On the Robustness of Space-Time Coding

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***Abstract*—Recently, space-time (ST) coding has emerged as one of the most promising technologies for meeting the challenges imposed by the wireless channel. This technology is primarily concerned with two-dimensional (2-D) signal design for multitransmit antenna wireless systems. Despite the recent progress in ST coding, several important questions remain unanswered.**

In a practical multiuser setting, one would expect different users to experience different channel conditions. This motivates the design of *robust* ST codes that exhibit satisfactory performance in various environments. In this paper, we investigate the robustness of ST codes in line-of-sight and correlated Rayleigh fading channels. We develop the design criteria that govern the performance of ST codes in these environments. Our analysis demonstrates that *full-diversity* ST codes are essential to achieving satisfactory performance in line-of-sight channels. We further show that a simple *phase randomization* approach achieves significant performance gains in the line-of-sight case without affecting the performance in Rayleigh fading channels. In the correlated fading environments, we characterize the achievable diversity order based on the number of diversity degrees of freedom in the channel. This characterization supports the recent experimental observations that suggest that the *quasistatic* model is not a *worst-case* scenario and establishes the *necessary* tradeoff between the transmission rate and performance robustness. Finally, we consider the design of ST codes using some *prior* knowledge about the channel *spatio-temporal* correlation function.

***Index Terms*—Fading channels, space-time codes, transmit diversity, wireless communication.**

I. Introduction

It is widely accepted that reliable communication in fading channels is possible only through the use of diversity techniques in which the receiver is afforded multiple replicas of the transmitted signal under varying channel conditions. Commonly used methods include

1) frequency diversity, in which the signal is transmitted on multiple RF carriers;
2) temporal diversity, in which channel coding and interleaving are used to replicate and distribute the signal in time;
3) antenna or spatial diversity, in which multiple antennas are used at the transmitter and/or the receiver to provide multiple replicas of the signal with decorrelated fading characteristics.

Recent information-theoretic studies have shown that spatial diversity allows for a significant increase in the capacity of wireless communication systems operating in a fading environment [1]–[3]. Tarokh *et al.* coined the term "space-time (ST) codes" to describe the two-dimensional (2-D) signals used in multiple transmit antenna systems. Earlier work on ST coding has focused on the quasistatic flat fading model (e.g., [4]–[6]). More recent works have considered extensions to frequency-selective and time-selective fading channels (e.g., [7]–[14]). Despite this progress, several fundamental issues remain open for further investigations.

In a typical wireless system, one would expect different users to experience different channel conditions that range from the spatially white Rayleigh fading channel[1] to the correlated Rayleigh fading channel with arbitrary *spatio-temporal* correlation function or the *dominant* line-of-sight channel[2]. Because the *user-dependent* channel statistics are not known *a priori* in most practical cases, it is necessary to develop a universal design paradigm for these different scenarios. The success of ST coding in practice, therefore, hinges on its ability to achieve *satisfactory* performance in all these environments. This paper takes a first step toward constructing *universal* ST codes for generalized channel models.

First, we analyze the performance of ST codes in dominant line-of-sight channels. We show that full diversity ST codes are *necessary* to achieve satisfactory performance in this case. This result limits the maximum transmission rate possible with robust ST coding as shown in Section III. We further propose a simple *phase randomization* strategy that allows for significant gains in dominant line-of-sight scenarios without affecting the performance in flat Rayleigh fading channels. As part of the review process for the initial submission of this paper, one of the reviewers pointed us to Kose and Wesel work [15], where a worst-case analysis for ST codes was developed. The differences and similarities between our analysis and that in [15] are outlined at the end of Section III-A.

Second, we investigate the worst-case performance of ST codes in Rayleigh fading channels with arbitrary *spatio-temporal* correlation functions. Most works in the literature used the quasistatic Rayleigh fading as a *worst-case* scenario (e.g., [4]). By characterizing the achievable diversity advantage based on the diversity degrees of freedom in the channel,[3] we will show that this is not true. Our analysis, therefore, explains the recent experimental observation in [16], where the *smart-greedy* codes of [4] were shown to yield worse performance in certain *spatio-temporal* correlated channels

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[1]This channel represents the most commonly used model in the ST coding literature.

[2]We will argue later that the Rice distribution used in [4] does not *faithfully* represent this scenario.

[3]The number of degrees of freedom in the channel will be defined later in the sequel.

than in the quasistatic scenario. Our analysis further characterizes the necessary tradeoff between the transmission rate and achievable diversity order in generalized *spatio-temporal* correlated fading channels. Finally, we consider the design of ST codes using some *prior* knowledge about the channel *spatio-temporal* auto-correlation function.

The focus in this paper will be devoted to *time-selective frequency-flat* fading channels. The duality between ST codes in time-selective channels and space-frequency codes in frequency-selective channels [9], however, means that our results apply to orthogonal frequency division multiplexing (OFDM) systems operated in frequency-selective channels.

The rest of this paper is organized as follows. Section II presents the multiantenna signaling model used throughout the paper. In Section III, we investigate the performance of ST codes in dominant line-of-sight channels. Section IV is devoted to correlated fading channels. Simulation results are reported in Section V for certain representative scenarios. Finally, some concluding remarks are offered in Section VI.

II. System Model

We consider a multiple antenna communication system with L_t transmit and L_r receive antennas. In this system, the source generates k information symbols from the discrete alphabet $\mathcal{X}$, which are encoded by an error control code C to produce code words of length NL_t. The encoded symbols are parsed among L_t transmit antennas and then mapped into constellation points from the discrete complex-valued signaling constellation Ω using the modulation operator "f,"[4] for transmission across the channel. For simplicity of notation, we assume that the coding alphabet size is the same as the size of the constellation, i.e., $|\mathcal{X}| = |\Omega|$. We assume that the channel is frequency nonselective and that the channel state information (CSI) is available *a priori* only at the receiver. At the receiver end, the signal y_t^j received by antenna j at time t is given by

$$y_t^j = \sqrt{\frac{\rho}{L_t}} \sum_{i=1}^{L_t} \alpha_t^{(ij)} s_t^{(i)} + n_t^j \tag{1}$$

where ρ is the signal-to-noise ratio (SNR) independent of the number of transmit antennas; $\alpha_t^{(ij)}$ is the normalized[5] complex path gain from transmit antenna i to receive antenna j at time t; $s_t^{(i)} = f\left(c_t^{(i)}\right)$ is the baseband symbol transmitted from antenna i at time t; and n_t^j is the additive white Gaussian noise sample for receive antenna j at time t. The noise samples are independent samples of a normalized[6] zero-mean complex Gaussian random variable. Unless otherwise stated, we assume that the 2-D code word matrix is obtained by mapping

$$\bar{c} = \left(c_1^{(1)}, c_1^{(2)}, \ldots, c_1^{(L_t)}, c_2^{(1)}, c_2^{(2)}, \ldots, c_2^{(L_t)} \right.$$
$$\left. \ldots, c_N^{(1)}, c_N^{(2)}, \ldots, c_N^{(L_t)}\right) \in C$$

to the matrix

$$\mathbf{c} = \begin{bmatrix} c_1^{(1)} & c_2^{(1)} & \cdots & c_N^{(1)} \\ c_1^{(2)} & c_2^{(2)} & \cdots & c_N^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{(L_t)} & c_2^{(L_t)} & \cdots & c_N^{(L_t)} \end{bmatrix}.$$

In this notation, it is understood that $c_t^{(i)}$ is the code symbol assigned to transmit antenna i at time t and $s_t^{(i)} = f\left(c_t^{(i)}\right)$. The collection of these code word matrices forms the ST code $\mathcal{C}$. Assuming the fading coefficients remain constant over one code word, we have the following model for the received signal in a matrix form:

$$\mathbf{Y} = \sqrt{\frac{\rho}{L_r}} \Upsilon \mathbf{f}(\mathbf{c}) + \mathbf{N} \tag{2}$$

where $\mathbf{Y}$ is the $L_r \times N$ received matrix, $\mathbf{f}(\mathbf{c})$ is the $L_t \times N$ baseband code word matrix, $\mathbf{N}$ is the $L_r \times N$ noise matrix, and

$$\Upsilon = \begin{bmatrix} \alpha^{(11)} & \alpha^{(21)} & \cdots & \alpha^{(L_t 1)} \\ \alpha^{(12)} & \alpha^{(22)} & \cdots & \alpha^{(L_t 2)} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{(1L_r)} & \alpha^{(2L_r)} & \cdots & \alpha^{(L_t L_r)} \end{bmatrix}. \tag{3}$$

III. Line of Sight Channels

In cellular systems, it is reasonable to assume that the channel characteristics will vary considerably from one user to the next. In addition, the channel characteristics for different users are in most cases unknown *a priori*, and hence, it is desirable to design a robust signaling strategy that offers a satisfactory performance in various scenarios. The significant gains offered by ST coding in slow flat fading channels makes this technology an excellent candidate for consideration (e.g., [4], [5]). What remains to be seen, however, is the robustness of these codes to different channel conditions. In the following, we investigate the design and performance of ST codes in line-of-sight channels.

