On the Design of Adaptive Space-Time Codes

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Abstract

In this paper, we investigate the design of adaptive space-time codes that exploit partial transmitter channel state information. We introduce the adaptive space-time parsing paradigm as a generalization of the transmitter selection diverisy approach. We then utilize this new framework to construct full diversity adaptive space-time codes for delay limited application with fixed rate transmission. The proposed codes allow for redcuded complexity decoders and are robust to inaccuracies in the transmitter channel state information.

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I. Introduction

In this paper, we develop an adaptive space-time coding framework that requires only partial transmitter channel state information (CSI). The proposed scheme can be viewed as a generalization of the selection diversity approach, and hence, is characterized by the need for a very low rate feedback channel. Furthermore, the proposed codes are shown, experimentally, to enjoy enhanced robustness to inaccuracies in the partial transmitter CSI.

In particular, we consider an $N_t \times N_r$ frequency non-selective multi-input-multi-output (MIMO) fading channel. The discrete-time signal y_{jk} observed by the j-th receive antenna at time k can be mathematically represented as

$$y_{jk} = \sum_{i=1}^{t} h_{ji} x_{ik} + n_{jk} \tag{1}$$

where h_{ji} represents the complex path gain between the i-th transmit antenna and the j-th receive antenna, x_{ik} denotes the code symbol transmitted from the i-th antenna at time k, and n_{jk} corresponds to the noise sample at the j-th receive antenna and time k. These noise samples are assumed to be independent and identically distributed samples from a circularly symmetric zero-mean complex Gaussian variable with unit variance per complex dimension. The fading coefficients h_{ji} are independent complex Gaussian random variables with zero mean and unit variance. The fading model of primary interest is the quasi-static model in which the path gains are constant during the transmission of a whole codeword and change independently from one code word to the next [1], [2]. The received signal can be expressed in matrix notation as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{N} \tag{2}$$

where $\mathbf{Y}$ denotes the $N_r \times T$ matrix of received signals, $\mathbf{N}$ represents the $N_r \times T$ white Gaussian noise matrix, $\mathbf{X}$ corresponds to the $N_t \times T$ matrix of transmitted symbols, T is the code word length, and $\mathbf{H}$ is the $N_r \times N_t$ matrix of channel coefficients. Finally, the following average power constraint is imposed on the transmitted signals

$$\frac{1}{T} \sum_{i=1}^{N_t} \sum_{k=1}^{T} ||x_{ik}||^2 \leq \gamma. \tag{3}$$

II. Adaptive Space-Time Parsing

The proposed adaptive parsing approach, depicted in Figure. 1, belongs to this category of algorithms that require only partial transmitter CSI. In this approach, the adaptive parser activates only a subset of the available transmit antennas, with equal power assigned to all active antennas, at any point of time. This subset of *active* antennas is chosen to maximize the instantaneous capacity, assuming white Gaussian signals, among all possible combinations of transmit antennas for this particular channel realization. For a given subset of active antennas, the instantaneous capacity is given by

$$C_{in} = \log_2 \Big(\det (\mathbf{I_r} + \frac{\gamma}{N_a} \mathbf{H_{ap}} \mathbf{H}^{\dagger}_{\mathbf{ap}}) \Big) \tag{4}$$

where $\mathbf{H_{ap}}$ represents the $N_r \times N_a$ matrix of fading coefficients corresponding to the active transmit antennas and N_a is the number of active transmit antennas. The receiver determines the best combination of *active* antennas and the corresponding transmission rate[1] and feed them back to the transmitter. We first observe that a much lower rate is required by the feedback channel in this approach compared to the perfect CSI scenario where "$N_t \times N_r$" complex fading coefficients must be sent to the transmitter for each channel realization. The average capacity of the adaptive parsing approach can be written as

$$C_{ap} = E\Bigg[\max_{\mathbf{H_{ap}} \epsilon \mathcal{H}} \Big(\log_2 \Big(\det (\mathbf{I_r} + \frac{\gamma}{n} \mathbf{H_{ap}} \mathbf{H}^{\dagger}_{\mathbf{ap}}) \Big) \Big) \Bigg] \tag{5}$$

where $\mathcal{H}$ is defined as the set of all possible truncated versions of $\mathbf{H}$ and the expectation is taken with respect to the distribution of the channel matrix $\mathbf{H}$.

The adaptive parsing approach just described suffers from high computational complexity that grows exponentially with the number of transmit antennas (i.e., the receiver needs to compute the instantaneous channel capacity for all combinations of active transmit antennas). To reduce this complexity, we propose a suboptimal adaptive parsing scheme where **only** those subsets that are composed of a **fixed number "N_a"** of antennas will be allowed.

[1]It is easy to see that only N_t bits are sufficient to determine the antennas that should be activated.

