On the Design and Maximum-Likelihood Decoding of Space–Time Trellis Codes

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***Abstract*—In this letter, we present a simple generalization of the maximum ratio combining principle for space–time coded systems. This result leads to a maximum-likelihood decoder implementation that does not depend on the number of receive antennas and avoids the loss in performance incurred in the decoders proposed by Tarokh and Lo and Biglieri *et al.* The insights offered by this decoding rule allow for a simple and elegant proof for the space–time code design criterion in systems with large number of receive antennas. We further present an upper bound on probability of error that captures the dependence of space–time code design on the number of receive antennas. Finally, we present a computationally efficient approach for constructing space–time trellis codes that exhibit satisfactory performance in systems with variable number of receive antennas.**

***Index Terms*—Code design, maximum ratio combining (MRC), multiple-input multiple-output (MIMO) systems, space–time coding.**

I. Introduction

RELIABLE communications over the fading wireless channel is made possible only through the use of diversity techniques where the receiver is afforded multiple opportunities to observe the transmitted signal under varying channel conditions. In particular, antenna arrays are playing an increasingly important role in wireless communication systems, since the spatial diversity provided by multiple transmit and/or receive antennas allows for a significant increase in system capacity [3], [4], [5]. The challenge of designing channel codes for high capacity coherent multiantenna systems has led to the development of space–time codes, where coding is performed across the spatial dimension (antenna) as well as time (e.g., [6], [7], [8]).

One of the obstacles that may limit the utility of space–time coding in practice is the complexity of the maximum-likelihood (ML) decoder. For space–time trellis codes, the straightforward computations of branch metrics grow with the number of transmit and receive antennas [1], [7], [9]. In [1] and [9], the authors proposed suboptimal decoding approaches that avoid the dependency on the number of receive antennas at the expense of a loss in performance. In this letter, we show that this compromise is not necessary. Through simple manipulations of the Euclidean distance metric, we derive a generalized maximum ratio combining (MRC) rule for ML space–time decoding. This optimal decoding rule linearly transforms the observations to a smaller dimensional vector, which allow a lower complexity ML search. This rule also offers an alternative way for obtaining the design criterion for space–time codes in systems with a large number of receive antennas, where the Euclidean distance between codewords is the dominant factor. A similar result, however, with a different proof, was obtained independently in [2], [10]. Other authors have also identified Euclidean distance as an important metric [11], [12]. In particular, the work in [12] showed the importance of Euclidean distance not only for a large number of receive antennas, but also for low signal-to-noise ratio (SNR).

The design criterion for a large number of receive antennas is significantly different from the design criteria proposed in [6] and [7] for systems with a small receive array size. In this letter, we investigate the effect of number of receive antennas on the design of space–time codes. We present an upper bound on pairwise error probability that explicitly captures the dependence of design criteria on the number of receive antennas and operating SNR. Furthermore, we present a canonical construction with the potential to simultaneously optimize different design criteria.

The rest of this paper is organized as follows. Section II outlines the multiple-input multiple-output (MIMO) system model. In Section III, we present the generalized space–time decoding rule. Section IV benefits from the generalized MRC result to derive the design criterion for space–time codes in systems with a large number of receive antennas. In Section V, we present an upper bound on the performance of space–time codes and a canonical construction that explicitly accounts for the different design criteria. Finally, we offer some concluding remarks in Section VI.

In this letter, boldface letters denote matrices and $\vec{x}$ denotes the vector variable x. For random variables, we use capital letters to denote random variables and lower-case letters to denote the realizations. Furthermore, $\mathbb{Z}_n$ denotes the ring of integers modulo n.

II. System Model

We consider a communication system with linear modulation that uses L_t transmit and L_r receive antennas and a frame length of N_f symbols. In this letter, we are interested in the scenario where the fading channel is frequency nonselective. In this MIMO system, a space–time encoder is used to encode and distribute the symbols among the L_t antennas. The transmissions from the L_t transmit antennas are simultaneous and synchronous. The signal received at each antenna is, therefore, a superposition of the L_t transmitted signals corrupted by additive white Gaussian noise (AWGN) and multiplicative fading. At the receiver end, the signal $Y_j(t)$ received by antenna j at

Paper approved by M. Z. Win, the Editor for Equalization and Diversity of the IEEE Communications Society. Manuscript received March 20, 2001; revised September 24, 2002. This work was supported in part by the National Science Foundation under Grant 0118859. This paper was presented in part at the 36th Annual Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November 2002.