A. Performance Analysis

In the line-of-sight scenario, the complex path gains can be modeled as

$$\alpha_t^{(ij)} = e^{j\theta_{ij}}. \tag{4}$$

This model indicates that all the path gains have the same modulus, and the small differences in path lengths only imply differences in the phase shifts. While the Rician fading model is more *realistic* than this line-of-sight model, which represents a Rician channel with a very high specular-to-diffuse ratio, this scenario was chosen because it allows for the most *insightful* analysis. One can easily extend the analysis to arbitrary Rician fading channels by combining the results presented in this section with that for Rayleigh fading channels (e.g., [4]). This extension, however, does not result in more insights in the code design.

Denote

$$\underline{\theta} = [\theta_{11}, \ldots, \theta_{L_t L_r}]^T \tag{5}$$

[4]For example, for BPSK modulation and binary codes $f(x) = (-1)^x$.

[5]$E\left(\left|\alpha_t^{(ij)}\right|^2\right) = 1$.

[6]$E\left(\left|n_t^j\right|^2\right) = 1$.

and the conditional pairwise probability of error that the decoder will choose $\mathbf{e}$ when $\mathbf{c}$ is transmitted as $P(\mathbf{c} \rightarrow \mathbf{e}|\underline{\theta})$. In fading channels, even when the user is stationary, one can still assume that the path gains change independently from one code word to the next due to the movements in the scattering medium (e.g., [4]). This assumption allows for using the *average* probability of error, with respect to the different channel realizations, as the design criterion. Contrary to fading channels, the complex path gains in the line-of-sight scenario remain *fixed* for each stationary user. These path gains, corresponding to one stationary user, depend on the angle of the direct path to the user and the geometry of the antenna array. This observation motivates the following code design criterion

$$\min_{\underline{\theta},\mathbf{c},\mathbf{e}\in\mathcal{C}} \max P(\mathbf{c} \rightarrow \mathbf{e}|\underline{\theta}). \tag{6}$$

This design criterion optimizes the worst-case performance with respect to all pairs of code words and *all* possible values for $\underline{\theta}$. One can think of the new criterion as minimizing the *worst-case* error rate with respect to the user position in the downlink of a cellular system. The following result establishes an upper bound on the worst-case pairwise probability of error in line-of-sight channels.

Proposition 1: In the line-of-sight scenario, the conditional pairwise probability of error is upper bounded by

$$\max_{\underline{\theta}} P(\mathbf{c} \rightarrow \mathbf{e}|\underline{\theta}) \le Q\left(\sqrt{\frac{\rho}{2}\lambda_{\min}L_r}\right) \tag{7}$$

where $\lambda_{\min}$ is the smallest eigenvalue of the matrix $\mathbf{A} = (\mathbf{f}(\mathbf{c})-\mathbf{f}(\mathbf{e}))(\mathbf{f}(\mathbf{c}) - \mathbf{f}(\mathbf{e}))^{\mathbf{H}}$.

Proof: Following in the footsteps of [4], we obtain

$$\begin{aligned} P(\mathbf{c} \rightarrow \mathbf{e}|\underline{\theta}) =& Q\left(\sqrt{\frac{\rho\|\mathbf{\Upsilon}(\mathbf{f}(\mathbf{c}) - \mathbf{f}(\mathbf{e}))\|^2}{2L_t}}\right) \\ =& Q\left(\sqrt{\frac{\rho}{2L_t}\mathrm{tr}(\mathbf{\Upsilon}\mathbf{A}\mathbf{\Upsilon}^{\mathbf{H}})}\right) \end{aligned} \tag{8}$$

where $\|\cdot\|^2$ denotes the Frobenius norm, and tr$(\cdot)$ denotes the trace of the matrix. One can now use the SVD decomposition of the positive semi-definite matrix $\mathbf{A}$ (i.e., $\mathbf{A} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathbf{H}}$, where $\mathbf{U}$ is the unitary eigenvectors matrix, and $\mathbf{\Lambda}$ is the diagonal eigenvalues matrix) to show that

$$\begin{aligned} \mathrm{tr}(\mathbf{\Upsilon}\mathbf{A}\mathbf{\Upsilon}^{\mathbf{H}}) =& \mathrm{tr}(\mathbf{\Upsilon}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathbf{H}}\mathbf{\Upsilon}^{\mathbf{H}}) \\ \ge& \lambda_{\min}\mathrm{tr}(\mathbf{\Upsilon}\mathbf{U}\mathbf{U}^{\mathbf{H}}\mathbf{\Upsilon}^{\mathbf{H}}) \\ =& \lambda_{\min}L_tL_r \end{aligned} \tag{9}$$

where $\lambda_{\min}$ is the minimum eigenvalue of $\mathbf{A}$. The second inequality follows from replacing $\mathbf{\Lambda}$ by $\lambda_{\min}\mathbf{I}$, whereas the last equality follows from the fact that $\mathbf{U}$ is unitary, and tr$(\mathbf{\Upsilon}\mathbf{\Upsilon}^{\mathbf{H}}) = L_tL_r$ in the line-of-sight scenario. Finally, we combine (8) and (9) to obtain

$$P(\mathbf{c} \rightarrow \mathbf{e}|\underline{\theta}) \le Q\left(\sqrt{\frac{\rho}{2}\lambda_{\min}L_r}\right). \tag{10}$$

The result then follows from the fact that this bound is independent on $\underline{\theta}$. □

Proposition 1 leads to the following code design criterion.

- *Line-of-Sight Criterion:* Maximize the smallest eigenvalue of the matrix $\mathbf{A} = (\mathbf{f}(\mathbf{c}) - \mathbf{f}(\mathbf{e}))\,(\mathbf{f}(\mathbf{c}) - \mathbf{f}(\mathbf{e}))^{\mathbf{H}}$ among all pairs of code words $\mathbf{c}, \mathbf{e} \in \mathcal{C}$.

Comparing this criterion to the quasistatic Rayleigh fading channel design criteria [4], [5], one notes the *generality* of the full diversity, i.e., $\lambda_{\min} > 0$, criterion. It is also clear that the coding advantage differs considerably in both scenarios. We can further strengthen the result in Proposition 1 by adopting a more generalized model for the path gains. In this model, the only constraint imposed on the path gains is

$$\sum_{i=1}^{L_t} |\alpha^{(ij)}|^2 = L_t. \tag{11}$$

Subject to this constraint only, one can easily show that

$$\max_{\underline{\alpha}} P(\mathbf{c} \rightarrow \mathbf{e}|\underline{\alpha}) = Q\left(\sqrt{\frac{\rho}{2}\lambda_{\min}L_r}\right). \tag{12}$$

The need for full diversity ST codes imposes the following bound on the transmission rate achieved by standard constellations (e.g., PSK, QAM)[7]

Lemma 2: The maximum transmission rate for ST codes in line-of-sight channels is one symbol/channel use.

Proof: This result follows directly from the Singleton bound [4], [8], [17]. □

In the rest of this paper, we will refer to ST codes that achieve the maximum transmission rate as full rate codes. These full rate codes are, however, generated by multiplexing rate $1/L_t$ codes across the L_t transmit antennas to achieve a throughput of one-symbol-per-channel use. For these full rate codes, we have the following upper bound on the coding gain in line-of-sight channels.

Lemma 3: For full rate ST codes, $\lambda_{\min} \le d^2_{\min}(\Omega)$, where $d^2_{\min}(\Omega)$ is the minimum squared Euclidean distance of the constellation Ω.

Proof: Using the fact that for the matrix $\mathbf{A} = \{a_{ij}\}$

$$\lambda_{\min}(\mathbf{A}) \le \min_{1\le i\le L_t} (a_{ii}) \tag{13}$$

where the a_{ii}s are the diagonal elements [18]. These diagonal elements are given by

$$a_{ii} = \sum_{k=1}^{N} \left| f\left(c_k^{(i)}\right) - f\left(e_k^{(i)}\right)\right|^2 \tag{14}$$

which results in

$$\begin{aligned} \lambda_{\min} &= \min_{\mathbf{c},\mathbf{e}\in\mathcal{C}} \lambda_{\min}(\mathbf{A}) \\ &\le \min_{\mathbf{c},\mathbf{e}\in\mathcal{C},1\le i\le L_t} \sum_{k=1}^{N} \left| f\left(c_k^{(i)}\right) - f\left(e_k^{(i)}\right)\right|^2. \end{aligned} \tag{15}$$

[7]This result does not apply to multidimensional rotated constellations.