The receiver will select the N_a antennas maximizing the instantaneous capacity with N_a chosen off-line as described next. It is straightforward to see that the computational complexity of this approach is of order $\mathcal{O}\left(\binom{N_t}{N_a}\right)$, and hence, of order $\mathcal{O}\left(N_t^{N_a}\right)$ for large N_t. We will refer to this low complexity variant as the *polynomial complexity parser* in the sequel. One can easily see that a large reduction in complexity is achieved for small values of N_a. This suggests that one should seek the minimum value of N_a that allows for realizing *most* of the gain possible with adaptive parsing. In fact, for systems with $N_r = 1$, we have the following result

Lemma 1: In systems with one receive antenna, the performance of the exponential complexity adaptive parser coincides with the polynomial complexity adaptive parser with $N_a = 1$ (i.e., selection diversity).

Proof: The proof follows from [3]. ■

On the other hand, for $N_r > 1$ the performance loss incurred by the selection diversity approach (i.e., $N_a = 1$) compared to the exponential complexity adaptive parser is significant. This can be attributed to the fact that the instantaneous capacity grows linearly with $\min\{N_a, N_r\}$. This observation suggests that $N_a = N_r$ is the minimum value of N_a that preserves the *degrees of freedom* in the channel. We currently do not have an analytical approach for optimizing the value of N_a for systems with arbitrary number of receive antennas. The numerical results presented later provide strong evidence, however, that the choice $N_a = N_r$ strikes a very favorable trade-off between performance and complexity. Therefore, unless otherwise stated, this choice will be adopted in the rest of the paper.

To further reduce the complexity of the adaptive space-time parsing scheme, an alternative design with linear complexity in N_t is considered next. This design is inspired by the following upper bound

Lemma 2: The instantaneous capacity of the polynomial complexity adaptive parsing scheme is upper bounded by

$$C_{in}^{(poly)} \leq N_a \log_2\left(1 + \frac{\gamma}{N_a}\lambda_{mean}\right) \tag{6}$$

where

$$\lambda_{mean} = \frac{1}{N_a}\sum_{l=1}^{N_a}\lambda_l = \frac{1}{N_a}\sum_{l=1}^{N_a}\sum_{j=1}^{N_r}|h_{ji_l}|^2, \tag{7}$$

$\{\lambda_1, \ldots, \lambda_{N_a}\}$ are the eigenvalues of $\mathbf{H_{ap}}\mathbf{H_{ap}^{\dagger}}$ and h_{ji_l} is the path gain from the l-th active transmit antenna to the j-th receive antenna.

Proof: The proof follows directly from Jensen's inequality. ■

This upper bound motivates the new design which aims at maximizing the trace of $\mathbf{H_{ap}}\mathbf{H_{ap}^{\dagger}}$. To maximize $\sum_{l=1}^{N_a}\left(\sum_{j=1}^{N_r}|h_{ji_l}|^2\right)$, the receiver selects the transmit antennas with the N_a largest antenna gains $\sum_{j=1}^{N_r}|h_{ji_l}|^2$. It is straightforward to see that the complexity of this approach grows linearly with the total number of transmit antennas N_t. It is worth noting that the number of bits needed in the feedback command of the polynomial and linear complexity parsers is only $\log_2\left(\binom{N_t}{N_a}\right)$.

III. Adaptive Space-Time Coding

The design presented in the previous section was based on information-theoretic results and assumed no delay or complexity constraints. We now utilize the insights gained in the previous section to construct an adaptive space-time coding (STC) framework for delay and complexity limited MIMO systems. The delay limitation is reflected in the constant rate transmission assumed in this section[2].

In our design, we utilize a **conditional** bound on the pairwise probability of error, rather than the average one, because it allows for exploiting the partial transmitter CSI to optimize the performance as shown next. The conditional pairwise probability of error $P(\mathbf{X_1} \rightarrow \mathbf{X_2}|\mathbf{H})$ corresponds to the probability of erroneously deciding in favor of the code word $\mathbf{X_2}$ when $\mathbf{X_1}$ was transmitted conditioned on the fading channel realization. The design rules for adaptive space-time coding will be extracted from the following upper bound on the conditional pairwise probability of error.

[2]We can see that the feedback information is now reduced to the N_t bits needed to determine the active antennas.

Lemma 3: The conditional pairwise probability of error is upper bounded by

$$P\Big(\mathbf{X_1} \rightarrow \mathbf{X_2}|\mathbf{H}\Big) \leq Q\Bigg(\sqrt{\lambda_{min}\sum_{j=1}^{N_r}\sum_{l=1}^{N_a}|h_{ji_l}|^2}\Bigg). \tag{8}$$

where λ_{min} denotes the minimum eigenvalue of the matrix $\mathbf{A}(\mathbf{X_1}, \mathbf{X_2})$, defined as

$$\mathbf{A}(\mathbf{X_1}, \mathbf{X_2}) = (\mathbf{X_1} - \mathbf{X_2})(\mathbf{X_1} - \mathbf{X_2})^{\dagger}. \tag{9}$$

