s = f(x) \in \{-1, 1\}$ according to the rule $s = (-1)^x$. Note that it is possible for the modulation format to include an arbitrary phase offset $e^{i\phi}$ [4]. Notationally, the circled operator $\oplus$ will be used to distinguish modulo 2 addition from real- or complex-valued ($+$, $-$) operations.

The baseband rank criterion does not allow for a systematic approach for designing algebraic space–time codes because it applies to the complex domain rather than the discrete domain in which codes are traditionally designed. In [4], the authors developed the following binary rank criterion to aid the design of algebraic full-diversity space–time codes for BPSK modulation

Theorem 2 (Binary Rank Criterion): Let $\mathcal{C}$ be a linear $L_t \times n$ space–time code with underlying binary code C of length $N = nL_t$ where $n \geq L_t$. Suppose that every nonzero code word $\mathbf{c}$ is a matrix of full rank over the binary field $\mathbb{F}$. Then, for BPSK transmission over the quasi-static fading channel, the space–time code $\mathcal{C}$ achieves full spatial transmit diversity L_t.

Now, let $C^{(s)}$ be the rate k/n binary convolutional code used in the single-antenna system. The encoder processes k binary input sequences $x_1(t), x_2(t), \ldots, x_k(t)$ and produces n coded output sequences $y_1^{(s)}(t), y_2^{(s)}(t), \ldots, y_n^{(s)}(t)$, which are multiplexed together to form the output code word. A sequence $\{x(t)\}$ is often represented by the formal series $X(D) = x(0) + x(1)D + x(2)D^2 + \ldots$. We refer to $\{x(t)\} \leftrightarrow X(D)$ as a D-transform pair. The action of the binary convolutional encoder is linear and is characterized by the so-called impulse responses $g_{i,j}^{(s)}(t) \leftrightarrow G_{i,j}^{(s)}(D)$ associating output $y_j^{(s)}(t)$ with input $x_i(t)$. Thus, the encoder action is summarized by the matrix equation

$$\mathbf{Y}^{(s)}(D) = \mathbf{X}(D)\mathbf{G}^{(s)}(D)$$

where $\mathbf{Y}^{(s)}(D) = [Y_1^{(s)}(D)\ Y_2^{(s)}(D)\ \cdots\ Y_n^{(s)}(D)]$, $\mathbf{X}(D) = [X_1(D)\ X_2(D)\ \cdots\ X_k(D)]$, and

$$\mathbf{G}^{(s)}(D) = \begin{bmatrix} G_{1,1}^{(s)}(D) & G_{1,2}^{(s)}(D) & \cdots & G_{1,n}^{(s)}(D) \\ G_{2,1}^{(s)}(D) & G_{2,2}^{(s)}(D) & \cdots & G_{2,n}^{(s)}(D) \\ \vdots & \vdots & \ddots & \vdots \\ G_{k,1}^{(s)}(D) & G_{k,2}^{(s)}(D) & \cdots & G_{k,n}^{(s)}(D) \end{bmatrix}.$$

We consider the space–time overlay code $\mathcal{C}$ in which the code word $\mathbf{Y}^{(i)}(D)$ transmitted from antenna i is obtained through the action of a rate k/n convolutionl encoder with transfer function $\mathbf{G}^{(i)}(D)$ on the k-tuple information stream $\mathbf{X}(D)$. It is clear that the overlay constraint (1) is satisfied if and only if $\mathbf{G}^{(1)}(D) = \mathbf{G}^{(s)}(D)$.

The following proposition establishes sufficient conditions on $(\mathbf{G}^{(1)}(D), \ldots, \mathbf{G}^{(L_t)}(D))$ which guarantee that the space–time overlay achieves full spatial transmit diversity L_t.

Proposition 3 (BPSK Overlay Construction): Let $\mathbf{G}^{(1)}(D), \mathbf{G}^{(2)}(D), \ldots, \mathbf{G}^{(L_t)}(D)$ be transfer functions for rate k/n convolutional codes, $n \geq k$, and let $\mathcal{C}$ be the $L_t \times n$ space–time code of dimension k consisting of the code words

$$\mathbf{C}(D) = \begin{bmatrix} \mathbf{X}(D)\mathbf{G}^{(1)}(D) \\ \mathbf{X}(D)\mathbf{G}^{(2)}(D) \\ \vdots \\ \mathbf{X}(D)\mathbf{G}^{(L_t)}(D) \end{bmatrix}$$

where $\mathbf{X}(D)$ denotes the formal series of k arbitrary binary information sequences and $L_t \leq n$. Then $\mathcal{C}$ satisfies the binary rank criterion, and thus, for BPSK transmission over the quasi-static fading channel, achieves full spatial transmit diversity L_t, if and only if $\mathbf{G}^{(1)}(D), \mathbf{G}^{(2)}(D), \ldots, \mathbf{G}^{(L_t)}(D)$ have the property that

$$\forall a_1, a_2, \ldots, a_{L_t} \in \mathbb{F}: \quad \mathbf{G}(D) = a_1\mathbf{G}^{(1)}(D) \oplus a_2\mathbf{G}^{(2)}(D) \oplus \cdots \oplus a_{L_t}\mathbf{G}^{(L_t)}(D)$$

is of rank k over $\mathbb{F}[[D]]$ (the space of all formal series) unless $a_1 = a_2 = \cdots = a_{L_t} = 0$.

Proof: ($\Rightarrow$) Let $\mathbf{G}(D)$ have rank k over $\mathbb{F}[[D]]$. Then, for

$$\mathbf{X}(D)\mathbf{G}(D) = \mathbf{X}(D)\left[a_1\mathbf{G}^{(1)}(D) \oplus a_2\mathbf{G}^{(2)}(D) \oplus \cdots \oplus a_{L_t}\mathbf{G}^{(L_t)}(D)\right]$$

to be equal to 0, we need $\mathbf{X}(D) = 0$ or $a_1 = a_2 = \cdots = a_{L_t} = 0$. Hence, $\mathcal{C}$ satisfies the binary rank criterion.