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Digital Object Identifier 10.1109/TCOMM.2003.813177

time t is then processed in a matched filter to the pulse shape. The output samples of the matched filter are given as a $1 \times L_r$ vector

$$\vec{Q}(k) = \sqrt{E_s}\vec{D}(k)\mathbf{H}(k) + \vec{N}(k) \quad (1)$$

where E_s is the energy per transmitted symbol, $H_{ij}(k)$ is the complex path gain from transmit antenna i to receive antenna j at time kT, $\vec{D}(k)$ is the $1 \times L_t$ vector of symbols transmitted for symbol k, and $\vec{N}(k)$ is the AWGN vector of size $1 \times L_r$. The noise samples are independent samples of circularly symmetric zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension. At each time kT, the different path gains $H_{ij}(k)$ are assumed to be statistically independent. Similar to [7], if we assume that the whole codeword is transmitted in a single coherence interval (i.e., $H_{ij}(k) = H_{ij}, \ \forall k$), the model reduces to the quasi-static fading model studied extensively in [5], [6], [7] and [8].

III. Generalized MRC

Assuming perfect channel state information at the receiver, the ML decoder, when $\mathbf{H}(k) = \mathbf{h}(k)$, is

$$\begin{aligned}\hat{\vec{I}} &= \arg\max_n T_n \\ &= \arg\min_n \sum_{k=1}^{N_f} \left(\vec{Q}(k) - \sqrt{E_s}\vec{d}^{(n)}(k)\mathbf{h}(k)\right) \\ &\quad \cdot \left(\vec{Q}(k) - \sqrt{E_s}\vec{d}^{(n)}(k)\mathbf{h}(k)\right)^H \\ &= \arg\max_n \sum_{k=1}^{N_f} 2\sqrt{E_s}\Re\left[\vec{Q}(k)\mathbf{h}^H(k)\left(\vec{d}^{(n)}(k)\right)^H\right] \\ &\quad - E_s\vec{d}^{(n)}(k)\mathbf{h}(k)\mathbf{h}^H(k)\left(\vec{d}^{(n)}(k)\right)^H \end{aligned} \quad (2)$$

where T_n is the ML metric, n is used to enumerate all possible transmitted bit sequences, and $\vec{d}^{(n)}(k)$ is used to denote the transmitted codeword at symbol k for transmitted bit sequence n. For space–time trellis codes, the straightforward computation of this metric requires a branch metric complexity that grows with the number of receive antennas. In [1], the authors proposed a suboptimal decoding algorithm that reduces the complexity by a factor of L_r (i.e., avoids the dependency on the number of receive antennas). This complexity reduction, however, entails a loss in performance which can be significant in some scenarios [1].

Here, following the lead of [13], we show that it is possible to formulate an ML decoding structure which has a linear combiner followed by an ML search that is not a function of the number of receive antennas. Consider the $1 \times L_t$ vector

$$\vec{V}(k) = \vec{Q}(k)\mathbf{h}(\mathbf{k})^H = \begin{bmatrix} \sum_{j=1}^{L_r} Q_j(k)h_{1j}^*(k) \\ \vdots \\ \sum_{j=1}^{L_r} Q_j(k)h_{L_tj}^*(k) \end{bmatrix}^T \quad (3)$$

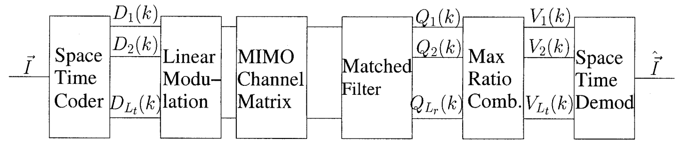


Fig. 1. Generalized MRC combiner implementation of the ML decoder.