For full rate ST codes, the transmissions from each antenna correspond to a rate one code, and hence, the Singleton bound ensures that

$$\min_{\mathbf{c},\mathbf{e}\in\mathcal{C},1\le i\le L_t} \sum_{k=1}^{N} \left| f\left(c_k^{(i)}\right) - f\left(e_k^{(i)}\right) \right|^2 = d_{\min}^2(\Omega) \quad (16)$$

which was to be proven. □

Lemma 3 indicates that the worst-case pairwise probability of error for ST codes in line-of-sight channels is lower bounded by that achieved with uncoded transmission from one antenna using the same constellation and transmission rate. This, however, does not necessarily mean that uncoded single antenna systems will give the best worst-case performance in line-of-sight channels. As shown in Section V-A, carefully constructed ST codes, with the same $\lambda_{\min}$ as the uncoded system, can outperform the single antenna case. This can be attributed to the superior *distance spectrum* [19] of the ST code compared with the uncoded scenario. Moreover, coding across multiple antennas is necessary to ensure satisfactory performance for users suffering from slow Rayleigh fading.

Remark 4: We also note that the Rician fading model used in [4] does not satisfy the constraint in (11). In [4], it was assumed that the energy in the diffuse components alone satisfies this constraint, and hence, the specular components always result in additional received signal energy. This explains the authors' assertion that the frame error rate performance in Rician fading channels is upper bounded by that in Rayleigh channels. Imposing the constraint in (11) on the total receiver energy, however, changes this conclusion, as argued by the previous results.

One can now compare the bound in (7) with the worst-case analysis in [15]. The main difference between our analysis and that in [15] is the *subset* of channel matrices over which the worst-case performance is investigated. Assuming that $L_t \ge L_r$, the subset considered in [15] is defined as

$$\bar{\mathbf{\Upsilon}}_{L_t,L_r} = \left\{ \mathbf{\Upsilon} \in \mathbf{C}^{L_r \times L_t} : I\left(\sqrt{\frac{\rho}{L_t}}\mathbf{\Upsilon}\right) \ge R \right\} \quad (17)$$

where $I(\cdot)$ denotes the mutual information, and R is the supported rate over this set of channel realizations. The worst-case performance of the ST code $\mathcal{C}$ over this set of channel realizations was shown to be equal to

$$Q\left(\sqrt{\frac{1}{2}G_{L_r}(2^R-1)^{1/L_r}}\right) \quad (18)$$

where G_{L_r} is the geometric mean of the smallest L_r eigenvalues of $\mathbf{A}$. Using the same formulation, one can write the subset of channel realizations corresponding to line-of-sight scenarios as

$$\tilde{\mathbf{\Upsilon}}_{L_t,L_r} = \left\{ \mathbf{\Upsilon} = \{\alpha^{(ij)}\} \in \mathbf{C}^{L_r \times L_t} : \|\alpha^{(ij)}\|^2 = 1 \right\}. \quad (19)$$

We first observe that with one receive antenna, the bounds in (7) and (18) coincide. For the general case, the bound in (18) is applicable to the line-of-sight scenario *only* if we choose the rate R such that $\tilde{\mathbf{\Upsilon}}_{L_r,L_t} \subseteq \bar{\mathbf{\Upsilon}}_{L_r,L_t}$. This approach, however, results in a very loose bound for asymptotically large SNRs with more than one receive antenna. This result follows from the fact that the class of line-of-sight channels includes many *rank-deficient* channel realizations whose mutual information

$$I\left(\sqrt{\frac{\rho}{L_t}}\mathbf{\Upsilon}\right) = \sum_{k=1}^{m} \log_2(1 + c_i\rho) \quad (20)$$

where $m < L_r$ is the number of nonzero eigenvalues of $\mathbf{\Upsilon}$, and c_i is a constant that depends on the ith nonzero eigenvalue. In fact, considering the all ones channel matrix, one can see that the minimum value of m is equal to one. Using this value to compute R in (18), we can see that the argument of the Q function in (18) will be proportional to $\rho^{1/2L_r}$ rather than $\rho^{1/2}$, as in the bound in (7). The utility of the worst-case analysis in [15] is, nevertheless, evident in its generality to include other scenarios beyond the line-of-sight case considered in this paper.

With respect to code construction, the two-design criteria, i.e., $\lambda_{\min}$ and G_{Lr}, become different only when the number of receive antennas is larger than one. In Section V-A, we present simulation results for a system with two receive antenna that suggest that optimizing the $\lambda_{\min}$ leads to superior worst-case performance in line-of-sight channels. One can further argue in favor of the $\lambda_{\min}$ criterion as follows. The two criteria suggest that optimal worst-case performance with one receive antenna is attained by maximizing $\lambda_{\min}$. By considering the line-of-sight channel where the path gains to the second receive antenna are scaled versions of the path gains to the first one, we can see that the *optimal* worst-case performance with two receive antenna in line-of-sight channels is *lower bounded* by the *optimal* worst-case performance with one receive antenna and an *additional* 3 dB (i.e., the second antenna acts like a repetition code). One can easily extend this lower bound to systems with L_r receive antennas [i.e., the worst-case performance with L_r receive antennas is lower bounded by that with one receive antenna and additional $(\log_{10}(L_r))$ dB]. This lower bound, along with the upper bound in (7), indicate that maximizing the $\lambda_{\min}$ design criterion results in optimal worst-case performance in line-of-sight channels with an *arbitrary* number of receive antennas.

B. Phase Randomization Strategy

In Section III-A, we showed that although all the paths do not suffer from deep fades in line-of-sight scenarios, the phase differences between the different paths can conspire to attenuate *many* of the mutually interfering channels for some users. This fact manifests itself in the need for full diversity codes to ensure adequate performance for *the worst-case* stationary user. The situation is further complicated by the fact that the phase shifts of the path gains are *fixed* for each stationary user and *do not* change from one code word to the next. This observation motivated a design criterion based on *the worst-case* probability of error with respect to $\underline{\theta}$ and suggests that performance gains may be realized by a *phase randomization* strategy where an independent random phase shift[8] is inserted in the signal assigned to each transmit antenna. These phase shifts are constant across the code word and change independently from one

[8]The phase shifts are known *a priori* at the transmitter and receiver.

code word to the next. Denote the random phase shift vector as $\underline{\theta}_r = [\theta_{1,r}, \ldots, \theta_{L_t,r}]$; then, the new design criterion is

$$\min_{p(\underline{\theta}_r)} \max_{\underline{\theta}, \mathbf{c}, \mathbf{e} \in \mathcal{C}} \int P(\mathbf{c} \to \mathbf{e} | \underline{\theta} + \underline{\theta}_r) p(\underline{\theta}_r) d\underline{\theta}_r. \tag{21}$$

Our objective now is to find the probability density function of the random phase shift vector "$p(\underline{\theta}_r)$" that minimizes the probability of error in (21). This optimum distribution is given by the following result in systems with one receive antenna.

Proposition 5: The worst-case pairwise probability of error in line-of-sight channels with one receive antenna is minimized by a phase randomization strategy with a multivariate i.i.d. uniform distribution for $\underline{\theta}_r$. Moreover, this phase randomization strategy does not affect the performance in Rayleigh fading channels.

Proof: In this scenario, we observe that

$$\begin{aligned} \max_{\mathbf{c}, \mathbf{e} \in \mathcal{C}} \iint P\left(\mathbf{c} \to \mathbf{e} | \underline{\theta} + \underline{\theta}_r\right) p(\underline{\theta}_r) d\underline{\theta} d\underline{\theta}_r &= \max_{\mathbf{c}, \mathbf{e} \in \mathcal{C}} \int g_1(\mathbf{c}, \mathbf{e}) \\ &\quad \times p(\underline{\theta}_r) d\underline{\theta}_r \\ &= \max_{\mathbf{c}, \mathbf{e} \in \mathcal{C}} g_1(\mathbf{c}, \mathbf{e}) \\ &= g_2(\mathcal{C}). \end{aligned} \tag{22}$$

This observation implies that the solution to the "$\min \max$" problem in (21) is the phase distribution that results in the same performance independent of $\underline{\theta}$. It is straightforward to see that this distribution is the multivariate i.i.d. uniform distribution. In Rayleigh fading channels, the phase introduced by the channel has a uniform distribution. Noting that the sum of two i.i.d. uniform random phases has the same uniform distribution, one concludes that this phase randomization strategy will not affect the *average* performance in Rayleigh fading channels. □

The optimal phase randomization strategy will therefore allow for the same performance for all stationary users, with one receive antenna, in the line-of-sight scenario, irrespective of their positions. In effect, this approach transforms the *stringent* maximum pairwise probability of error criterion into the *more relaxed* average probability of error, with respect to $\underline{\theta}$, criterion. We also conjecture that this phase randomization strategy will yield the optimal performance in systems with arbitrary numbers of receive antennas. A similar phase randomization strategy was recently proposed in [20] and [21]. However, in [20] and [21], the phase randomization strategy was used to improve the performance of ST systems with concatenated codes[9] in slow Rayleigh fading channels.