($\Leftarrow$) Let $\mathbf{G}(D)$ have rank less than k over $\mathbb{F}[[D]]$. Then, there is nonzero $\mathbf{X}(D)$ such that

$$\mathbf{X}(D)\mathbf{G}(D) = a_1\mathbf{X}(D)\mathbf{G}^{(1)}(D) \oplus a_2\mathbf{X}(D)\mathbf{G}^{(2)}(D) \oplus \cdots \oplus a_{L_t}\mathbf{X}(D)\mathbf{G}^{(L_t)}(D) = 0$$

for given $a_1, a_2, \ldots, a_{L_t}$, other than the all-zero case. Hence, $\mathcal{C}$ does not satisfy the binary rank criterion. □

For the special case where $C^{(s)}$ is a rate $1/n$ convolutional code, it is sufficient to choose $(G_j^{(1)}, \ldots G_j^{(L_t)})$ for any single arbitrary j, $1 \leq j \leq n$, according to the stacking construction proposed in [4] to ensure that the resulting space–time code achieves full diversity. However, it is more intuitively appealing to construct $(G_j^{(1)}, \ldots, G_j^{(L_t)})$ for all j, $1 \leq j \leq n$, according to the stacking construction.

Except for the constraint that $\mathbf{G}^{(1)}(D) = \mathbf{G}^{(s)}(D)$, Theorem 3 does not impose upper bounds on the constraint lengths of the other transfer functions $\mathbf{G}^{(2)}(D), \ldots, \mathbf{G}^{(L_t)}(D)$. However, restricting these constraint lengths serves to limit the trellis complexity of the overall space–time code. More specifically, we can leverage the existing Viterbi decoder for the single antenna code $C^{(s)}$ through limiting the maximum constraint length of $\mathbf{G}^{(1)}(D), \ldots, \mathbf{G}^{(L_t)}(D)$ to be equal to that of $\mathbf{G}^{(s)}(D)$. This way, the resulting space–time code has the same trellis complexity as $\mathbf{G}^{(s)}(D)$ and the only modification required is to change the branch metric computations of the single-antenna

Viterbi decoder. The branch metric computations depend on the number of transmit and receive antennas, as described in [1].

B. QPSK Modulation

For QPSK modulation, the natural discrete symbol alphabet $\mathcal{Y}$ is the ring $\mathbb{Z}_4 = \{0, \pm 1, 2\}$ of integers modulo 4. Modulation is performed by mapping the symbol $x \in \mathbb{Z}_4$ to the constellation point $s \in \{\pm 1, \pm i\}$ according to the rule $s = i^x$, where $i = \sqrt{-1}$. Again, the absolute phase reference of the QPSK constellation could have been chosen arbitrarily without affecting the performance [4]. Since the binary rank criterion developed in [4] for QPSK modulated space–time codes pertains to certain projections of the $\mathbb{Z}_4$-valued matrix $\mathbf{c}$ over the binary field, we need the following definitions.

Let $\mathbf{c}$ be a $\mathbb{Z}_4$-valued matrix that consists of exactly ℓ rows and p columns that are not multiples of two. After suitable row permutations if necessary, it has the following row structure:

$$\mathbf{c} = \begin{bmatrix} \underline{c}_1 \\ \vdots \\ \underline{c}_\ell \\ 2\underline{c}'_{\ell+1} \\ \vdots \\ 2\underline{c}'_{L_t} \end{bmatrix}.$$

The row-based indicant projection (Ξ-projection) is then defined as

$$\Xi(\mathbf{c}) = \begin{bmatrix} \beta(\underline{c}_1) \\ \vdots \\ \beta(\underline{c}_\ell) \\ \beta(\underline{c}'_{\ell+1}) \\ \vdots \\ \beta\left(\underline{c}'_{L_t}\right) \end{bmatrix}$$

where $\beta(\underline{c}_i)$ is the binary projection (i.e., remainder after division by two) of the $\mathbb{Z}_4$ vector $\underline{c}_i$. Similarly, the column-based indicant projection (Ψ-projection) is defined as

$$[\Psi(\mathbf{c})]^{\mathrm{T}} = \Xi(\mathbf{c}^{\mathrm{T}}). \tag{3}$$

The row and column indicant projections serve to indicate certain aspects of the binary structure of the $\mathbb{Z}_4$ matrix in which multiples of two are ignored. Using these binary indicants, we developed the following binary rank criterion for QPSK-modulated codes [4].

Theorem 4 (QPSK Binary Rank Criterion): Let $\mathcal{C}$ be a linear $L_t \times n$ space–time code over $\mathbb{Z}_4$, with $n \geq L_t$. Suppose that for every nonzero $\mathbf{c} \in \mathcal{C}$, the row-based indicant $\Xi(\mathbf{c})$ or the column-based indicant $\Psi(\mathbf{c})$ has full rank L_t over $\mathbb{F}$. Then, for QPSK transmission, the space–time code $\mathcal{C}$ achieves full spatial diversity L_t.

The common practice in most single antenna communication systems is to use binary convolutional codes with optimal free distances d_{free}. The encoder output is then mapped to the $\mathbb{Z}_4$ alphabet according to the Gray mapping rule (i.e., $00 \rightarrow 0, 01 \rightarrow 1, 11 \rightarrow 2, 10 \rightarrow 3$). The resulting code is known to maximize the minimum Hamming distance between any two distinct code words and hence, maximizes the minimum Euclidean distance among the class of codes based on binary convolutional code.

1) General Design Approach: Exploiting the structure of the single-dimensional binary code in designing QPSK space–time overlays is more complicated than the BPSK scenario, due to the nonlinearity of the Gray mapped binary code over the $\mathbb{Z}_4$ ring of integers. In this case, the QPSK binary rank criterion only applies to differences between code words which increases the difficulty involved in extracting an algebraic framework for constructing overlays. Therefore, our proposed solution for the general problem entails the use of systematic inner space–time codes that satisfy the overlay constraint and achieve full spatial diversity. The stacking construction in [4] can be the basis for constructing systematic inner block or convolutional codes achieving full diversity [4]. For the sake of completeness, we will provide a brief description of the inner convolutional code design principle.