then the ML decoder, when $\mathbf{H}(k) = \mathbf{h}(k)$, has the form

$$\begin{aligned}\hat{\vec{I}} = \arg\max_n \sum_{k=1}^{N_f} 2\sqrt{E_s}\Re\left[\vec{V}(k)\left(\vec{d}^{(n)}(k)\right)^H\right] \\ -E_s\vec{d}^{(n)}(k)\mathbf{h}(k)\mathbf{h}^H(k)\left(\vec{d}^{(n)}(k)\right)^H. \end{aligned} \quad (4)$$

In fact, once $\vec{V}(k)$ and $\tilde{\mathbf{h}}(k) = \mathbf{h}(k)\mathbf{h}^H(k)$ are computed, the complexity of the ML decoding is completely independent of L_r. The complexity of computing $\vec{V}(k)$ is $O(L_rL_t)$ and must be done for every matched filter output. The complexity of computing $\tilde{\mathbf{h}}(k)$ is $O(L_r^2L_t^2)$ and must be done for every matched filter output. If the fading is quasi-static, the computation of $\tilde{\mathbf{h}}$ need only be done once for each frame. It should be noted that the ith component of $\vec{V}(k)$ is, in fact, the output of a MRC [14] for transmission from the ith antenna. This demodulation structure is shown in Fig. 1.

IV. Design Criterion for Systems With a Large Number of Receive Antennas

The implementation presented in the last section for the ML decoder allows for some interesting insights in the asymptotic case of a large number of receive antennas. To allow for some meaningful discussion, a quasi-static channel will be assumed. We further note that a decision will not change if the ML metric is multiplied by a constant that is not a function of the observations or data. Denoting a new ML decoder metric

$$\begin{aligned}\tilde{T}_n = \frac{T_n}{L_r} = \frac{1}{L_r}\sum_{k=1}^{N_f} 2\sqrt{E_s}\Re\left[\vec{V}(k)\left(\vec{d}^{(n)}(k)\right)^H\right] \\ -\frac{1}{L_r}E_s\vec{d}^{(n)}(k)\mathbf{H}\mathbf{H}^H\left(\vec{d}^{(n)}(k)\right)^H \end{aligned} \quad (5)$$

the question of interest is how does the decoder metric behave with respect to the random channels in the limit as the number of receive antennas gets large. This limit is expressed as

$$\begin{aligned}\lim_{L_r\to\infty}\tilde{T}_n = \sum_{k=1}^{N_f} 2\sqrt{E_s}\Re\left[\left(\lim_{L_r\to\infty}\frac{\vec{V}(k)}{L_r}\right)\vec{d}_n^H(k)\right] \\ -E_s\vec{d}_n(k)\left(\lim_{L_r\to\infty}\frac{\tilde{\mathbf{H}}}{L_r}\right)\vec{d}_n^H(k). \end{aligned} \quad (6)$$

The two limits are easily evaluated by noting that $\vec{V}(k) = \sqrt{E_s}\vec{D}(k)\tilde{\mathbf{H}} + \tilde{\vec{N}}(k)$ where $\tilde{\vec{N}}(k)$ is a Gaussian noise with covariance matrix $\tilde{\mathbf{H}}N_0$, and using the strong law of large numbers

$$\begin{aligned}\lim_{L_r\to\infty}\frac{\tilde{\mathbf{H}}}{L_r} &= \mathbf{I}_{L_t} \\ \lim_{L_r\to\infty}\frac{\vec{V}(k)}{L_r} &= \sqrt{E_s}\vec{D}(k) + \lim_{L_r\to\infty}\frac{\tilde{\vec{N}}(k)}{L_r} \end{aligned} \quad (7)$$

where in the limit the covariance matrix of $\vec{\tilde{N}}(k)/L_r$ is $N_0\mathbf{I}_{L_t}$.