Proof: Assuming perfect fading estimates at the receiver, the conditional pairwise probability of error can be expressed as

$$P\Big(\mathbf{X_1} \rightarrow \mathbf{X_2}|\mathbf{H}\Big) = Q\Big(\sqrt{d^2(\mathbf{X_1}, \mathbf{X_2})}\Big) \tag{10}$$

where $Q(.)$ is the Marcum function, and $d^2(\mathbf{X_1}, \mathbf{X_2})$ denotes the squared Euclidian distance between the **received** signals corresponding to the two codewords $\mathbf{X_1}$ and $\mathbf{X_2}$. This distance is given by [1]

$$d^2(\mathbf{X_1}, \mathbf{X_2}) = \sum_{j=1}^{N_r} \underline{h}_j \mathbf{A}(\mathbf{X_1}, \mathbf{X_2}) \underline{h}_j^{\dagger} \tag{11}$$

where $\underline{h}_j$ denotes the j-th row of the truncated channel matrix $\mathbf{H_{ap}}$ containing the fading coefficients experienced by the active transmit antennas, i.e., $\underline{h}_j = [h_{ji_1} \; h_{ji_2} \; \dots \; h_{ji_n}]$, and the matrix $\mathbf{A}(\mathbf{X_1}, \mathbf{X_2})$ is constructed from the pair of distinct codewords $\mathbf{X_1}$ and $\mathbf{X_2}$ as defined in (9). This matrix is Hermitian, and therefore, has real nonnegative eigenvalues $\{\lambda_1, \dots, \lambda_{N_a}\}$. The eigen-decomposition of the matrix $\mathbf{A}(\mathbf{X_1}, \mathbf{X_2})$ is employed:

$$\mathbf{A}(\mathbf{X_1}, \mathbf{X_2}) = \mathbf{V}^{\dagger}\mathbf{D}\mathbf{V} \tag{12}$$

where $\mathbf{D}$ is a real diagonal matrix collecting the eigenvalues of $\mathbf{A}(\mathbf{X_1}, \mathbf{X_2})$, and $\mathbf{V}$ is a unitary matrix whose rows are given by the eigenvectors of $\mathbf{A}(\mathbf{X_1}, \mathbf{X_2})$. Following in the footsteps of [1], we define the row vectors

$$\underline{\zeta}_j = \underline{h}_j \mathbf{V}^{\dagger} = (\zeta_{ji_1} \; \zeta_{ji_2} \; \dots \; \zeta_{ji_n}) \quad j = 1, \dots, N_r. \tag{13}$$

It is then straightforward to see that

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{j=1}^{N_r}\sum_{l=1}^{N_a} \lambda_l |\zeta_{ji_l}|^2 \geq \lambda_{min} \sum_{j=1}^{N_r}\sum_{l=1}^{N_a} |\zeta_{ji_l}|^2 \tag{14}$$

where λ_{min} denotes the minimum eigenvalue of the matrix $\mathbf{A}(\mathbf{X_1},\mathbf{X_2})$. Now, since the elements ζ_{ji_l} are derived from the fading gains h_{ji_l} through a unitary transformation, the following property holds

$$\sum_{j=1}^{N_r}\sum_{l=1}^{N_a} |\zeta_{ji_l}|^2 = \sum_{j=1}^{N_r}\sum_{l=1}^{N_a} |h_{ji_l}|^2, \tag{15}$$

and leads to

$$P\Big(\mathbf{X_1} \rightarrow \mathbf{X_2}|\mathbf{H}\Big) \leq Q\Bigg(\sqrt{\lambda_{min} \sum_{j=1}^{N_r}\sum_{l=1}^{N_a} |h_{ji_l}|^2}\Bigg). \tag{16}$$

■

Lemma 3 suggests that adaptive space-time codes should be constructed to maximize

$$\lambda_{min} \sum_{j=1}^{N_r}\sum_{l=1}^{N_a} |h_{ji_l}|^2 = \lambda_{min}\sigma. \tag{17}$$

This design criterion can be decomposed into two **independent** optimization problems. The first one is the construction of space-time codes that maximize λ_{min} whereas the second one is concerned with maximizing the equivalent channel gain "σ". The decomposition of σ as a sum of antenna gains

$$\sigma = \sum_{l=1}^{N_a} \sigma_l, \tag{18}$$

where $\sigma_l = \sum_{j=1}^{N_r} |h_{ji_l}|^2$ is the *partial* gain corresponding to the i_l-th transmit antenna, implies that the equivalent channel gain is maximized by the linear complexity adaptive parser discussed in the previous section. Now, we turn our attention to maximizing λ_{min} over all distinct pairs of codewords. We observe that the requirement that $\lambda_{min} > 0$ [3] is equivalent to the *full diversity* criterion utilized for code design with only receiver CSI, however, in a system with only N_a transmit antennas (e.g., [1], [2]). The need for full diversity codes entails the following bound on the maximum transmission rate with standard constellations[4]

[3] Otherwise the bound in Proposition 3 is trivial.

[4] This refers to constellations like QAM or PSK and excludes the rotated multi-dimensional constellations.

Lemma 4: The maximum transmission rate for the proposed adaptive space-time codes with standard constellations is one symbol/channel use.