Remark 6: We note that the worst-case analysis proposed in Section III-A does not predict the performance enhancement allowed by the phase randomization strategy. The phase randomization is, in fact, intended to enhance the worst-case performance to match that of the average user. The bound in (7) is still an upper bound on the performance of the average user. In fact, if all the eigenvalues of the code are equal, as in Alamouti code for example, the worst-case performance is the same as the average performance with phase randomization in line-of-sight channels. At the moment, we do not have a tighter upper bound on the performance with phase randomization in the general case. Nevertheless, simulation results, presented in Section V, show that ST codes with large $\lambda_{\min}$ still offer the best performance with phase randomization in various line-of-sight scenarios.

Finally, it is worth noting that the need for robust codes is tightly coupled with the absence of CSI at the transmitter and the desire to construct signaling schemes that yield satisfactory performance in various scenarios. The availability of this information at the transmitter will change the whole design paradigm as one should consider adaptive techniques that exploit the transmitter CSI.

IV. Correlated Rayleigh Fading Channels

The diversity and coding advantage design criteria for ST coding were developed for the spatially white quasistatic flat Rayleigh model [4]. In this model, the path gains are assumed to be independent samples of a zero-mean complex Gaussian random variable. The gains are further assumed to be constant across one code word and change independently from one code word to the next. Assuming that the quasistatic model is *a worst-case* scenario, Tarokh *et al.* presented "smart greedy" codes that achieve full spatial diversity in the quasistatic scenario and strive to exploit the additional temporal diversity in the fast fading scenario [4]. Recently, Siwamogasatham and Fitz observed experimentally that "smart-greedy" codes achieve *lower* diversity advantages in certain *time-selective* correlated channels than that in quasistatic channels. This surprising observation calls for further investigations of the performance of ST codes in correlated fading channels. In the following, we characterize the achievable diversity advantage in fading channels with *arbitrary spatio-temporal* correlation functions based on the *number of degrees in freedom* in the channel. Our analysis argues that the quasistatic assumption is not a worst-case model. In particular, we show that within the class of channels with the same number of degrees of freedom as the quasistatic channel, the minimum diversity advantage can be lower than L_t (the full spatial diversity scenario). We argue that *robust* codes should be constructed to maximize the diversity advantage, assuming the *independent* block fading multi-input multi-output (MIMO) model in [22][10]. This result, along with the result for the line-of-sight channels, suggest that robust ST codes should be constructed to achieve full spatial diversity in quasistatic channels *and* maximize the diversity advantage in independent block fading channels.[11] We further propose a new framework for exploiting *prior* knowledge about the channel correlation function to enhance the code robustness. The proposed design approach encompasses the smart-greedy principle as a special case.

[9]Phase randomization was used to approximate a fast fading at the input of the outer decoder.

[10]Code design rules for *independent* (MIMO) block fading channels are presented in [8].

[11]Code design in independent block fading channels only requires the transmitter to know the channel coherence time or an upper bound on it in order to estimate the number of independent fading blocks per code word.

A. Fundamental Limit on Code Robustness

The fading model of primary interest in this section is that of a block flat Rayleigh fading process in which the code word encompasses M fading blocks, where complex fading gains are constant over one fading block. For simplicity of notation, we assume that the number of receive antennas $L_r = 1$; the extension to systems with arbitrary numbers of receive antennas is straightforward. We denote $\alpha_k^{(i)}$ as the zero-mean unit-variance complex Gaussian fading coefficient from the ith transmit antenna in the kth block, and $\kappa_{k_1,k_2}^{(i_1,i_2)} = E\left[\alpha_{k_1}^{(i_1)}\alpha_{k_2}^{(i_2)*}\right]$. The independent quasistatic and fast-fading models are special cases of the block fading model in which $\kappa_{k_1,k_2}^{(i_1,i_2)} = \delta(i_1-i_2)\delta(k_1-k_2)$, and $M=1$ and $M=N$, respectively [4]–[6]. For simplicity, it is also assumed here that M divides N.

By partitioning the $(L_t \times N)$ code word matrix into "M" $(L_t \times N/M)$ components, i.e.,

$$\mathbf{c} = [\mathbf{c}[1], \mathbf{c}[2], \dots, \mathbf{c}[M]] \tag{23}$$

we have the following *correlated quasistatic* model with L_tM *virtual* transmit antennas for the received signal in (24), shown at the bottom of the page, where $\mathbf{f}(\mathbf{c}[m]^T)$ is the baseband version of $\mathbf{c}[m]^T$, and $\mathbf{0}_{N/M\times L_t}$ refers to the $(N/M \times L_t)$ all-zero matrix. The only difference between the model in (24) and that used to develop the design criteria in [4] and [5] is the correlation between the different elements of $\underline{\alpha}$. Denote the correlation matrix of the path gains in the quasistatic model as $R = \{r_{i,j}\} = E[\underline{\alpha}\underline{\alpha}^H]$; then, it is easy to see that R is a Hermitian positive semi-definite matrix, and

$$r_{(k_1-1)L_t+i_1,(k_2-1)L_t+i_2} = \kappa_{k_1,k_2}^{(i_1,i_2)}. \tag{25}$$

We can then factor $R = AA^H$, where the rank of R is equal to that of A (e.g., [4]). This allows for rewriting (24) as

$$\underline{\mathbf{y}} = \sqrt{\frac{\rho}{L_t}}\mathcal{H}(\mathbf{c})A\underline{\alpha}_{\text{ind}} + \underline{\mathbf{n}} \tag{26}$$

where $\underline{\alpha}_{\text{ind}}$ is a $(L_tM \times 1)$ vector with i.i.d. zero-mean and unit variance complex Gaussian components. Since the model in (26) corresponds to a quasistatic Rayleigh fading channel with L_tM *independent* transmit antennas, it follows that the diversity advantage is given by

$$r = \min_{\mathbf{c},\mathbf{e}\in\mathcal{C}} \text{Rank}((\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e}))A). \tag{27}$$

In many practical scenarios, the correlation matrix R is not known *a priori* at the transmitter, and hence, it is desirable to construct codes that maximize the diversity advantage for a wide class of *spatio-temporal* correlation matrices. Using the fact that [18]

$$\text{Rank}((\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e}))A) \le \min(\text{Rank}(A), \text{Rank}(\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e})) \tag{28}$$

we classify the ST channels according to the *degrees of freedom* in the channel as

$$\mathcal{A}_d = \{A: \text{Rank}(A) = d\} \text{ for } 1 \le d \le L_tM \tag{29}$$

where d is the number of degrees of freedom in the channel (i.e., the maximum achievable diversity advantage). As a side note, we observe that $\mathcal{A}_d$ does not contain all matrices with rank equal to "d" since AA^H is a covariance matrix for unit variance random variables. For example, A can not contain an all-zero row. This classification allows for defining the *parameterized robustness* of the ST code as

$$r_d = \min_{\mathcal{A}_d}\left(\min_{\mathbf{c},\mathbf{e}\in\mathcal{C}} \text{Rank}((\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e}))A)\right). \tag{30}$$

The following result characterizes the code robustness based on the diversity advantage achieved in the independent fading scenario.

Proposition 7: The robustness of the ST code $\mathcal{C}$, with rate k/N symbols per channel use, is lower bounded by

$$r_d \ge \max\left(\min_{\mathbf{c},\mathbf{e}\in\mathcal{C}}\sum_{m=1}^{M}\text{Rank}(\mathbf{f}(\mathbf{c}[m]) - \mathbf{f}(\mathbf{e}[m])) + d - L_tM, 1\right) \tag{31}$$

and can be upper bounded by

$$r_d \le \max\left(\left\lfloor L_tM\left(1 - \frac{k}{NL_t}\right)\right\rfloor + 1 + d - L_tM, 1\right). \tag{32}$$

Proof: The first result follows directly from the fact that for $\mathbf{U} \in \mathsf{C}^{n\times m}$ and $\mathbf{V} \in \mathsf{C}^{m\times l}$

$$\text{Rank}(\mathbf{UV}) \ge \text{Rank}(\mathbf{U}) + \text{Rank}(\mathbf{V}) - m \tag{33}$$

and

$$\text{Rank}(\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e})) = \sum_{m=1}^{M}\text{Rank}(\mathbf{f}(\mathbf{c}[m]) - \mathbf{f}(\mathbf{e}[m])) \tag{34}$$