Equations (6) and (7) offer some interesting insights into this asymptotic scenario. From (7), it is seen that the large number of receive antennas not only averages out the fading effect associated with each transmit antenna, but also when MRC is used, it effectively decorrelates the signals from different transmit antennas. This MRC essentially transforms the MIMO fading channel into L_t parallel equal-power AWGN channels. The design criterion for this scenario is well understood [15], [16], and amounts to maximizing the Euclidean distance between different codewords. One can also see from (6) that the decoder converges to the ML decoder for the case of parallel AWGN channels. This asymptotic design criterion was obtained independently using a different approach in [2] and [10].

V. Space–Time Code Design for an Arbitrary Receive Array Size

As argued in the previous section, in quasi-static fading channels, the design criterion for space–time codes in systems with a large number of receive antennas is to maximize the Euclidean distance between all distinct pairs of codewords. This criterion is significantly different from the criteria for systems with a small receive array size. Let $\mathbf{d}_n$ be the two-dimensional codeword matrix corresponding to the transmitted bit sequence n, and the space–time code $\mathcal{D}$ be the collection of these codewords. Then, it was shown in [6] and [7] that the code design criteria, for systems with a small receive array size, are as follows.

• *Diversity Advantage*: Maximize $\Delta_H(n_1, n_2) = \text{rank}(\mathbf{d}_{n_1} - \mathbf{d}_{n_2})$ over all pairs of distinct codewords $\mathbf{d}_{n_1}, \mathbf{d}_{n_2} \in \mathcal{D}$.

• *Coding Gain*: Maximize the geometric mean of the nonzero eigenvalues of $\mathbf{C}_s(n_1, n_2) = (\mathbf{d}_{n_1} - \mathbf{d}_{n_2})(\mathbf{d}_{n_1} - \mathbf{d}_{n_2})^H$ over all distinct pairs of codewords $\mathbf{d}_{n_1}, \mathbf{d}_{n_2} \in \mathcal{D}$.

A great deal of work has gone into designing codes based on these design criteria. Some noteworthy papers are [17] and [18].

The Euclidean distance criterion can also be written as follows.

• *Euclidean Distance*: Maximize over all distinct pairs of codewords $\mathbf{d}_{n_1}, \mathbf{d}_{n_2} \in \mathcal{D}$ the arithmetic mean of the eigenvalues of $\mathbf{C}_s(n_1, n_2) = (\mathbf{d}_{n_1} - \mathbf{d}_{n_2})(\mathbf{d}_{n_1} - \mathbf{d}_{n_2})^H$.

One should first note that the boundary between the two scenarios is not well defined. Furthermore, in the typical downlink of a wireless system, one would expect different users to have different numbers of receive antennas. It is, therefore, desirable to develop a general framework that can provide satisfactory performance in various scenarios. In this section, we first present an upper bound on the probability of error that captures the dependence of code design on the number of receive antennas. We then present a space–time trellis code construction that explicitly accounts for the different design criteria.

A. An Upper Bound on Pairwise Error Probability

Following in the footsteps of [6] and [7], one can simply derive the following upper bound on pairwise error probability (PWEP) of the decoder favoring the erroneous codeword $\mathbf{d}_{n_2}$ over the transmitted codeword $\mathbf{d}_{n_1}$

$$\begin{aligned} P(n_1, n_2) &\le \left(\prod_{i=1}^{L_t}\left(1 + \lambda_i \frac{E_s}{4N_0}\right)\right)^{-L_r} \\ &\le \Bigg(1 + \Delta_E(n_1, n_2)\frac{E_s}{4N_0} \\ &\qquad + \Delta_P(n_1, n_2)\left(\frac{E_s}{4N_0}\right)^{\Delta_H(n_1, n_2)}\Bigg)^{-L_r} \end{aligned} \tag{8}$$

where λ_i is the ith nonzero eigenvalue of $\mathbf{C}_s(n_1, n_2)$, and the Euclidean distance (Δ_E) and the product measure (Δ_P) associated with this error event are given as

$$\Delta_E(n_1, n_2) = \text{trace}(\mathbf{C}_s(n_1, n_2)) = \sum_{i=1}^{\Delta_H(n_1, n_2)} \lambda_i \tag{9}$$