The coded $\mathbb{Z}_4$ output stream $\mathbf{X}^{z_4}(D)$ after Gray mapping is presented at the input of the inner $\mathbb{Z}_4$ rate $1/L_t$ convolutional code $C^{(z_4)}$ with the $\mathbb{Z}_4$ transfer function

$$\mathbf{G}^{(z_4)}(D) = \left[G_1^{(z_4)}(D) \quad G_2^{(z_4)}(D) \cdots G_{L_t}^{(z_4)}(D)\right]. \tag{4}$$

In the natural space–time formatting of $C^{(z_4)}$, the output sequence corresponding to $Y_i^{(z_4)}(D) = \mathbf{X}^{(z_4)}(D)G_i^{(z_4)}(D)$ is assigned to the ith transmit antenna. This construction satisfies the overlay constraint if and only if $C^{(z_4)}$ is a systematic code (i.e., $G_1^{(z_4)} = 1$). The resulting space–time code $\mathcal{C}$ satisfies the QPSK binary rank criterion under relatively mild conditions on the generator polynomials.

Proposition 5 (QPSK Overlay Construction I): Let $\mathbf{G_c}$ be the $\mathbb{Z}_4$ coefficients matrix corresponding to the natural space–time code $\mathcal{C}$ associated with the rate $1/L_t$ nonrecursive convolutional code $C^{(z_4)}$. Then $\mathcal{C}$ satisfies the QPSK binary rank criterion, and thus achieves full spatial transmit diversity L_t for QPSK transmission, if the binary projection $\beta(\mathbf{G_c})$ has full rank L_t as a matrix of coefficients over the binary field $\mathbb{F}$.

Proof: Please refer to [4]. □

It is worth noting that the popular delay diversity transmission format is a special case of Proposition 5. Since the condition in Proposition 5 is related to the binary projection of the transfer function, the linear $\mathbb{Z}_4$ codes can be obtained by lifting full diversity binary convolutional codes to the $\mathbb{Z}_4$ domain (i.e., each 1 in the binary code coefficients matrix can be replaced with either 1 or 3 and each 0 with either 0 or 2). As introduced in [4], binary rate $1/L_t$ convolutional codes with optimal d_{free} are promising candidates for this application as their associated natural space–time codes usually satisfy the binary rank criteria. Furthermore, as shown in [6], these codes outperform the best space–time trellis codes found so far by extensive computer search methods, especially for increasing numbers of antennas. [4, Table 1] provides a list of the full-diversity nonsystematic feed-forward binary convolutional codes with optimal free distances covering a wide range of constraint lengths and numbers of antennas. The desired full diversity inner systematic codes can be obtained by lifting the recursive version of those optimal free distance codes to the $\mathbb{Z}_4$ domain.

The main limitation of this overlay construction is the additional complexity required at the decoder. Joint ML decoding of the outer single-dimensional code $C^{(s)}$ and the inner systematic space–time code $\mathcal{C}$ is expected to be of prohibitive complexity, especially for a large number of transmit antennas, due to the large number of states in the joint trellis diagram. Fortunately, this does not impose a major obstacle, since this coding scheme allows for a straightforward application of the turbo processing architecture [7]. A soft input/soft output decoder can be used for both $C^{(s)}$ and $\mathcal{C}$, and the decoding process should be iterated with soft information passing between the two decoders [7], [8]. A random interleaver should be used to scramble the output stream of $C^{(s)}$ before passing it to $\mathcal{C}$. This is necessary to aid the turbo decoder convergence [8], [9], and does not affect the diversity advantage achieved by the inner space–time code. Guided by the excellent performance exhibited by this architecture in various applications [8], [10], [11], one would expect this receiver to offer a very close performance to ML decoding with a reasonable complexity.

2) Rate $1/n$ Coded Systems: The added complexity required to decode the space–time overlay construction in Proposition 5 can be avoided when $C^{(s)}$ is a rate $1/n$ convolutional code. In this special case, we provide a space–time overlay construction with the same trellis complexity as that of $C^{(s)}$. For simplicity of presentation, we will first consider the case where $C^{(s)}$ is a rate 1/2 binary convolutional code. The extension to arbitrary rate $1/n$ codes is then briefly outlined.

We assume that the two output branches from the encoder $Y_1^{(s)}(D)$, $Y_2^{(s)}(D)$ are grouped according to the Gray mapping rule to form the $\mathbb{Z}_4$ stream $Y_{z4}^{(s)}(D)$. The only implication of this assumption is that temporal interleaving has to be performed on a QPSK symbol-by-symbol basis. Based on the Gray mapping rule, we have the following relation:

$$Y_{z_4}^{(s)}(D) = \left(Y_1^{(s)}(D) \oplus Y_2^{(s)}(D)\right) + 2Y_2^{(s)}(D) \tag{5}$$

and hence

$$\begin{aligned}\beta\left(Y_{z_4}^{(s)}(D)\right) &= Y_1^{(s)}(D) \oplus Y_2^{(s)}(D)\\ &= \mathbf{X}(D)\left(G_1^{(s)}(D) \oplus G_2^{(s)}(D)\right).\end{aligned} \tag{6}$$

Therefore, the binary projection of the $\mathbb{Z}_4$ stream is equivalent to a rate 1 convolutionally encoded stream with the generator polynomial $G_1^{(s)}(D) \oplus G_2^{(s)}(D)$. This observation inspires the following overlay construction.