(see [18]). Using the Singleton bound [8], [17], it is easy to see that there exist two code words $\mathbf{c}$ and $\mathbf{e}$ such that $(\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e}))$ has at maximum $\lfloor L_tM(1 - (k/NL_t))\rfloor + 1$ nonzero columns. Within $\mathcal{A}_d$, one can construct a matrix A_c with $LM_t - d + 1$ equal

$$\underline{\mathbf{y}} = \sqrt{\frac{\rho}{L_t}}\mathcal{H}(\mathbf{c})\underline{\alpha} + \underline{\mathbf{n}}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \sqrt{\frac{\rho}{L_t}}\begin{bmatrix} \mathbf{f}(\mathbf{c}[1]^T) & \mathbf{0}_{N/M\times L_t} & \mathbf{0}_{N/M\times L_t} & \cdots & \mathbf{0}_{N/M\times L_t} \\ \mathbf{0}_{N/M\times L_t} & \mathbf{f}(\mathbf{c}[2]^T) & \mathbf{0}_{N/M\times L_t} & \cdots & \mathbf{0}_{N/M\times L_t} \\ \vdots & \vdots & \ddots & \vdots & \\ \mathbf{0}_{N/M\times L_t} & \mathbf{0}_{N/M\times L_t} & \cdots & \mathbf{0}_{N/M\times L_t} & \mathbf{f}(\mathbf{c}[M]^T) \end{bmatrix}\begin{bmatrix} \alpha_1^{(1)} \\ \alpha_1^{(2)} \\ \vdots \\ \alpha_M^{(L_t)} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \tag{24}$$

rows whose indices have the largest overlap with the indices of the nonzero columns in $(\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e}))$. The second result now follows directly from observing that

$$\min_{\mathbf{c},\mathbf{e}\in\mathcal{C}} \text{Rank}((\mathcal{H}(\mathbf{c}) - \mathcal{H}(\mathbf{e}))A_c) \le \max\left(\left\lfloor L_t M\left(1 - \frac{k}{NL_t}\right)\right\rfloor + 1 + d - L_t M, 1\right). \quad (35)$$

□

This result suggests the following design criterion for optimizing the code robustness

$$\text{Maximize}\left(\min_{\mathbf{c},\mathbf{e}\in\mathcal{C}} \sum_{m=1}^{M} \text{Rank}(\mathbf{f}(\mathbf{c}[m]) - \mathbf{f}(\mathbf{e}[m]))\right). \quad (36)$$

This design criterion for robust ST codes, in the absence of any knowledge about the channel correlation matrix, is the same as that for maximizing the diversity advantage in the *independent* ST block fading channel [8]. This observation suggests that maximizing the diversity advantage, assuming independent fading, is important to ensuring satisfactory performance in channels with small numbers of degrees of freedom. For codes that achieve the maximum diversity advantage in independent fading, one can see that the upper and lower bounds in Proposition 7 coincide. Moreover, Proposition 7 proves that when ST codes are interleaved in time-selective channels, e.g., smart-greedy codes, it is no longer sufficient to guarantee full diversity in the quasistatic scenario. The rank of the correlation matrix in the quasistatic scenario is $d = L_t$, and the robustness of full rate codes $(k = N)$ for channels with this number of degrees of freedom is

$$r_d = L_t + 1 - M < L_t \text{ for } M > 1. \quad (37)$$

This means that there exists a channel with the same number of degrees of freedom as the quasistatic channel, where the code will achieve a lower diversity advantage than L_t. Finally, Proposition 7 establishes the necessary tradeoff between the transmission rate and the robustness of the code, as is evident in (32).

Except for the mild assumption about the known coherence time at the transmitter, Proposition 7 assumes no prior knowledge about the channel correlation matrix at the transmitter. It is now reasonable to expect that the availability of more information about the correlation at the transmitter would allow for constructing codes that are *more* robust to certain types of correlation. In the following, we will investigate this topic in some detail for binary ST trellis codes with BPSK modulation. QPSK-modulated codes can be easily constructed by *lifting* the binary codes to the Z_4 ring using the approach proposed in [6].

B. Robust ST Trellis Codes

To simplify the presentation, we consider binary convolutional codes that can be *represented* as k/ML_t codes. This class of codes includes the full transmission rate scenario where the rate $1/L_t$ code are represented as M/ML_t codes. The details of the representation depend on the multiplexing of the encoder output across the different fading blocks (for more details, see [23]). The proposed approach can be further extended to convolutional codes with arbitrary rates. This extension, however, will entail additional technical details and will not contribute further insights into the problem.

Let C be a rate k/ML_t binary convolutional code. The encoder processes k binary input sequences $x_1(t), x_2(t), \ldots, x_k(t)$ and produces ML_t coded output sequences $y_1(t), y_2(t), \ldots, y_{ML_t}(t)$, which are multiplexed together to form the output code word. A sequence $\{x(t)\}$ is often represented by the formal series $X(D) = x(0) + x(1)D + x(2)D^2 + \cdots$. We refer to $\{x(t)\} \leftrightarrow X(D)$ as a D-transform pair. The action of the binary convolutional encoder is linear and is characterized by the so-called impulse responses $g_{i,j}(t) \leftrightarrow G_{i,j}(D)$ associating output $y_j(t)$ with input $x_i(t)$. Thus, the encoder action is summarized by the matrix equation

$$\mathbf{Y}(D) = \mathbf{X}(D)\mathbf{G}(D)$$

where $\mathbf{Y}(D) = [Y_1(D) \quad Y_2(D) \quad \ldots \quad Y_{ML_t}(D)]$, $\mathbf{X}(D) = [X_1(D) \quad X_2(D) \quad \ldots \quad X_k(D)]$, and

$$\mathbf{G}(D) = \begin{bmatrix} G_{1,1}(D) & G_{1,2}(D) & \cdots & G_{1,ML_t}(D) \\ G_{2,1}(D) & G_{2,2}(D) & \cdots & G_{2,ML_t}(D) \\ \vdots & \vdots & \ddots & \vdots \\ G_{k,1}(D) & G_{k,2}(D) & \cdots & G_{k,ML_t}(D) \end{bmatrix}. \quad (38)$$

We consider the natural ST formatting of C in which the output sequence corresponding to $Y_{(m-1)L_t+l}(D)$ is assigned to the lth transmit antenna in the mth fading block and wish to construct generator polynomials that maximize the code robustness to spatio-temporal correlations. To simplify our development, we also define the matrices

$$\mathbf{F}_{(m-1)L_t+l}(D) = \begin{bmatrix} 0_{1\times(m-1)} & G_{1,(m-1)L_t+l}(D) & 0_{1\times(M-m)} \\ 0_{1\times(m-1)} & G_{2,(m-1)L_t+l}(D) & 0_{1\times(M-m)} \\ 0_{1\times(m-1)} & \vdots & 0_{1\times(M-m)} \\ 0_{1\times(m-1)} & G_{k,(m-1)L_t+l}(D) & 0_{1\times(M-m)} \end{bmatrix} \quad (39)$$

where $0_{1\times(m-1)}$ is the all-zero row vector of length $(m-1)$. The insertion of zeros in the matrices will simplify the presentation of our approach, as shown later. Assuming that the transmitter only knows M, then Proposition 7 suggests that the code should be constructed to maximize the diversity advantage, assuming independent fading. Sufficient conditions that guarantee the maximum diversity advantage in this scenario are provided in [8] for arbitrary values of L_t, M, and k. This design approach, however, only maximizes the worst-case diversity advantage as predicted by r_d for the class of channels with d degrees of freedom. This code robustness can be enhanced when the transmitter has more information about the channel correlation matrix, as we show next.

We assume that the prior knowledge only allows for partitioning the fading coefficients into "P" mutually independent classes,[12] where $1 \le P \le ML_t$. Thus, this classification ensures that the number of degrees of freedom is $d \ge P$. Our objective is to exploit this knowledge, about the correlation matrix *structure*, to maximize the diversity advantage assuming that

[12]We believe that in many practical scenarios, sufficiently spaced antennas and/or fading blocks can be assumed independent, which will facilitate this partitioning.

$P = d$.[13] This worst-case assumption implies that all the fading coefficients in the same class are equal. It is worth noting that the generality of this model allows for coefficients in the same class to span different spatial *and/or* temporal coordinates and, hence, allows for modeling the *2-D* correlation functions encountered in [16].

Let $\{\alpha_{p_1}, \ldots, \alpha_{p_\nu}\}$ be the fading coefficients in the $p \in \{1, \ldots, P\}$ class that contains ν coefficients and

$$\mathcal{F}_p(D) = \mathbf{F}_{p_1}(D) \oplus \cdots \oplus \mathbf{F}_{p_\nu}(D) \tag{40}$$

where the modulo two addition $\oplus$ is performed componentwise. We now have the following result that establishes sufficient conditions for achieving *full diversity* in the worst-case scenario.