$$\Delta_P(n_1, n_2) = \prod_{i=1}^{\Delta_H(n_1, n_2)} \lambda_i. \tag{10}$$

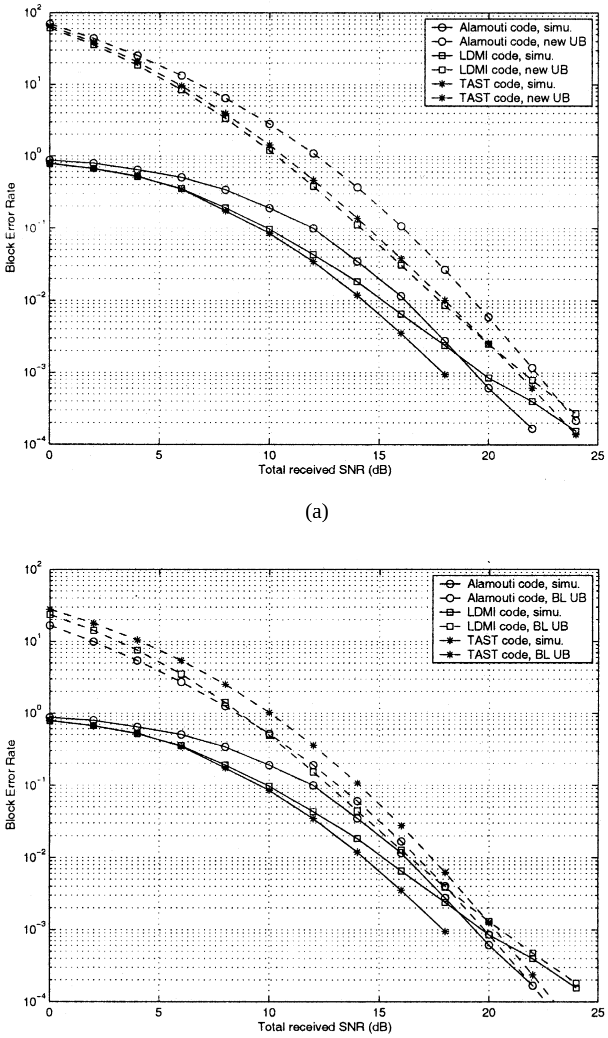
From this bound, one can see that the performance of a space–time code depends on the number of receive antennas and operating SNR. For example, for large number of receive antennas, the frame error rates of interest (e.g., 10^{-3}–10^{-2}) are typically achieved at small SNRs. For such small SNRs (i.e., less than one), it is clear the Euclidean distance term will dominate the bound. For asymptotically high SNR, however, the diversity advantage and coding gain criteria will be important. This observation is also made independently in [9].

One can further use this upper bound in a union bound analysis to obtain the following upper bound on frame-error probability

$$\begin{aligned} P_f \le \sum_{\Delta_H}\sum_{\Delta_E}\sum_{\Delta_P} N(\Delta_H, \Delta_E, \Delta_P) \\ \cdot\left(1 + \Delta_E\frac{E_s}{4N_0} + \Delta_P\left(\frac{E_s}{N_0}\right)^{\Delta_H}\right)^{-L_r} \end{aligned} \tag{11}$$

where $N(\Delta_H, \Delta_E, \Delta_P)$ represents the relative frequency of error events with rank Δ_H, Euclidean distance Δ_E, and product measure Δ_P.

To demonstrate the utility of this bound, we consider three space–time block codes with $R = 4$ bits per channel use (PCU) for $L_t = 2$: the Alamouti code [19], the linear dispersion code optimized with respect to mutual information criterion (LDMI) [20], and the threaded algebraic space–time code (TAST) [21]. In Fig. 2, the simulated block error performance of the three codes are compared with the union bound given in (11) for the $L_r = 2$ case. In the same figure, the simulated error performance is compared with the union bound obtained by using the upper bound on PWEP proposed by Byun and Lee (BL) in [22]. The BL PWEP upper bound is only a function of the product measure and argued to be the tightest upper bound for a given Δ_P. From the figure, it is observed that even though the new bound is not as tight as the BL union bound, it provides a more accurate characterization of the relative ordering of these three codes at low-to-medium SNR range. More simulation results that support this observation are presented in [23] and removed



(a)

(b)

Fig. 2. Comparison of block error performance of $R = 4$ bits PCU linear space–time block codes. (a) Union bound in (11). (b) BL union bound.

from here for brevity. Since this bound only requires the Euclidean distance and product measure spectra, which can be computed by the technique presented in [24], one can use it as a computationally efficient search tool for good space–time codes.