Proof: This result follows directly from the Singleton bound [4], [1], [5]. ■

We will refer to space-time codes that achieve the maximum transmission rate as full rate codes. These full rate codes are, however, generated by multiplexing rate $1/N_a$ codes across the N_a active transmit antennas to achieve a throughput of one symbol per channel use. It is worth noting, however, that unlike the coherent scenario (only receiver CSI) where one should seek full diversity space-time codes for N_t transmit antennas, we only need full diversity codes for N_a transmit antennas in this design approach to achieve $d = N_t \times N_r$ levels of diversity[5]. As the trellis complexity of full diversity space-time codes grows exponentially with the number of *active* transmit antennas [1], the efficient utilization of partial transmitter CSI in the proposed approach results in a significant reduction in decoding complexity for trellis space-time codes, especially for systems with $N_t \gg N_r$.

The robustness of the proposed approach can be best illustrated in the scenario with **zero** signal-to-noise ratio, or equivalently infinite delay, in the feedback channel. In this case, the N_a active transmit antennas are selected **randomly**, and the system is reduced to a multi-antenna systems with $N_t = N_a$ and only receiver CSI. In this case, the worst case diversity level is equal to $d_w = N_a \times N_r$. The proposed approach is therefore *universal* in the sense that it exploits *reliable* partial CSI whenever available. This observation implies that the choice of N_a offers a trade-off between the worst case performance, the complexity of the decoder, and the adaptive gain. For example, setting $N_a = N_t$ is the preferred choice in terms of worst case performance with $d_w = N_t \times N_r$ but offers no adaptive gain. On the other hand, setting $N_a = N_r$, i.e., the minimum value that preserves the degrees of freedom, yields a diversity advantage of only $d_w = N_r^2$ in the worst case, but achieves a significant reduction in the decoder complexity and a significant adaptive gain as illustrated by the numerical results. Moreover, in the absence of the transmitter CSI [6], we can cycle through

[5]One can easily verify that the proposed approach achieves full diversity if $\Lambda_{min} > 0$.

[6]This scenario is different than the unreliable transmitter CSI since the transmitter now *knows* that no CSI is

the different transmit antennas with N_a antennas active simultaneously to transform the quasi-static fading channel into a block fading channel with $M = \lfloor \frac{N_t}{N_a} \rfloor$ blocks per codeword, and hence, allow for even higher diversity advantages than "d_w" [5].

IV. Numerical Results

We first present numerical results for the average capacity achieved by the different variants of the adaptive parsing approach. In Figures 2 and 3, we report the average capacity[7] of the exponential complexity adaptive parser, the polynomial complexity adaptive parser with different values of N_a, and the coherent case (only receiver CSI "R-CSI"). It is shown in the figure that the gain of the adaptive parser increases with N_t for a fixed N_r. It is also observed that the capacity of the polynomial complexity adaptive parser obtained with $N_a = N_r$ is very close to the capacity achieved by the exponential complexity adaptive parser in all the reported cases. In fact, the same observation holds for other values of receive antennas as well. The results for those scenarios, however, were not reported here for brevity. The significant loss incurred by the selection diversity approach when $N_r > 1$ is also evident in the figure. When the number of receivers N_r increases, slightly better results are achieved with larger values for N_a, but the loss of performance incurred by choosing $N_a = N_r$ is very marginal. The saving in computational complexity is, however, significant (i.e., from $\mathcal{O}\left(2^{N_t}\right)$ in the exponential complexity parser to $\mathcal{O}\left(\binom{N_t}{N_r}\right)$ in the polynomial complexity parser).

The performance of the adaptive space-time coding approach is reported in Figure 4 with BPSK modulation, two receive antennas, and different numbers of transmit antennas (i.e., $2, 4, 8$). In the proposed approach, we select the best **two** transmit antennas and use the same 8-state full diversity code in all cases. The 8-state code is the one reported in [6] which was found to maximize the minimum eigenvalue. We also report the performance of the 8-state delay diversity scheme with four transmit antenna. When $N_t = 2$, the adaptive scheme also coincides with the coherent scenario. From the figure, it is clear that the gain offered by

available.

[7]The expectation in (5) is computed using Monte-Carlo integration.

the proposed approach increases as the number of transmit antennas increases (i.e., as the number of antennas grows, the parser is more likely to select antennas with favorable channel conditions). Furthermore, this gain does not entail any **additional decoding complexity** since a full diversity code for $N_a = 2$ transmit antennas is employed in all cases. This can be contrasted to the case with CSI only at the receiver, where the trellis complexity of full diversity codes grows exponentially with the number of transmit antennas N_t.

In Figure 5, we report the performance of the proposed approach with $N_a = N_r = 2$ and outdated fading estimates at the transmitter. The correlation between the outdated estimates and the correct CSI is referred to as ρ in the figure. We also report the performance of the selection diversity approach (i.e., $N_a = 1$) with the same constellation and throughput. The significant gain offered by the proposed approach is apparent in the figure for different ρ. One can also observe that the gain offered by the proposed approach, compared to selection diversity, increases as ρ decreases. This observation confirms the enhanced robustness of the proposed approach to the feedback channel delay.