Proposition 6 (QPSK Overlay Construction II): Let $\mathcal{C}$ be a $\mathbb{Z}_4$ $L_t \times n$ space–time code obtained by grouping the two output branches from L_t rate 1/2 binary convolutional encoders $(\mathbf{G}^{(1)}(D), \ldots, \mathbf{G}^{(L_t)}(D))$ according to the Gray mapping rule. Then, for QPSK transmission over the quasi-static fading channel, $\mathcal{C}$ satisfies the QPSK binary rank criterion, and hence, achieves full spatial diversity if

$$\begin{aligned}\forall a_1, a_2, \ldots, a_{L_t} \in \mathbb{F} : a_1 &\left(G_1^{(1)}(D) \oplus G_2^{(1)}(D)\right)\\ &\oplus a_2\left(G_1^{(2)}(D) \oplus G_2^{(2)}(D)\right) \oplus \cdots\\ &\oplus a_{L_t}\left(G_1^{(L_t)}(D) \oplus G_2^{(L_t)}(D)\right) \neq 0\end{aligned}$$

unless $a_1 = a_2 = \cdots = a_{L_t} = 0$.

Proof: Let $a_1(G_1^{(1)}(D) \oplus G_2^{(1)}(D)) \oplus a_2(G_1^{(2)}(D) \oplus G_2^{(2)}(D)) \oplus \ldots \oplus a_{L_t}(G_1^{(L_t)}(D) \oplus G_2^{(L_t)}(D)) \neq 0$ unless $a_1 = a_2 = \cdots = a_{L_t} = 0$. Based on the Gray mapping rule, the two output branches from the encoder $Y_1^{(i)}(D)$ and $Y_2^{(i)}(D)$ that correspond to antenna i, $i = 1, 2, \ldots, L_t$, are grouped to yield

$$Y_{z_4}^{(i)}(D) = \left(Y_1^{(i)}(D) \oplus Y_2^{(i)}(D)\right) + 2Y_2^{(i)}(D).$$

The binary projection of $Y_{z4}^{(i)}(D)$ is

$$\begin{aligned}\beta\left(Y_{z_4}^{(i)}(D)\right) &= Y_1^{(i)}(D) \oplus Y_2^{(i)}(D)\\ &= \mathbf{X}(D)\left(G_1^{(i)}(D) \oplus G_2^{(i)}(D)\right)\end{aligned}$$

for $i = 1, 2, \ldots, L_t$. Therefore, the row-based indicant projection is given by

$$\Xi(\mathbf{C}(D)) = \begin{bmatrix} \beta\left(Y_{z_4}^{(1)}(D)\right)\\ \beta\left(Y_{z_4}^{(2)}(D)\right)\\ \vdots\\ \beta\left(Y_{z_4}^{(L_t)}(D)\right)\end{bmatrix} = \begin{bmatrix} \mathbf{X}(D)\left(G_1^{(1)}(D) \oplus G_2^{(1)}(D)\right)\\ \mathbf{X}(D)\left(G_1^{(2)}(D) \oplus G_2^{(2)}(D)\right)\\ \vdots\\ \mathbf{X}(D)\left(G_1^{(L_t)}(D) \oplus G_2^{(L_t)}(D)\right)\end{bmatrix}.$$

Now, $\forall a_1, a_2, \ldots, a_{L_t} \in \mathbb{F}$, we have

$$\begin{aligned}&a_1\beta\left(Y_{z_4}^{(1)}(D)\right) \oplus a_2\beta\left(Y_{z_4}^{(2)}(D)\right) \oplus \cdots \oplus a_{L_t}\beta\left(Y_{z_4}^{(L_t)}(D)\right)\\ &\quad= a_1\mathbf{X}(D)\left(G_1^{(1)}(D) \oplus G_2^{(1)}(D)\right) \oplus \cdots\\ &\qquad\oplus a_{L_t}\mathbf{X}(D)\left(G_1^{(L_t)}(D) \oplus G_2^{(L_t)}(D)\right)\\ &\quad= \mathbf{X}(D)\left[a_1\left(G_1^{(1)}(D) \oplus G_2^{(1)}(D)\right) \oplus \cdots\right.\\ &\qquad\left.\oplus a_{L_t}\left(G_1^{(L_t)}(D) \oplus G_2^{(L_t)}(D)\right)\right] \neq 0\end{aligned}$$

unless $\mathbf{X}(D) = 0$ or $a_1 = a_2 = \cdots = a_{L_t} = 0$. Hence, $\Xi(\mathbf{C}(D))$ has full rank for all nonzero codewords. Therefore, $\mathcal{C}$ satisfies the QPSK binary rank criterion, and hence, achieves full spatial diversity. □

Proposition 6 implies that it is sufficient to choose $G_1^{(1)}(D) \oplus G_2^{(1)}(D), \ldots, G_1^{(L_t)}(D) \oplus G_2^{(L_t)}(D)$ according to the stacking condition in [4] to ensure that the $\mathbb{Z}_4$ code achieves full diversity. By restricting the maximum constraint length of any component in $\mathbf{G}(D)$ to be equal to that of $C^{(s)}$, it is easy to see that $\mathcal{C}$ has the same trellis complexity as $C^{(s)}$.

It is clear that Proposition 6 can be easily extended to construct space–time overlays for systems with rate $1/2m$ codes. For rate $1/(2m+1)$ codes, the condition in Proposition 6 should be slightly modified. In this case, the code needs to be represented in a rate $2/2(2m+1)$ form, and the condition for full diversity is that all the linear combinations of the $2 \times (2m+1)$ transfer functions resulting from the binary projection operator β must have full rank 2 over the space of all formal series. The trellis diagram of the new representation has four branches coming out of each state. However, the number of branches per decoding bit remains the same as that in the single-dimensional code $C^{(s)}$.