B. Canonical Space–Time Trellis Coding

We consider the code construction in Fig. 3. In this construction, a single-dimensional trellis code is first used to encode the data stream. The encoded stream is then distributed among the L_t antennas using the spatial parser. It is straightforward to see that, in this construction, the spatial parser will not affect the Euclidean distance between codewords of the single-dimensional code. Hence, the single-dimensional trellis code should be designed to maximize the Euclidean distance over all distinct pairs of codewords. This will also maximize

$$\Delta_E\,(\min) = \frac{1}{L_t} \min_{\mathbf{d}_{n_1},\, \mathbf{d}_{n_2} \in \mathcal{D}} \operatorname{trace}\left(\mathbf{C}_s(n_1,\, n_2)\right). \tag{12}$$

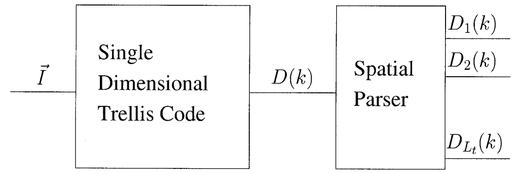


Fig. 3. Canonical construction for space–time coding.

Subject to this constraint on $\operatorname{trace}(\mathbf{C}_s(n_1,\, n_2))$, the optimum parser, which ensures full diversity and maximizes the coding gain, will distribute the symbols among the different antennas with a goal of maximizing

$$\Delta_P\,(\min) = \min_{\mathbf{d}_{n_1},\, \mathbf{d}_{n_2} \in \mathcal{D}} \det\left(\mathbf{C}_s(n_1,\, n_2)\right). \tag{13}$$

It should be noted that $\Delta_P\,(\min) \leq (\Delta_E\,(\min))^{L_t}$. One of the innovations in this construction is that it allows for utilizing the large body of work on trellis code design for AWGN channels. The problem is then reduced to optimum spatial parser design. Some progress toward this goal has been reported in [25], where spatial parsers that ensure full spatial diversity for general classes of binary phase-shift keying (BPSK) and quaternary phase-shift keying (QPSK) codes were presented.

More generally, the new construction allows for a computationally efficient searching procedure for identifying space–time codes that provide satisfactory performance in systems with arbitrary numbers of transmit and receive antennas. The code search first identifies optimum Euclidean distance codes (i.e., optimum with a large number of receive antennas) and then down-selects codes based on diversity advantage and coding gain criteria. We focus only on canonical constructions with natural parsing of convolutional codes where the output of a rate $1/L_t$ code is multiplexed among the antennas. This offers the additional advantage that the space–time ML decoder has the same trellis complexity as the single-dimensional code. Other parsers may, however, offer better performance at the expense of increased decoding complexity.

In general, trellis codes are not geometrically uniform with respect to the Euclidean distance. This means that for a trellis code with 2^ν states, one needs to search over a trellis with $2^{2\nu}$ states to find the minimum Euclidean distance of the code. However, by restricting ourselves to single-dimensional rate $1/L_t$ convolutional codes that are linear over the appropriate polynomial ring, the complexity of the minimum Euclidean distance computation can be significantly reduced (a search over trellis with 2^ν states). This complexity reduction highlights one of the advantages of our design approach. We avoid the large computational complexity involved in computing the nonlinear product measure for all possible encoders, since the nonlinear product measure is only computed for the small set of codes with optimal minimum Euclidean distance.

The space–time trellis codes for QPSK and 8-PSK modulated systems with $L_t = 2,\, 3$ transmit antennas identified with this searching procedure are reported in Tables I and II. QPSK and 8-PSK codes are linear over the $\mathbb{Z}_4$ and $\mathbb{Z}_8$ rings, respectively.