Proposition 8: The ST trellis code $\mathcal{C}$ described by (38)–(40) will achieve full diversity in the P-classes channel if $\forall a_1, a_2, \ldots, a_P \in \{0, 1\}$

$$a_1\mathcal{F}_1(D) \oplus a_2\mathcal{F}_2(D) \oplus \cdots \oplus a_P\mathcal{F}_P(D) \tag{41}$$

has full row rank k over the integral domain $F[[D]]$ unless $a_1 = a_2 = \cdots = a_P = 0$.

Proof: Let $\underline{\gamma} = [\gamma_1, \ldots, \gamma_P]^T$ be the fading coefficients corresponding to the P classes; then,

$$\gamma_p = \alpha_{p_1} = \cdots = \alpha_{p_\nu}. \tag{42}$$

The received signal is given by

$$\underline{y} = \mathcal{T}(\mathbf{c})\underline{\gamma} + \underline{n}. \tag{43}$$

Using (42), we can obtain $\mathcal{T}(\mathbf{c})$ from $\mathcal{H}(\mathbf{c})$ as

$$\underline{t}_p(\mathbf{c}) = \underline{h}_{p_1}(\mathbf{c}) + \cdots + \underline{h}_{p_\nu}(\mathbf{c}) \tag{44}$$

where $\underline{t}_p(\mathbf{c})$ is the pth row in $\mathcal{T}(\mathbf{c})$, and $\underline{h}_{p_1}(\mathbf{c})$ is the p_1th row in $\mathcal{H}(\mathbf{c})$. This model corresponds to a quasistatic Rayleigh fading channel with P independent transmit antennas and one receive antenna. Because the entries in $\mathcal{H}(\mathbf{c})$ belong to $\{0, -1, 1\}$, it is straightforward to see that the entries in $\mathcal{T}(\mathbf{c})$ are integers. This observation allows for using the binary rank criterion to test for full diversity [6].[14] Denote $\xi(\cdot)$ as the binary projection operator; then, we have

$$\begin{aligned} \xi(\underline{t}_p(\mathbf{c}) - \underline{t}_p(\mathbf{e})) &= \xi(\underline{h}_{p_1}(\mathbf{c} \oplus \mathbf{e})) \oplus \cdots \oplus \xi(\underline{h}_{p_\nu}(\mathbf{c} \oplus \mathbf{e})) \\ \xi(\underline{t}_p(\mathbf{c}) - \underline{t}_p(\mathbf{e}))(D) &= (\mathbf{X}_{\mathbf{c}}(D) \oplus \mathbf{X}_{\mathbf{e}}(D))\mathcal{F}_p(D) \end{aligned} \tag{45}$$

where $\mathbf{c}, \mathbf{e}, \mathbf{c} \oplus \mathbf{e} \in \mathcal{C}$, $\mathbf{X}_{\mathbf{c}}(D)$, and $\mathbf{X}_{\mathbf{e}}(D)$ are the D-transforms of the inputs corresponding to $\mathbf{c}$ and $\mathbf{e}$, respectively. The result now follows by applying the stacking construction conditions [6] to the binary convolutional code $\{\mathcal{F}_1(D), \ldots, \mathcal{F}_P(D)\}$. □

While the previous result is stated only for full diversity codes, one can easily extend it to codes that maximize the diversity advantage, subject to a rate constraint for example, using the machinery in [8]. Using the singleton bound, it is straightforward to see that the achievable diversity advantage using this approach is given by

$$\tilde{r}_d \leq \left\lfloor P\left(1 - \frac{k}{NL_t}\right)\right\rfloor + 1 \tag{46}$$

whereas from Proposition 7, we can see the code robustness without exploiting the knowledge about the correlation matrix is upper bounded by

$$r_d \leq \left\lfloor ML_t\left(1 - \frac{k}{NL_t}\right)\right\rfloor + 1 + d - ML_t. \tag{47}$$

The gain allowed by exploiting the structure of the correlation matrix can be quantified by comparing (46) and (47) in certain representative scenarios. For example, full rate codes, i.e., $k = N$, have $\tilde{r}_d \leq L_t$ and $r_d \leq L_t + 1 - M$ in channels with $P = L_t$, where we can see the significant gains that are *possible* when the knowledge about the correlation matrix is exploited, especially for large values of M.

Within the different classes of correlation matrices, we can identify the following special case of spatially white channels. The main assumption in this scenario is that the antennas are sufficiently spaced to ensure independence, and hence, robust codes need only to consider temporal correlations. One can easily see that the *smart-greedy* design principle [4] was proposed for this case where it was assumed that we have L_t classes, and each class contains all the coefficients corresponding to a particular transmit antenna. The codes were then constructed to ensure full spatial diversity in the quasistatic scenario (i.e., all coefficients in the same class are equal) and maximize the diversity advantage, assuming that the coefficients in the same class are independent.

In summary, code robustness in line-of-sight channels entails using *full spatial diversity* codes, whereas code robustness in correlated fading channels entails maximizing the diversity advantage in *independent* MIMO block fading channels (only a mild assumption about the channel coherence time needs to be enforced in order to compute M in this scenario). When more information about the channel correlation function is available at the transmitter, one can utilize the new approach proposed here to further enhance the code robustness in correlated fading channels.

V. Simulation Results

A. Line-of-Sight Channels

Fig. 1 reports the performance of the full diversity four-state binary ST code with generator polynomials $(5_8, 7_8)$[15] in a system with $L_t = 2$, $L_r = 1$, and BPSK modulation in line-of-sight channels. In the figure, “phase randomization” refers to the performance obtained with the optimal phase randomization strategy, and “LB” refers to a lower bound on the worst-case performance, without phase randomization. Throughout the simulation study, this lower bound corresponds

[13]This represents a worst-case scenario given the prior knowledge about the channel statistics.

[14]The binary rank criterion states that if the binary projections of the differences between all the code word matrices have full rank over the binary field, then the code achieves full diversity.

[15]This code was shown in [6] to achieve full diversity while maximizing the minimum Hamming distance, i.e., the sum of eigenvalues.

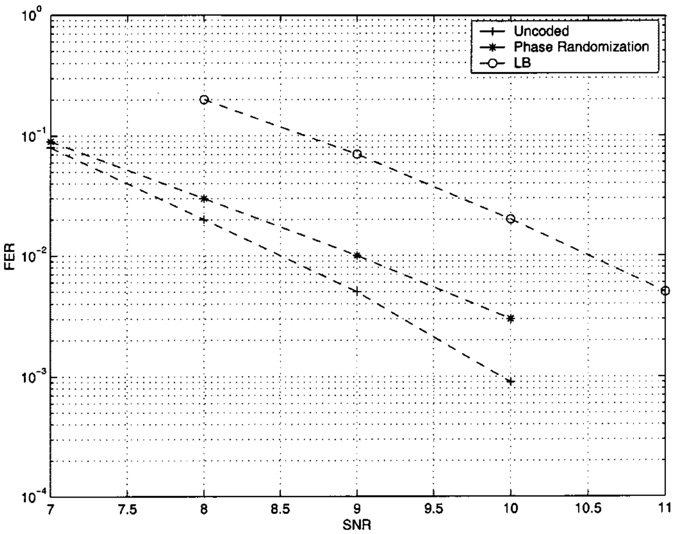

Fig. 1. Performance of the four-state (5, 7) BPSK ST code in line-of-sight channels with $L_t = 2$ and $L_r = 1$.

to the worst-case performance over the *subset* of line-of-sight channels defined as[16]

$$\tilde{\Upsilon}_{\rm LB} = \left\{ \Upsilon = \{\alpha^{(ij)}\} \in \mathbf{C}^{L_r \times L_t} : \alpha^{(ij)} = \pm 1 \right\}. \tag{48}$$

In the optimal phase randomization policy, an i.i.d. uniform random phase was assigned to each transmit antenna. This phase is constant in each code word and changes from one code word to the next. We further assume that these random phases are known at the receiver. In practice, this policy can be implemented using a long pseudo-random sequence that is known to both the transmitter and the receiver. In the figure, we also report the performance of uncoded transmission from one antenna and refer to it as "uncoded." The frame error rates correspond to a frame length of 100 bits. The significant gain possible with the phase randomization strategy is apparent in this example. We can also see that the performance with the phase randomization strategy is close to, but still worse than, the uncoded transmission from a single antenna (i.e., a loss of 0.5 dB at 10^{-2} frame error rate) in this scenario.