V. Conclusions

In this paper, we considered the design of adaptive space-time techniques that exploit partial transmitter channel state information. In particular, we presented the adaptive space-time parsing strategy as a generalization of the selection diversity approach. This strategy selects a subset of active transmit antennas presenting optimal fading conditions. The adaptive parsing strategy was shown to achieve substantial performance improvements over nonadaptive schemes with a very low rate feedback link. Furthermore, low complexity variants of the adaptive parser that offer a compromise between performance and complexity were investigated. Using this new framework, adaptive space-time codes that exploit the partial transmitter channel state information, whenever available, were presented. The performance gains offered by the proposed codes were quantified experimentally in certain representative scenarios.

References

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank. Space-time codes for high data rate wireless communication: Performance criterion and code construction. *IEEE Trans. Info. Theory*, IT-44:744–765, March 1998.

[2] J.-C. Guey, M. R. Bell M. P. Fitz, and W.-Y. Kuo. Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels. *IEEE Vehicular Technology Conference*, pages 136–140, Atlanta, 1996.

[3] R. Knopp and P. A. Humblet. Information capacity and power control in single cell multi-user communcations. *presented at the IEEE Intrenational Conference in Communications*, Seatle, USA, June 1995.

[4] A. Lapidoth. The performance of convolutional codes on the block erasure channel using various finite interleaving techniques. *IEEE Trans. Info. Theory*, IT-43:1459–1473, September 1997.

[5] H. El Gamal and A. R. Hammons Jr. On the design of algebraic space-time codes for MIMO block fading channels. *IEEE Trans. Info. Theory*, vol. 49:151–163, Jan. 2003.

[6] Q. Yan and R. Blum. Optimum space-time convolutional codes. *Presented at the WCNC'00*, pages Chicage, U.S.A, September 2000.

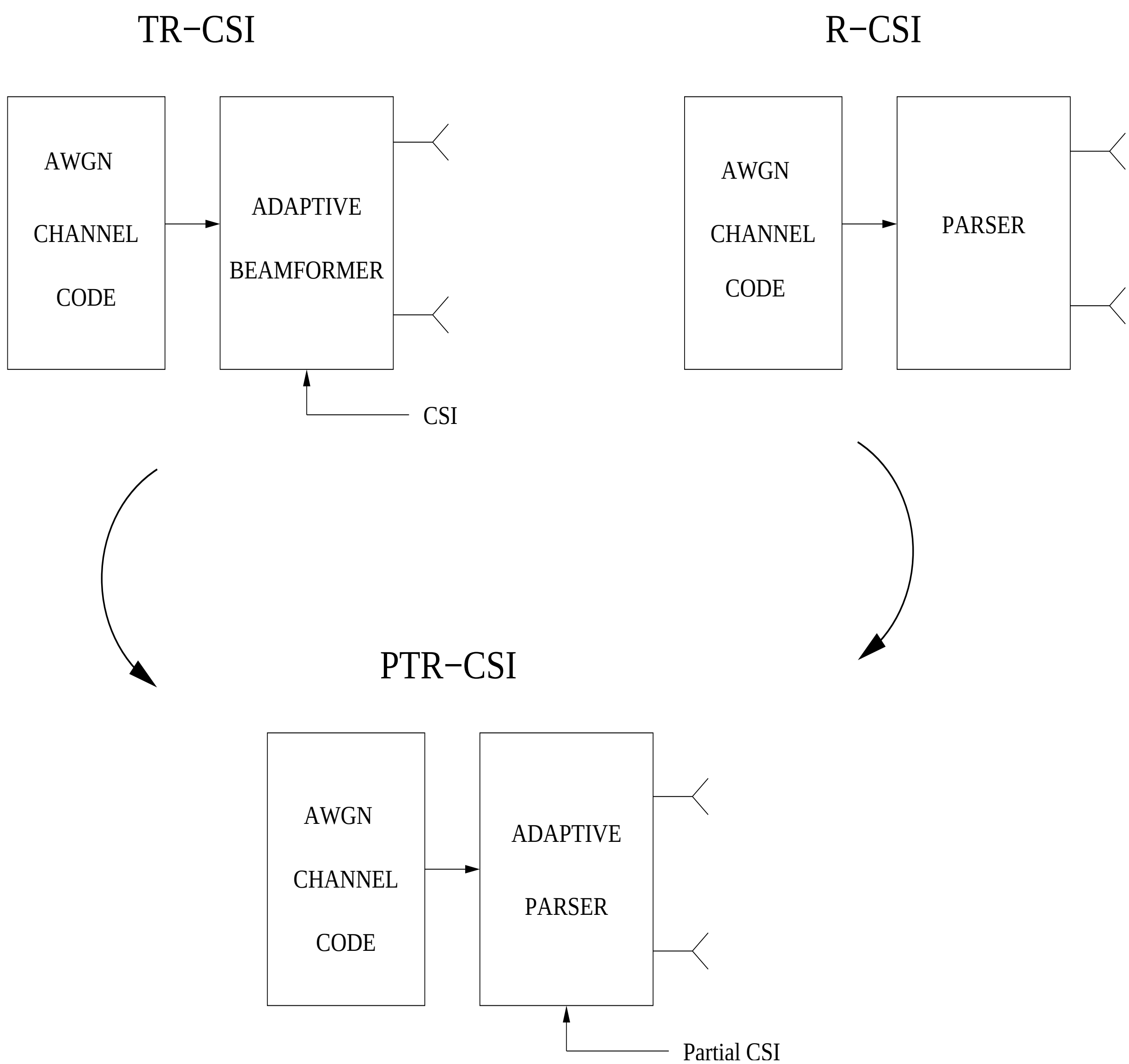

Fig. 1. The proposed scheme with partial transmitter CSI (PTR-CSI) as a compromise between the scheme using perfect CSI at the transmitter and receiver (TR-CSI) and the one with only receiver (R-CSI).