TABLE I
PROPOSED CODES FOR $L_t = 3$

Rate	Numb. states	$\mathbf{G}$	$\Delta_E(\min)$	Rate	Numb. states	$\mathbf{G}$	$\Delta_E(\min)$
2	16	$\begin{pmatrix} 3 & 3 & 1 \\ 3 & 2 & 2 \\ 3 & 2 & 1 \end{pmatrix}^\star$	24	2	64	$\begin{pmatrix} 3 & 3 & 2 & 3 \\ 3 & 1 & 1 & 3 \\ 2 & 1 & 2 & 2 \end{pmatrix}^\star$	32
2	32	$\begin{pmatrix} 3 & 3 & 3 & 2 \\ 3 & 2 & 3 & 0 \\ 3 & 1 & 0 & 2 \end{pmatrix}^\star$	26	3	64	$\begin{pmatrix} 7 & 7 & 5 \\ 6 & 4 & 1 \\ 5 & 5 & 6 \end{pmatrix}^\star$	20

TABLE II
PROPOSED CODES FOR $L_t = 2$

Rate	Numb. states	$\mathbf{G}$	$\Delta_E(\min)$	$\Delta_P(\min)$	Rate	Numb. states	$\mathbf{G}$	$\Delta_E(\min)$	$\Delta_P(\min)$
2	4	$\begin{pmatrix} 3 & 3 \\ 2 & 3 \end{pmatrix}$	10	4	3	8	$\begin{pmatrix} 7 & 6 \\ 4 & 5 \end{pmatrix}$	7.171	2
2	8	$\begin{pmatrix} 3 & 3 & 2 \\ 2 & 3 & 0 \end{pmatrix}$	12	12	3	16	$\begin{pmatrix} 7 & 7 & 4 \\ 6 & 7 & 0 \end{pmatrix}^\star$	7.757	0.686
2	16	$\begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$	16	8	3	32	$\begin{pmatrix} 7 & 7 & 6 \\ 5 & 6 & 2 \end{pmatrix}$	10	2.059
2	32	$\begin{pmatrix} 3 & 3 & 3 & 2 \\ 2 & 0 & 3 & 0 \end{pmatrix}^\star$	16	32	3	64	$\begin{pmatrix} 7 & 4 & 1 \\ 6 & 5 & 6 \end{pmatrix}$	11.515	1.373
2	64	$\begin{pmatrix} 3 & 3 & 3 & 2 \\ 2 & 1 & 0 & 3 \end{pmatrix}^\star$	18	28					

The generator matrix $\mathbf{G}$ is constructed such that the output sequence for the ith transmit antenna for input symbol sequence $x(k)$ is given by

$$d_i(k) = \sum_{j=1}^{\nu+1} g_{ij} x(k-j+1) \tag{14}$$

where g_{ij} is the element in the ith row and jth column of $\mathbf{G}$ and the sum is over the appropriate ring. It should be noted that the search resulted in many pareto optimal codes and we have only reported one of them. In the tables, codes marked with $\star$ are not optimized with respect to coding gain due to the large search space. Therefore, linear codes with the same minimum Euclidean distance but better product measure spectrum may exist in these cases.

Fig. 4 compares the performance of the 32-state QPSK code for $L_t = 3$ in Table I to two other codes with the same trellis complexity [26]. The two codes from [26] are constructed such that one has good Euclidean distance (good for a large number of receive antennas) but is rank deficient (bad for a small number of receive antennas), while the other is full diversity but has worse minimum Euclidean distance. The newly proposed code provides uniformly better performance than the two codes in [26]. Since the new code has a good product measure spectrum, it provides better performance with a small number of receive antennas. Additionally, the code was optimized to have a good Euclidean distance characteristic in order to facilitate good performance with a large number of receive antennas. This figure demonstrates that universal codes, with good performance in wide variety of applications, do exist.