IV. Orthogonal Block Codes

The algebraic framework developed in Sections III-A and III-B encompasses wide range of convolutional-based space–time overlays. All space–time codes within this framework achieve full spatial diversity. The second criterion that determines the performance of space–time codes in quasi-static fading channels is the product distance (coding advantage) which does not affect the asymptotic slope, but results in a shift of the asymptotic performance curve [1], [4], [12]. The question now arises, Which of those full diversity space–time codes enjoys the best product distance, and hence, achieves optimal asymptotic performance? In this section, we provide an upper bound on the product distance of linear BPSK space–time codes in quasi-static fading channels in terms of the free distance of the underlying binary code. This bound suggests that using the full diversity orthogonal space–time codes introduced by Tarokh *et al.* [5] as space–time appliques in concatenation with outer optimal d_{free} convolutional codes provides close-to-optimal performance within the class of BPSK convolutional overlays presented in the previous section. In the next section, we show that the optimality of this concatenated coding approach is limited to quasi-static fading channels and does not extend to time-varying channels. Except for systems with two transmit antennas, the inner orthogonal block coding approach is not suitable for QPSK modulation because it requires a larger constellation size to achieve the same throughput which violates the overlay constraint [5].

In quasi-static fading channels, the product distance η of a space–time code $\mathcal{C}$ is defined as the minimum, over all distinct pairs of code words $\mathbf{c}, \mathbf{e} \in \mathcal{C}$, of the geometric mean of the eigenvalues of $\mathbf{A} = (f(\mathbf{c})-f(\mathbf{e}))(f(\mathbf{c})-f(\mathbf{e}))^H$ [1], [3]. Now, we have the following upper bound on the product distance of the class of linear BPSK space–time codes in Definition 1.

Lemma 7: Let $\mathcal{C}$ be a linear full diversity $L_t \times n$ space–time code with underlying binary code C of length $N = nL_t$, where $n \geq L_t$, and free distance d_{free}. Then, for BPSK transmission over the quasi-static fading channel, the space–time code product distance η is upper bounded by $(4E_s d_{\text{free}}/L_t)$ (i.e., $\eta \leq (4E_s d_{\text{free}}/L_t)$).

Proof: Let $\lambda_1, \ldots, \lambda_{L_t}$ be the eigenvalues of the full rank matrix $\mathbf{A} = (f(\mathbf{c}) - f(\mathbf{e}))(f(\mathbf{c}) - f(\mathbf{e}))^H$, then

$$\sum_{j=1}^{L_t} \lambda_j = tr\mathbf{A} = 4E_s d_{e,c} \tag{7}$$

where $d_{e,c}$ is the binary distance between the code words e, $c \in C$, and

$$\min_{\mathbf{c},\mathbf{e}\in\mathcal{C}} \sum_{j=1}^{L_t} \lambda_j = 4E_s d_{\text{free}}. \tag{8}$$

Subject to this constraint on the sum of the eigenvalues, it is easy to see that the product distance obtained by the optimal parsing function is upper bounded by

$$\eta \leq \lambda_1^o = \lambda_2^o = \ldots = \lambda_{L_t}^o = \frac{4E_s d_{\text{free}}}{L_t}. \tag{9}$$

□

TABLE I
Comparison Between the Product Distance of the Concatenated Coding Approach and the Upper Bound for a BPSK System With Rate 1/2 Single-Dimensional Code and Optimal Free Distance

L_t	K	η	Upper Bound
2	3	20	20
2	4	24	26
2	5	28	32
2	6	32	36
2	7	40	40
3	3	20	21
3	4	24	26
3	5	28	32
3	6	32	36
3	7	40	45

Orthogonal space–time codes are particularly appealing because of the simplicity of their ML decoder [5]. This simplicity is a result of the orthogonality between the rows of the space–time code-word matrix $\mathbf{c}$. It is straightforward to see that using a slightly modified version of the real orthogonal space–time codes [5]—some columns are multiples by (-1) to adjust the sign of the first entry—as inner appliques to upgrade single-antenna BPSK-modulated systems satisfies the overlay constraint (1). The following result establishes the remarkable product distance achieved by this overlay design which rivals that of the optimal convolutional based space–time overlay with the same constraint length. Quite interestingly, this near-optimal performance is also facilitated by the orthogonality between the rows of the resulting space–time code.

Lemma 8: Let $\mathcal{C}$ be a full diversity $L_t \times n$ concatenated space–time code with single-dimensional outer code $C^{(s)}$ of length n and inner orthogonal block code of length L_t, and let $d_{\text{free}}^{(s)}$ be the free distance of $C^{(s)}$. Then, for BPSK transmission over the quasi-static fading channel, the product distance of $\mathcal{C}$ is $\eta = 4E_s d_{\text{free}}^{(s)}$.

Proof: The orthogonality between the different rows of $(f(\mathbf{c}) - f(\mathbf{e}))$ results in a diagonal $\mathbf{A} = (f(\mathbf{c}) - f(\mathbf{e}))(f(\mathbf{c}) - f(\mathbf{e}))^H$ for all distinct pairs of code words $\mathbf{c}, \mathbf{e} \in \mathcal{C}$. Hence, for the code $\in C^{(s)}$ with the minimum distance separation $d_{\text{free}}^{(s)}$, we have

$$\eta = \lambda_1 = \lambda_2 = \ldots = \lambda_{L_t} = 4E_s d_{\text{free}}^{(s)} \tag{10}$$

which was to be shown. □

Table I compares the product distance achieved by the concatenated coding approach and the upper bound in Lemma 7 for some exemplary scenarios. In this comparison, the constraint lengths of C and $C^{(s)}$ are the same to allow for the same decoder complexity. In all considered cases, it is shown that the concatenated coding approach achieves either optimal or very-near-optimal performance. It is also worth noting that the same optimality argument for this overlay design approach holds for QPSK-modulated systems with only two transmit antennas in quasi-static fading channels.

V. Comment on the Extension to Block Fading Channels

In block fading channels, the code word is composed of multiple blocks. The fading coefficients are constant over one fading block but are independent from block to block. The number of fading blocks per code word M can be regarded as a measure of the interleaving delay allowed in the system, so that systems subject to a strict delay constraint are usually characterized by a small number of independent blocks [13].