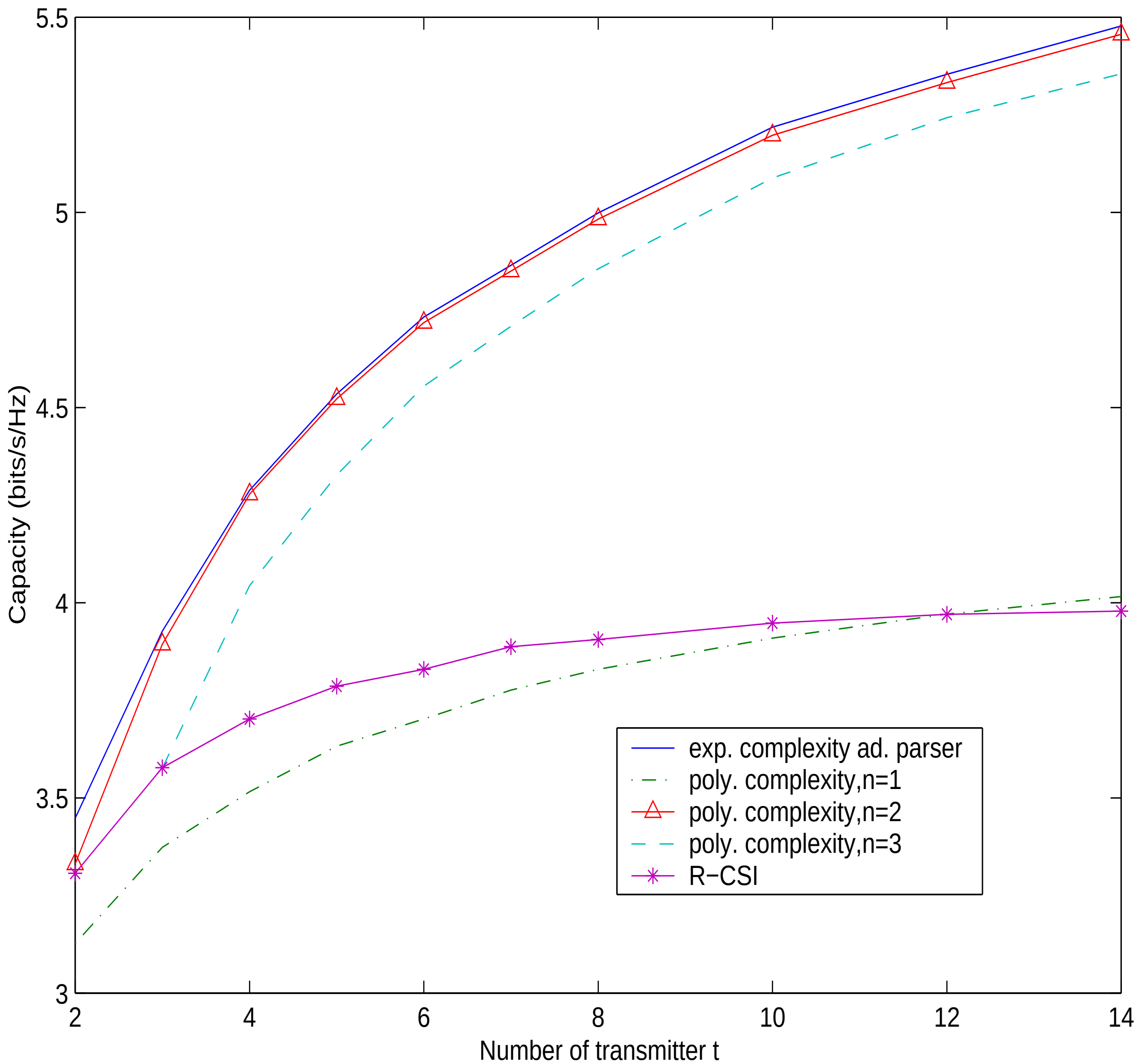

Fig. 2. Capacity obtained with the polynomial complexity adaptive parser for different values of n, compared with the capacity of the exponential complexity adaptive parser (SNR = 5 dB and two receive antennas).