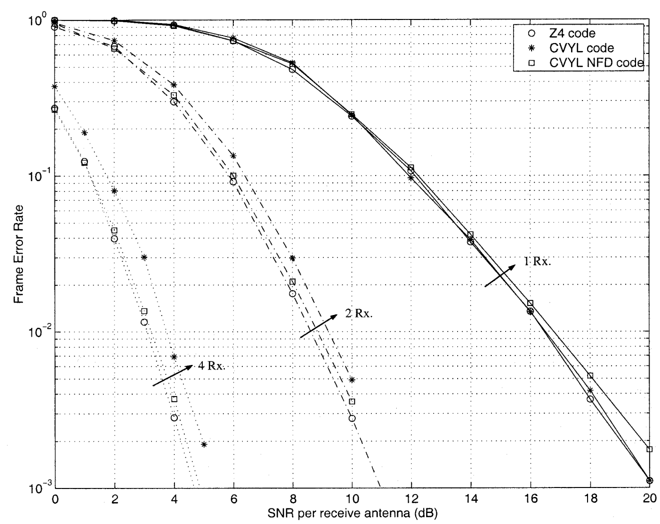


Fig. 4. Performance of space–time code designed for a more universal application, 32 states, $R = 2$ bits PCU, $L_t = 3$.

In Fig. 5, we compare the performance of three eight-state 8-PSK code for $L_t = 2$. The proposed code in Table II ($\mathbb{Z}_8$), the code in [12] (TC) which is optimized with respect to $\det(\mathbf{I} + \mathbf{C}_s)$ (equivalent to optimizing performance for $E_s/N_0 = 4L_t$) and is rank deficient, and the code in [7] (TSC) which is full diversity and claimed to have optimal $\Delta_P(\min)$ in [12]. From

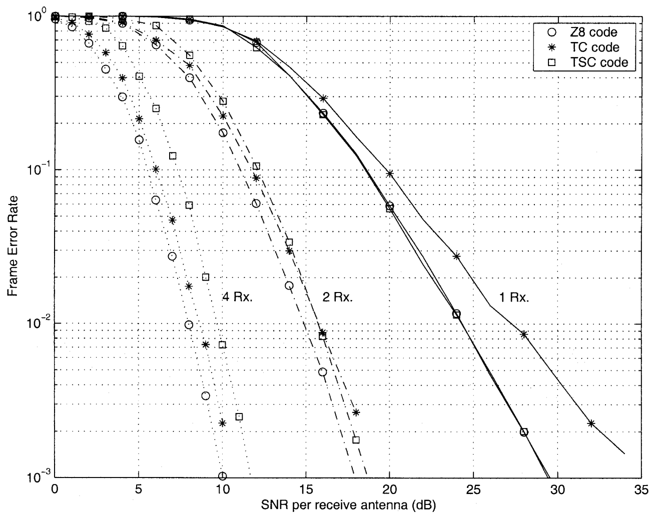


Fig. 5. Performance of space–time code designed for a more universal application, eight states, $R = 3$ bits PCU, $L_t = 2$.

the figure, it is observed that the code identified by our method provides uniformly better performance than the other two codes. The proposed code has the same $\Delta_P(\min)$ as the TSC code and it has the largest minimum Euclidean distance. This result demonstrates that our search method provide codes with better performance than the design criterion in [12] in this scenario.

As a final note, comparing the minimum Euclidean distance of the proposed codes to the codes in [10] and [26], it is observed that restricting the search to linear codes, over the appropriate ring of polynomials, does not result in a loss in the minimum Euclidean distance, except for the 16-state 8-PSK code for $L_t = 2$, where a marginal loss (7.75 for the new code versus 8 for the CYV code [10]) is observed.

VI. Conclusions

In this letter, we presented a simple generalization to the MRC principle for space–time coded systems. This generalization allowed for a significant reduction in the ML decoder complexity, especially for systems with a large number of receive antennas. Contrary to the approaches in [1] and [9], this reduction does not entail any loss in performance. Furthermore, this decoder implementation allowed for some interesting insights into the asymptotic scenario with a large number of receive antennas. We presented an upper bound on error performance that allows for optimizing the code design based on the number of receive antennas. Finally, we presented a canonical space–time code construction with the potential to optimize the performance in various scenarios.

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