It is easy to see that the framework developed in Section III for the quasi-static fading channel can be extended to block fading channels using the machinery introduced in [2]. The objective in this scenario is to exploit both temporal and spatial diversity available in the system. In such channels, the maximum transmit diversity advantage possible with space–time overlays (without factoring in the receive antennas effect) is given by [2], [8], [14]

$$d_m = \left\lfloor L_t M \left(1 - \frac{r}{L_t \log_2 |\Omega|}\right)\right\rfloor + 1 \tag{11}$$

where L_t is the number of transmit antennas, M is the number of fading blocks per code word, r is the transmission rate, and $|\Omega|$ is the size of the constellation alphabet. It is interesting to compare this result with the maximum diversity advantage possible for the single-antenna system supporting the same transmission throughput [14]

$$d_s = \left\lfloor M \left(1 - \frac{r}{\log_2 |\Omega|}\right)\right\rfloor + 1 \tag{12}$$

where it is clear that $d_m \geq L_t \times d_s$. This inequality suggests that design approaches optimized for quasi-static fading channels may not yield the maximum possible diversity advantage for block fading channels. The primary example is the concatenated coding approach with inner block orthogonal space–time codes discussed in the previous section. In the previous section, we argued that this approach yields excellent performance in quasi-static fading channels. Unfortunately, this excellent performance does not carry on to block fading channels in general. The reason is that the simple ML decoder dictates the transmission of a complete inner code word in the same fading state [5]. This limits the maximum possible diversity advantage to $d_{\text{conc}} = L_t \times d_s$. Table II compares d_{conc}, d_m for some exemplary scenarios.

TABLE II
Possible Diversity Advantages of the Algebraic Overlay Approach and the Concatenated Coding Approach in a BPSK System With 0.5 b/s/Hz

L_t	M	d_{conc}	d_m
2	1	2	2
2	2	4	4
2	3	4	5
2	4	6	7
2	5	6	8
3	1	3	3
3	2	6	6
3	3	6	8
3	4	9	11
3	5	9	13

TABLE III
Space–Time Overlays for Rate 1/2 Coded Systems

$K=\nu+1$	$L_t=1$	$L_t=2$	$L_t=3$
4	15,17	15,17,13,15	15,17,13,15,17,13
5	23,35	23,35,25,37	23,35,25,37,27,33
6	53,75	53,75,67,71	53,75,67,71,55,57
7	133,171	133,171,117,165	133,171,117,165,151,137

TABLE IV
Space–Time Overlays for Rate 1/3 Coded Systems

$K=\nu+1$	$L_t=1$	$L_t=2$
4	13,15,17	13,15,17,17,13,15
5	25,33,37	25,33,37,35,27,35
6	47,53,75	47,53,75,65,57,73
7	133,145,175	133,145,175,175,175,133
8	225,331,367	225,331,367,277,263,355

VI. Numerical Results

In this section, we present results for the proposed algebraic space–time overlays for convolutionally coded systems. We present the search results for algebraic space–time overlays obtained from underlying rate 1/2 and rate 1/3 convolutional codes [15]. In particular, Table III presents algebraic overlays for systems with underlying single-dimensional rate 1/2 convolutional codes. We consider $L_t = 1, 2, 3$ transmit antennas and convolutional codes with constraint lengths of $K = 4, \ldots, 7$. All codes achieve full diversity for both BPSK and QPSK transmissions (with Gray mapping) over the quasi-static fading channel, i.e., they satisfy the BPSK and QPSK rank criteria. Furthermore, with the exception of the constraint length $K = 5$ convolutional code which achieves free distance of $d_{\text{free}} - 1$, those codes achieve optimal values of the free distance d_{free}.

In Table IV, we present overlays obtained for systems with underlying rate 1/3 convolutional codes. We consider $L_t = 1, 2$ transmit antennas and convolutional codes with constraint lengths $K = 4, \ldots, 7$. All codes provide optimal values of d_{free} while achieving full diversity for both BPSK and QPSK transmissions.

Next, we present simulation results for the proposed algebraic convolutional space–time overlays. These results demonstrate the excellent performance achieved by the proposed designs and quantify the possible improvements with increasing numbers of transmit antennas. In all the examples, one frame corresponds to 130 transmissions for all antennas. We focus on the scenario with rate 1/2 single-dimensional code, and hence, the system achieves a spectral efficiency of 0.5 and 1 b/s/Hz in the case of BPSK and QPSK modulation, respectively.

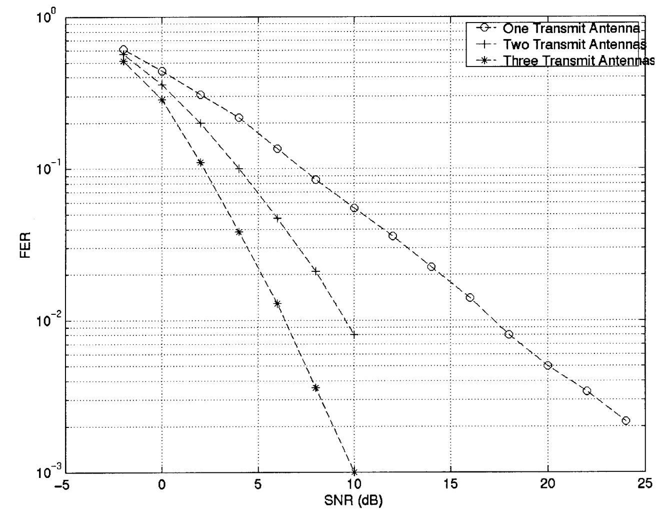

Fig. 1. Performance of $K = 7$ algebraic BPSK space–time overlays with different number of transmit antennas $(L_r = 1)$.