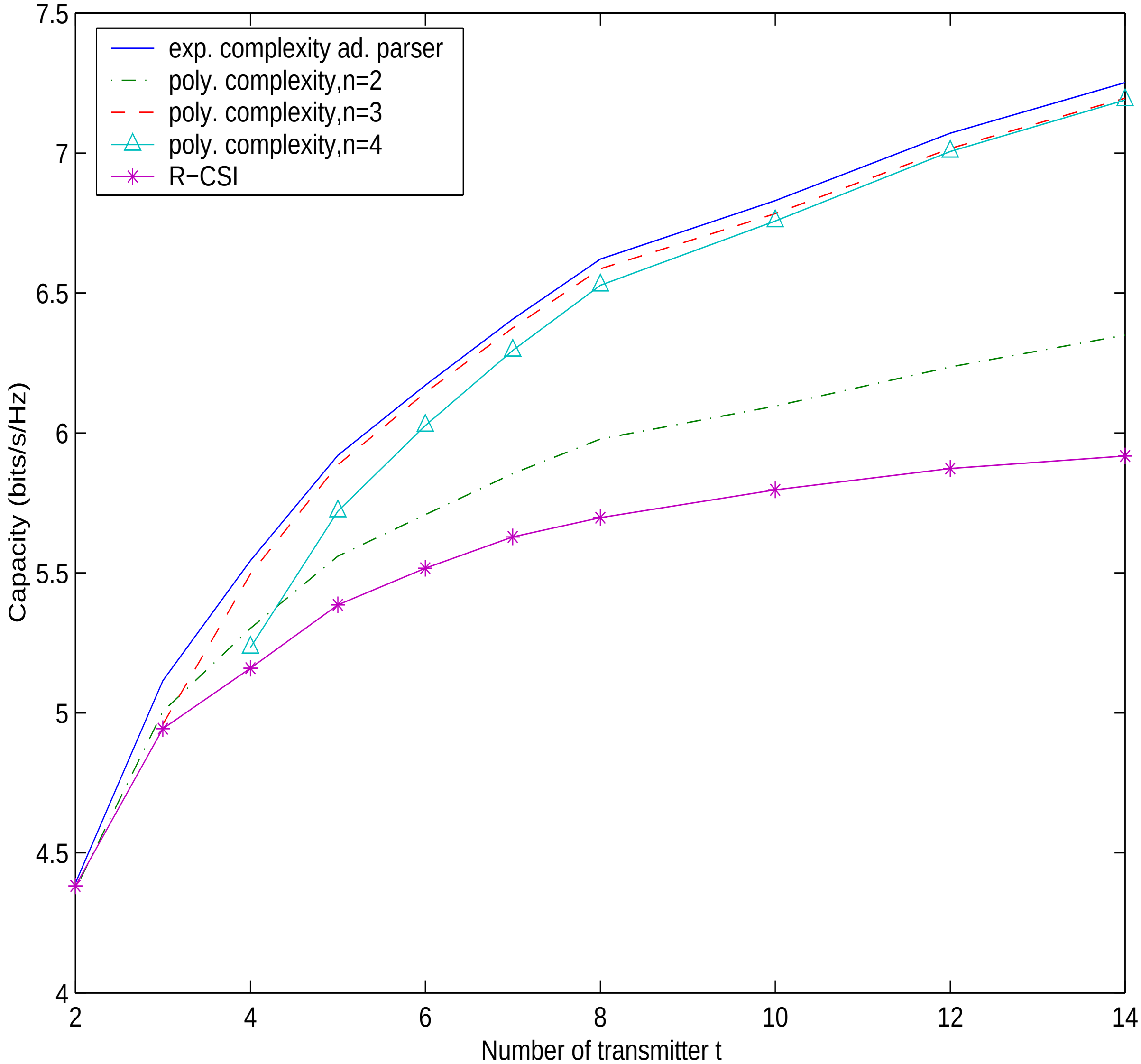

Fig. 3. Capacity obtained with the polynomial complexity adaptive parser for different values of n, compared with the capacity of the exponential complexity adaptive parser (SNR = 5 dB and three receive antennas).