In Table I, we report $\lambda_{\min}$ for the two transmit antennas BPSK convolutional ST codes found for Rayleigh fading channels in [6], [24]. As a reference, we know from Lemma 3 that $\lambda_{\min}$ is upper bounded by the minimum squared Euclidean distance of uncoded BPSK transmission, which is equal to 4 with the normalization adopted in the table.[17] In Fig. 2, we report the LB performance of the four-state $(5_8, 7_8)$, the eight-state (64, 74) code, the eight-state (64, 54) code, and uncoded transmission. The same comparison is repeated in Fig. 3 with phase randomization. From the two figures, one can first see that the *relative ordering* of the three codes comes in agreement with the predictions of the design criterion, i.e., codes with larger $\lambda_{\min}$ exhibit superior performance in line-of-sight channels. It is also shown that through the use of robust[18] ST codes coupled with the phase randomization strategy, the multiantenna system can outperform the corresponding single antenna system (i.e., with the same throughput) in line-of-sight channels.[19]

TABLE I
$\lambda_{\min}$ AND G_2 FOR THE TWO TRANSMIT ANTENNA BPSK CONVOLUTIONAL CODES IN [6], [24] AND THE ORTHOGONAL CODE [25]

Connection Polynomials	λ_{min}	G_2
5, 7	1.73	$\sqrt{32}$
64, 74	1.83	$\sqrt{48}$
54, 64	4	$\sqrt{80}$
46, 72	4	$\sqrt{96}$
26, 52	3.57	$\sqrt{128}$
Orthogonal Code	4	4

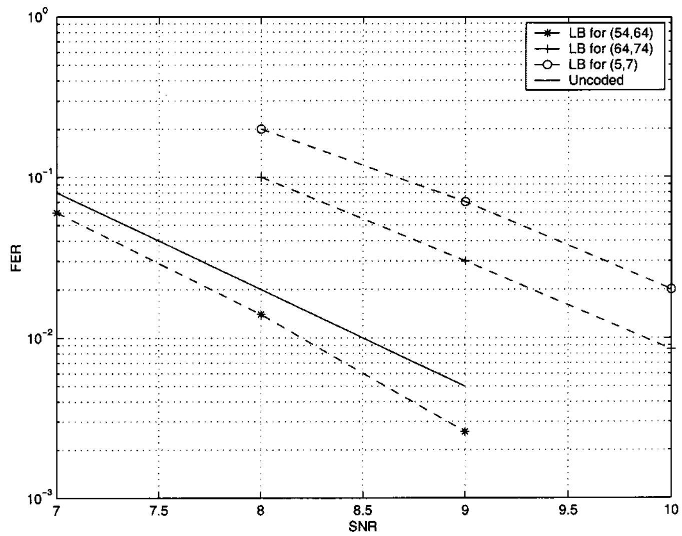

Fig. 2. Lower bound on the worst-case performance of BPSK ST codes with different $\lambda_{\min}$ in line-of-sight channels with $L_t = 2$ and $L_r = 1$.

In order to compare the $\lambda_{\min}$ and G_{L_r} criteria in line-of-sight channels, we consider a system with two receive antennas (i.e., $L_r = 2$). In Fig. 4, we compare the LB performance for the two eight-state codes considered earlier along with the orthogonal ST code [25]. The minimum eigenvalues $\lambda_{\min}$ and G_2 for the three codes are reported in Table I. It is worth noting that the orthogonal code has equal eigenvalues, and hence, its worst-case performance is the same as its average performance. The equality of the two eigenvalues also reduces the *inequality* in (7) to an *equality*. One can see from the figure that, although the (64, 74) code has a superior G_2 compared to the orthogonal code, its worst-case performance is worse. The *relative ordering* of the three codes, however, is in agreement with the $\lambda_{\min}$ criterion. The slight performance advantage offered by the (64, 54)

[16]Since this corresponds to the worst-case performance over only a subset of channel realizations, it is guaranteed to be a lower bound on the overall worst-case performance.

[17]This normalization corresponds to a unit transmitted symbol energy.

[18]In terms of the $\lambda_{\min}$ criterion.

[19]The superiority of multiantenna systems has already been established for Rayleigh fading channels in many earlier works.

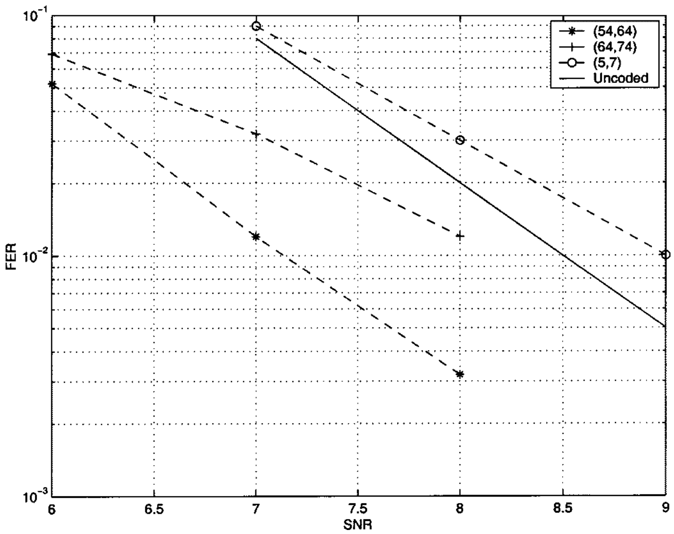

Fig. 3. Performance with phase randomization of BPSK ST codes with different $\lambda_{\min}$ in line-of-sight channels with $L_t = 2$ and $L_r = 1$.

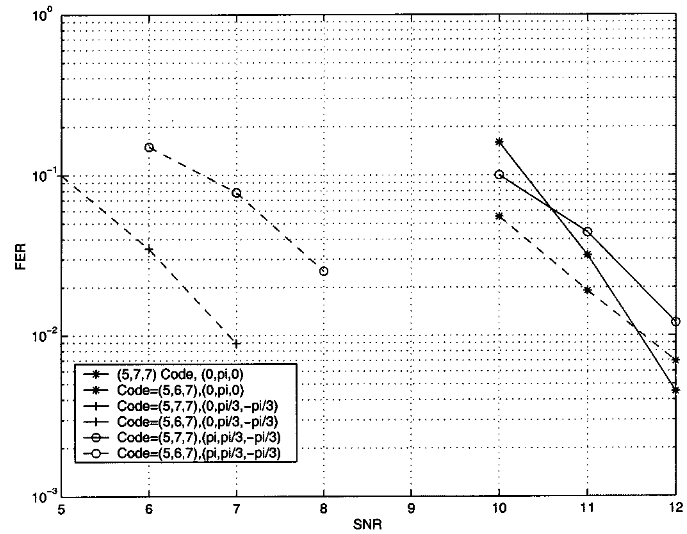

Fig. 5. Performance of four-state BPSK ST codes in line-of-sight channels with $L_t = 3$, and $L_r = 1$ [the dashed lines correspond to the (5, 6, 7) code and the solid lines correspond to the (5, 7, 7) code].

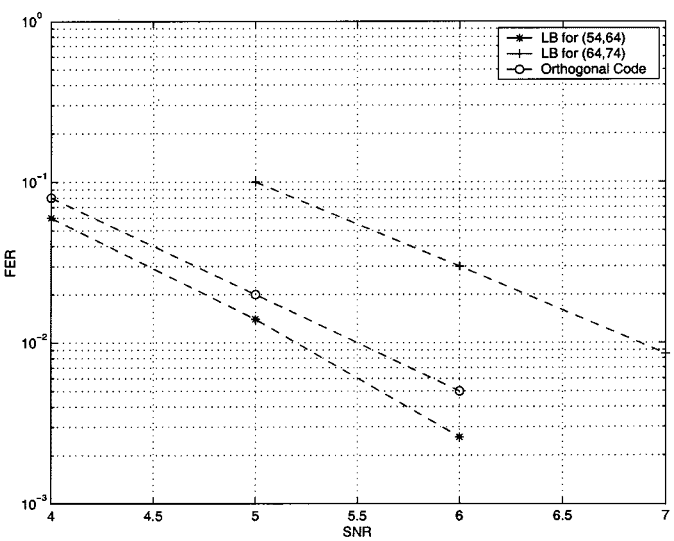

Fig. 4. Lower bound on the worst-case performance of BPSK ST codes with different $\lambda_{\min}$ and G_2 in line-of-sight channels with $L_t = 2$, and $L_r = 2$.