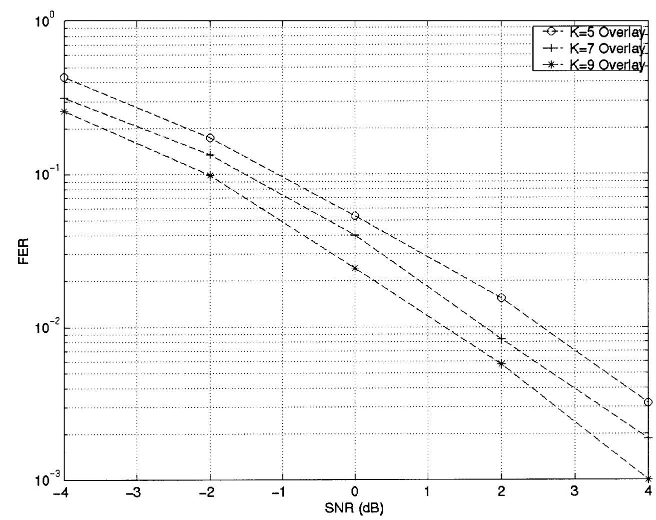

Fig. 2. Performance of algebraic BPSK space–time overlays with different constraint lengths $(L_t = 2, L_r = 2)$.

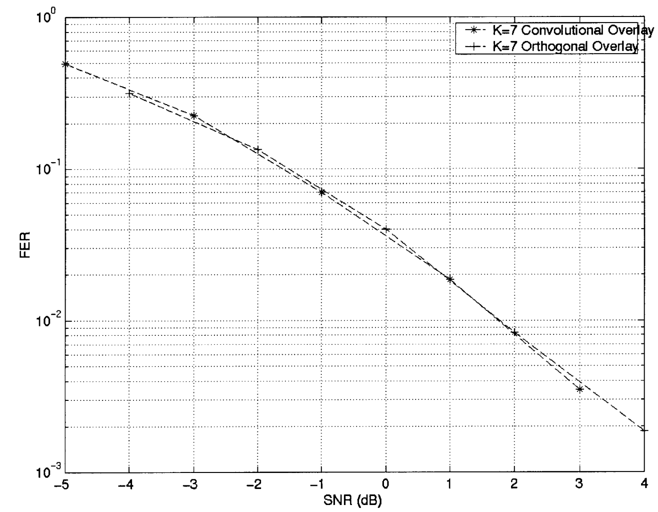

Fig. 3. Performance of the $K = 7$ algebraic BPSK space–time overlay and the concatenated coding approach with inner orthogonal code and outer $K = 7$ convolutional code with optimal free distance $(L_t = 2, L_r = 2)$.

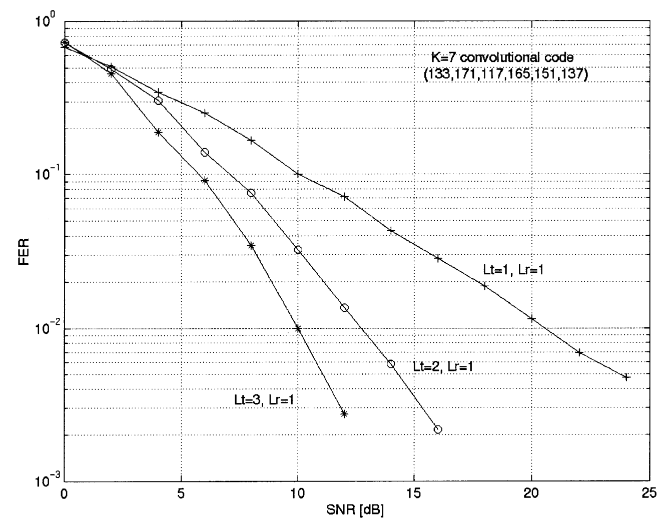

Fig. 4. Performance of $K = 7$ algebraic QPSK space–time overlays with different number of transmit antennas $(L_r = 1)$.

In the first set of figures, we consider BPSK-modulated systems. Fig. 1 provides performance comparisons for the constraint length 7 algebraic space–time overlays with one, two, and three transmit antennas. The number of receive antennas is one in the three cases. We observe that at a frame-error rate (FER) of 10^{-1} the systems with two and three transmit antennas provide gains of approximately 3 and 5 dB over the underlying single antenna system. At a FER of 10^{-2}, the gains of the convolutional space–time overlays with two and three antennas compared to the single-antenna system are even higher: 8 and 10.5 dB, respectively. In Fig. 2, we compare the performance of space–time overlays with different constraint lengths. We consider the case with two transmit and two receive antennas. The performance of convolutional space–time codes is shown to improve as the constraint length of the code increases. For example, the constraint length $K = 9$ convolutional code outperforms the constraint length $K = 5$ code by 1.5 dB. Fig. 3 compares the performance of the algebraic convolutional space–time overlay and that of the concatenated coding approach with inner orthogonal codes. In this particular scenario, we can see that both approaches provide identical performance.

The same comparisons are then repeated for QPSK-modulated systems. In Fig. 4, we quantify the gain obtained by increasing the number of transmit antennas when algebraic space–time overlays are used. At a FER of 10^{-1}, the systems with two and three transmit antennas provide gains of approximately 3 and 4.5 dB, whereas at a FER of 10^{-2}, the gains increase to 7.5 and 10 dB, respectively. Fig. 5 compares the performance of space–time overlays with different constraint lengths in a system with two transmit and two receive antennas. It is shown that the constraint length $K = 9$ space–time code outperforms the constraint length $K = 5$ code by 1.5 dB, similar to the BPSK scenario. The performance of the $K = 5$ and $K = 7$ algebraic overlays is compared in Fig. 6 for the case of three transmit and three receive antennas, where it is shown that the $K = 7$ convolutional code outperforms the $K = 5$ code by 1 dB at a FER of 10^{-2}.