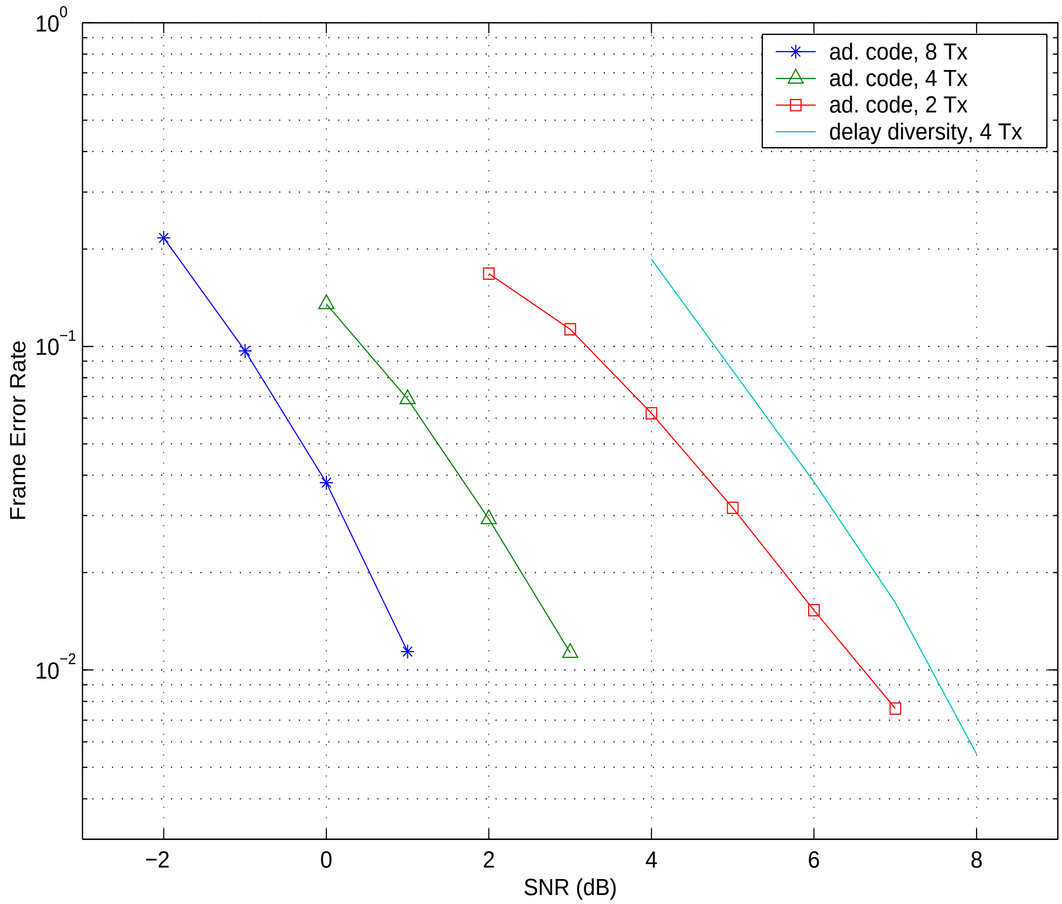

Fig. 4. Performance of the proposed adaptive coding scheme when $n = r = 2$ and different total numbers of transmit antennas with an 8-state BPSK space-time code.

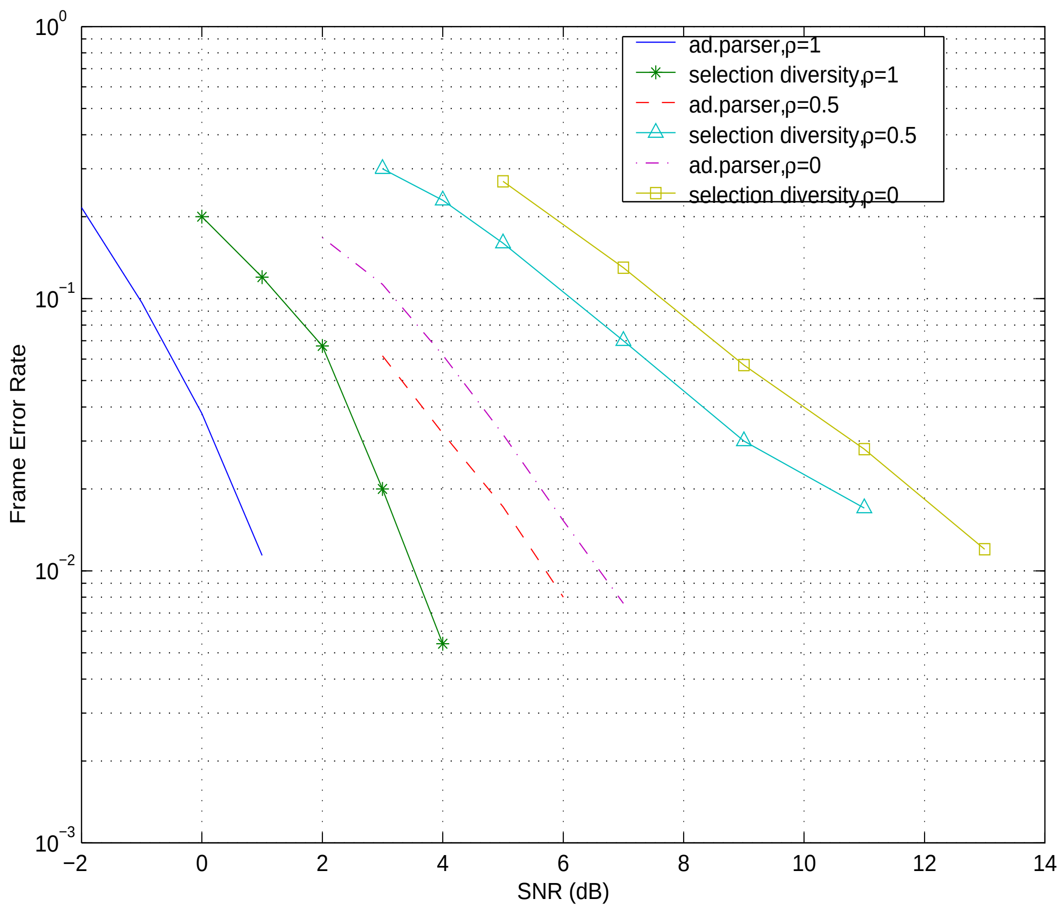

Fig. 5. Comparison of the performance achieved by the adaptive code and selection diversity with imperfect feedback ($n = r = 2$ and $t = 8$).