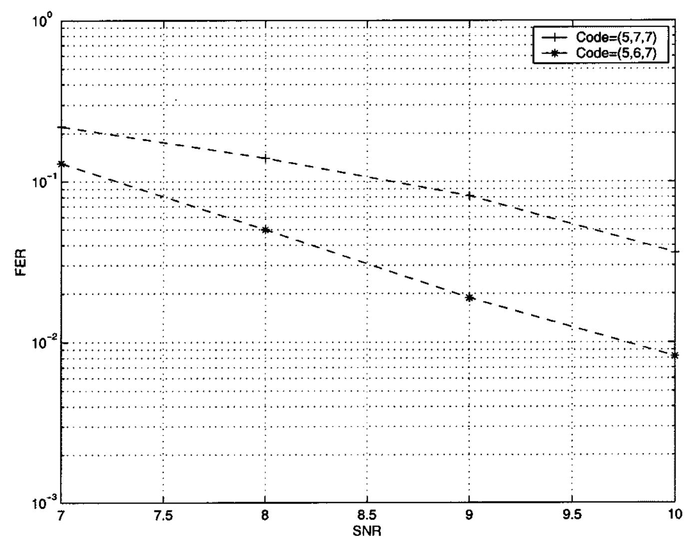

Fig. 6. Performance of four-state BPSK ST codes with phase randomization in line-of-sight channels with $L_t = 3$ and $L_r = 1$.

code over the orthogonal code can be attributed to two reasons: 1) While the two codes have the same $\lambda_{\min}$, the (64, 54) code is expected to have a better *distance spectrum* compared with the *simple* orthogonal code,[20] and 2) the reported performance of the (64, 54) code only corresponds to a lower bound on the worst-case performance, whereas for the orthogonal code, the reported performance corresponds to the *true* worst-case performance.

Fig. 5 compares the performance, without phase randomization, of the four-state ST codes $(5_8, 7_8, 7_8)$ and $(5_8, 6_8, 7_8)$ with $L_t = 3$, and $L_r = 1$ in different line-of-sight channels (i.e., $\underline{\theta} = [0, \pi, 0]$, $\underline{\theta} = [0, \pi/3, -\pi/3]$, and $\underline{\theta} = [\pi, \pi/3, -\pi/3]$). The code $(5_8, 7_8, 7_8)$ achieves the best minimum Hamming distance [26] but fails to achieve full diversity,[21] whereas the second code achieves full diversity. Focusing on the worst-case performance in this subset of channel realizations, one can easily see the gain offered by the full diversity code. It is also interesting to note that within this subset of channel realizations, the scenario resulting in the worst-case performance is different for the two codes (i.e., the worst-case channel realization is *code dependent*). The performance of the two code with phase randomization is then compared in Fig. 6, where the gain offered by the full diversity code is still evident. Finally, in Fig. 7, we compare the best minimum Hamming distance eight-state code "$(54_8, 64_8, 64_8, 74_8)$" and the full diversity code "$(54_8, 64_8, 60_8, 74_8)$" with $L_t = 4$, $L_r = 1$, and phase randomization. Again, it is clear that the full diversity code offers better performance in this scenario.

In summary, these exemplary results validate the $\lambda_{\min}$ design criterion and show that carefully chosen ST codes with phase randomization can offer the diversity needed for users suffering from slow fading and the *robust* performance needed for stationary users in line-of-sight channels.

[20]One can think of the distance spectrum of the code as a second-order effect.

[21]One can easily check this using the binary rank criterion [6].

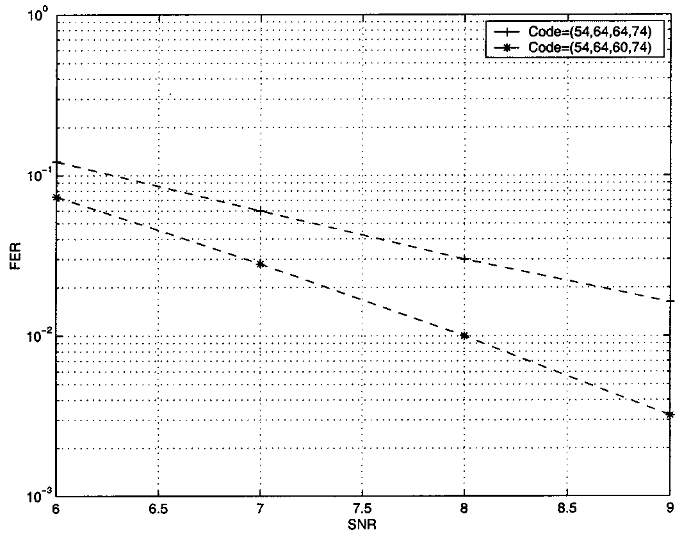

Fig. 7. Performance of eight-state BPSK ST codes with phase randomization in line-of-sight channels with $L_t = 4$ and $L_r = 1$.

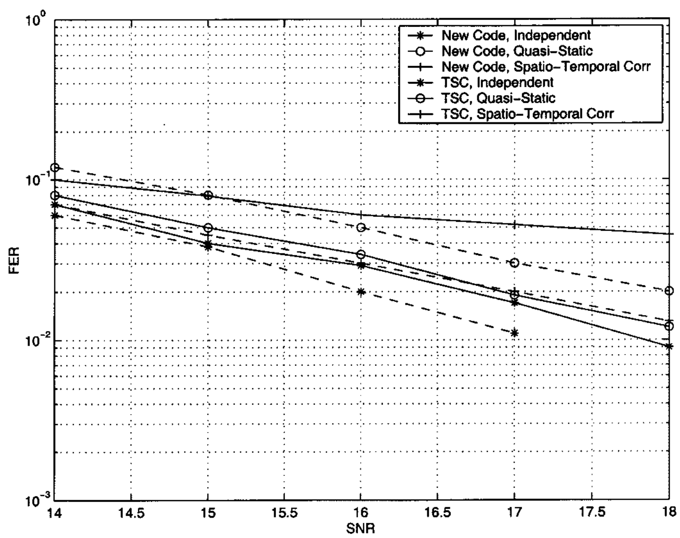

Fig. 8. Performance comparison of the new 16-state QPSK ST code and the TSC ST code with $L_t = 2$, $M = 2$, and $L_r = 1$ (the solid lines correspond to the TSC code and the dashed lines correspond to the new code).

B. Correlated Rayleigh Fading Channels

Fig. 8 compares the performance of the 16-state Z_4 code $(1+D+2D^2, 1+D+D^2)$ and the 16-state code proposed by Tarokh, Seshadri, and Calderbank (TSC) [4] in a system with $L_t = 2$, $M = 2$, $L_r = 1$, 100 simultaneous transmissions, and QPSK modulation. The Z_4 code was obtained by lifting the binary four-state $(6_8, 7_8)$ code as described in [6]. This new code was shown to achieve the maximum diversity advantage for full rate codes with independent fading in this scenario (i.e., $r = 3$) [8]. By inspecting the smallest error event for the TSC code, it is easy to see that it achieves $r = 2$ in independent fading. The performance of the two codes is reported for independent fading, quasistatic fading (i.e., the two fading blocks are temporally correlated), and in a channel with the following 2-D spatio-temporal correlation matrix

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{49}$$

which can be factored to $R = AA^H$, where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{50}$$

has a rank $d = 3$.

One can easily check that both codes achieve full spatial diversity in quasistatic channels. Proposition 7 shows that the new code achieves $r = 2$ in the spatio-temporal correlated channel. By inspecting the error events of the TSC code, one can easily see that it achieves $r = 1$ in this scenario. This validates our claim that the quasistatic channel is not a worst-case scenario (i.e., the TSC code achieves $r = 2$ in the quasistatic channel with $d = 2$, and $r = 1$ in this channel with $d = 3$).

The diversity gain offered by the proposed code in independent fading is shown in the steeper probability of error curve. This results in performance gains at high signal-to-noise ratios (e.g., 0.8 dB at 10^{-2} frame error rate). The difference in the diversity advantage in the spatio-temporal correlated channel is, again, evident in the superior slope of the probability of error curve for the new code. The performance gain offered by the new code is even more significant in this scenario (i.e., 3 dB at 5×10^{-2} frame error rate). In the quasistatic channel, the two performance curves have the same slope. The TSC code, however, offers an advantage of 1 dB in coding gain. Focusing on the worst-case performance in these three channels, we can see the significant gain offered by the new code. This example supports our claim that achieving full diversity in quasistatic channels is not enough to avoid catastrophic performance in certain spatio-temporal correlated channels. Maximizing the diversity advantage in independent block fading, however, results in a more predictable performance, as given by Proposition 7. In fact, as illustrated in the example, maximizing the diversity advantage in *independent* fading may even result in significantly higher gains in *correlated* fading scenarios.

VI. CONCLUSIONS

In this paper, we investigated the robustness of space-time codes in the line-of-sight and correlated space-time fading channels. Our analysis revealed the design criterion for optimizing the worst-case performance of stationary users in line-of-sight channels. We further proposed a simple *phase randomization* approach that was shown to achieve significant performance gains in the line-of-sight scenario without affecting the performance in Rayleigh fading channels. In the correlated fading environments, we characterized the achievable diversity order based on the number of degrees of freedom in the channel. This characterization resulted in valuable insights on the tradeoff between transmission rate and performance robustness and the design of space-time codes using some *prior* knowledge about the channel *spatio-temporal* correlation function. Finally, simulation results that demonstrate the significant gains possible with the proposed approaches were presented.

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