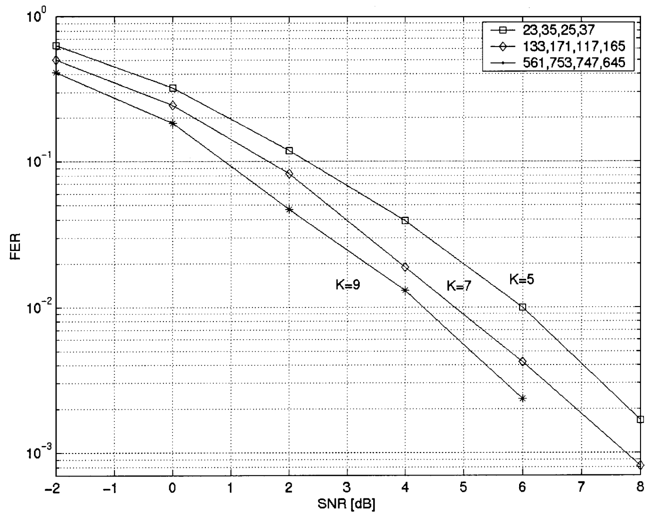

Fig. 5. Performance of algebraic QPSK space–time overlays with different constraint lengths ($L_t = 2, L_r = 2$).

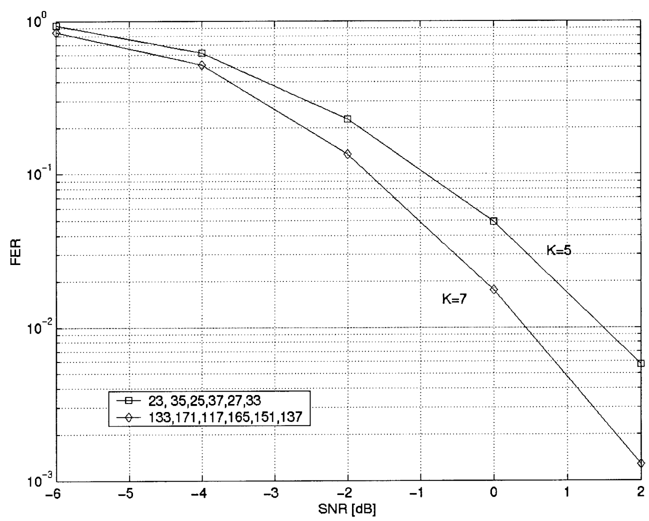

Fig. 6. Performance of algebraic QPSK space–time overlays with different constraint lengths ($L_t = 3, L_r = 3$).

Finally, a comparison between the proposed QPSK overlays and the best BPSK space–time convolutional codes [4] is presented in Fig. 7. We illustrate the FER performance of the constraint length $K = 7$ convolutional codes. Both systems achieve a spectral efficiency of 1 b/s/Hz. We consider two transmit and one or two receive antennas. In both cases, the QPSK space–time overlay outperforms the best known BPSK space–time convolutional code by about 1 dB. This gain can be attributed to the lower coding rate of the proposed overlay. Clearly, the space–time overlays for convolutionally coded systems are not only suitable as an upgrade of existing single-antenna systems, but they represent an appealing solution due to their excellent performance in the multiantenna system.

VII. Conclusions

In this paper, we considered the design of space–time overlays to upgrade convolutionally coded single-antenna wireless communication systems. We developed an algebraic framework for constructing convolutional space–time overlays that achieve full spatial diversity in quasi-static fading channels without altering the signal transmitted from the first antenna. For BPSK-modulated systems, we presented a general approach for constructing space–time overlay codes with the same trellis complexity as the code used in the single-antenna system. The general approach for QPSK-modulated systems involves the use of systematic inner space–time codes that require separate soft input/soft output decoders at the receiver. For QPSK-modulated systems using rate $1/n$ binary convolutional codes with Gray mapping, we presented an alternative space–time construction with the same trellis complexity as the single-dimensional convolutional code.

We also investigated the use of orthogonal designs as inner space–time block codes that achieve full spatial diversity [5]. For BPSK-modulated signals, we showed that this approach satisfies the overlay constraint, while achieving optimal or very-near-optimal performance in quasi-static fading channels. On the other hand, this approach only satisfies the overlay constraint for QPSK-modulated systems with two transmit antennas. For more than two transmit antennas, larger constellation sizes are needed to allow for the same transmission throughput [5].

In Section V, we offered a brief comment on the extension of the design framework to block fading channels. This extension is straightforward using the machinery introduced in [2]. We showed that in such channels the orthogonal inner space–time coding approach does not achieve the maximum diversity advantage possible using the algebraic framework presented in this paper. As a final remark, it is worth noting that the choice of convolutional codes as the basis of our presentation was only motivated by their widespread use in most practical applications. The framework established in this paper for constructing algebraic space–time overlays can be easily extended to block coded systems.

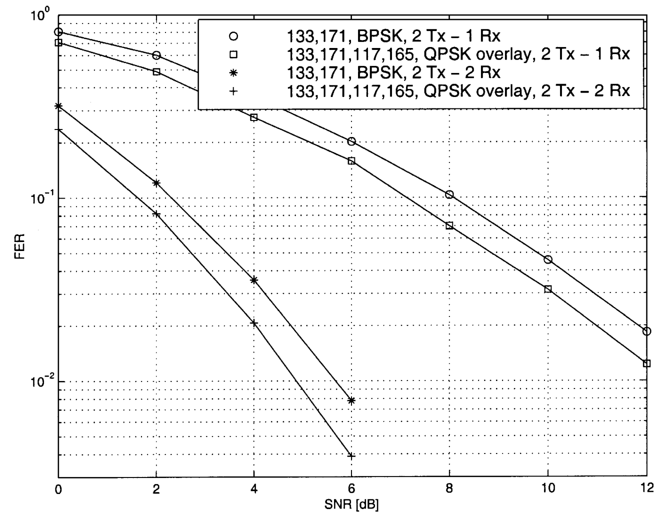

Fig. 7. FER performance comparison for the $K = 7$ BPSK space–time convolutional code and the $K = 7$ QPSK space–time overlay for $L_t = 2$ transmit antennas and $L_r = 1$ or $L_r = 2$ receive antennas.

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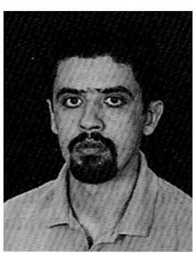

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