

# Space-Time Coding for MIMO Systems with Co-Channel Interference

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## Abstract

We consider the design of space-time codes for multi-input-multi-output (MIMO) systems operated in the presence of co-channel interference (CCI). Based on the pairwise probability of error analysis, we develop a new design criterion that determines the code robustness to CCI (CCI diversity gain). We further develop an algebraic framework for constructing space-time codes that jointly optimize the fading and CCI diversity gains. The proposed framework is general for arbitrary numbers of transmit antennas and quadrature amplitude modulation (QAM) constellations. Numerical results that quantify the performance gains offered by the proposed techniques are also presented.

**Index terms:** wireless communication, space-time codes (STCs), block fading (BF) channels, co-channel interference (CCI), channel state information (CSI) and erasure channel (EC).

## 1 Introduction

In this paper, we consider the design of space-time codes (STCs) for coherent multi-input-multi-output (MIMO) systems that operate in the presence of co-channel interference (CCI). **As a first step, we use a simplified two state model for the CCI in this paper.** By adopting this model, we gain valuable insights that guide code design. In particular, we introduce a new design criterion that determines the STC robustness to CCI. We refer to the new criterion as the *CCI diversity gain* to differentiate it from the well known fading diversity gain (e.g., [1]). We further benefit from the algebraic design framework proposed in [2, 3] to construct trellis space-time codes that **jointly optimize both diversity gains. In quasi-static**

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**channels, these codes exploit the spatial dimension to smooth out the fluctuations induced by multi-path fading and the temporal dimension to combat the CCI.** The proposed design framework is general for arbitrary numbers of transmit antennas and single dimensional QAM constellations<sup>1</sup>. For this class of coded modulation schemes, we also discuss the necessary trade-off between transmission rate and code robustness. The rest of the paper is organized as follows. Section 2 introduces the system model. The new design criterion is introduced in Section 3. Section 4 develops an algebraic framework for constructing STCs that optimize the CCI diversity gain. Simulation results are presented in Section 5 and concluding remarks in Section 6.

## 2 System Model

We consider an  $L_t \times L_r$  MIMO system where the source generates  $K$  information symbols which are encoded by the error control code  $C$  to produce code words of length  $NL_t$  over the alphabet  $\mathcal{C}$ . The encoded symbols are parsed among  $L_t$  transmit antennas and then mapped by the modulator into constellation points from the normalized<sup>2</sup> discrete complex-valued QAM constellation  $\mathcal{S}$  for transmission across the channel. Therefore, a code word  $\underline{c} = (c_1^1, c_1^2, \dots, c_1^{L_t}, c_2^1, c_2^2, \dots, c_2^{L_t}, \dots, c_N^1, c_N^2, \dots, c_N^{L_t}) \in C$  is first mapped to the matrix

$$\mathbf{c} = \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_N^1 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{L_t} & c_2^{L_t} & \cdots & c_N^{L_t} \end{bmatrix} = \begin{bmatrix} \underline{c}_1 \\ \vdots \\ \underline{c}_{L_t} \end{bmatrix}$$

before modulation. In this notation,  $c_k^i$  is the code symbol assigned to transmit antenna  $i$  at time  $k$  and  $\underline{c}_i$  is the set of  $N$  symbols transmitted by antenna  $i$ . With a slight abuse of notation, we will now refer to the  $(L_t \times N)$  STC as  $C$  in the rest of the paper. Let  $f : \mathcal{C} \rightarrow \mathcal{S}$  be the modulator mapping function. Then

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<sup>1</sup>A single dimension here refers to one complex dimension.

<sup>2</sup>Unit energy.

$\mathbf{s} = f(\mathbf{c})$  is the base-band version of the code word as transmitted across the channel. For this system, we have the following baseband model of the received signal:

$$r_k^j = \sqrt{E_s} \sum_{i=1}^{L_t} \alpha_k^{ji} s_k^i + n_k^j, \quad (1)$$

where  $E_s$  is the energy per transmitted symbol;  $\alpha_k^{ji}$  is the complex path gain from transmit antenna  $i$  to receive antenna  $j$  at time  $k$ ;  $s_k^i = f(c_k^i)$  is the transmitted constellation point from antenna  $i$  at time  $k$ ;  $n_k^j$  is the additive noise, resulting from the sum of the thermal and CCI noises, for receive antenna  $j$  at time  $k$ . The different path gains  $\alpha_k^{ji}$  are assumed to be statistically independent. The fading model of primary interest is that of a block flat Rayleigh fading process in which the code word encompasses  $M$  fading blocks. The complex fading gains are constant over one fading block but are independent from block to block. We model the CCI as a white Gaussian process whose variance is constant across one block and changes independently from block to block. We further adopt a simple two state model for the CCI noise variance. The additive noise samples are, therefore, independent samples of a zero-mean complex Gaussian random variable with variance  $(N_0 + Z_m N_1)/2$  per dimension, where  $Z_m$  is the output of an i.i.d Bernoulli process with probability  $\rho$ . The variance of the noise is assumed to be constant across the fading block but changes independently from block to block. For simplicity, it is assumed that  $M$  divides  $N$  and  $N \geq L_t$ . While this model may not be very realistic, we believe that the desirable properties of codes constructed according to this model will carry over when those codes are used in realistic wireless channels. In fact, this model is widely used in the literature to model the CCI in frequency hopping systems. In the sequel, we present numerical results with a multi-state CCI model that validate our claim. Throughout the paper, we assume that the channel state information (CSI) is known *a-priori* only at the receiver.

The received signal expressed in vector notation is

$$\underline{r}_k = \sqrt{E_s} \mathbf{H}_m \underline{s}_k + \underline{n}_k, \quad (2)$$

where  $\underline{r}_k$  denotes the  $L_r \times 1$  vector of received signals at time  $k$ ,  $\underline{n}_k$  is the  $L_r \times 1$  white Gaussian noise

vector,  $\underline{s}_k$  is the  $L_t \times 1$  vector of transmitted symbols and  $\mathbf{H}_m$  the  $L_r \times L_t$  complex matrix of fading coefficients corresponding to fading block  $m$  given by

$$\mathbf{H}_m = \begin{bmatrix} h_m^{11} & h_m^{12} & \dots & h_m^{1L_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_m^{L_r 1} & h_m^{L_r 2} & \dots & h_m^{L_r L_t} \end{bmatrix}, h_m^{ji} = \alpha^{\frac{ji}{\left(\frac{(m-1)N}{M} + 1\right)}} = \dots = \alpha^{\frac{ji}{\left(\frac{mN}{M}\right)}}. \quad (3)$$

### 3 CCI Diversity Gain

Let  $\mathbf{c}$  be the transmitted code word, then the conditional pairwise probability of error (PWE<sup>3</sup>) that the decoder will prefer the alternate code word  $\mathbf{e}$  to  $\mathbf{c}$  is given by (e.g., [1])

$$P(\mathbf{c} \rightarrow \mathbf{e} | \{\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_M, Z_1, \dots, Z_M\}) \leq \exp \left\{ -\frac{E_s}{4} \sum_{m=1}^M \frac{d_m^2(\mathbf{c}, \mathbf{e})}{(N_0 + z_m N_1)} \right\}, \quad (4)$$

where  $d_m^2(\mathbf{c}, \mathbf{e})$  denotes the squared Euclidean distance given by

$$d_m^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^{L_r} \underline{h}_m^j \mathbf{A}_m \underline{h}_m^{jH},$$

$\mathbf{A}_m(\mathbf{c}, \mathbf{e}) = (f(\mathbf{c}[m]) - f(\mathbf{e}[m]))(f(\mathbf{c}[m]) - f(\mathbf{e}[m]))^H$ ,  $\mathbf{c}[m]$  is the sub-matrix of code word  $\mathbf{c}$  assigned to the  $m^{\text{th}}$  fading block, and  $\underline{h}_m^j = [h_m^{j1} \ h_m^{j2} \ \dots \ h_m^{jL_t}]$ . We further denote the eigen-values of  $\mathbf{A}_m(\mathbf{c}, \mathbf{e})$  as  $\{\lambda_{A1}[m], \lambda_{A2}[m], \dots, \lambda_{AL_t}[m]\}$ .

Following in the footsteps of [1, 4], we now average (4) w.r.t.  $\{\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_M, Z_1, \dots, Z_M\}$  to get,

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \left( \frac{\mu E_s}{4(N_0 + N_1)} \right)^{-d_{fad} L_r} \left\{ \prod_{m=1}^M \left( \rho + (1 - \rho) \left( 1 + \frac{N_1}{N_0} \right)^{-d_{fad}^{(m)} L_r} \right) \right\}. \quad (5)$$

where  $d_{fad}^{(m)} = \text{rank}(f(\mathbf{c}[m]) - f(\mathbf{e}[m]))$ ,  $d_{fad} = \sum_{m=1}^M d_{fad}^{(m)}$  and  $\mu = (\prod_{m=1}^M \lambda_{A1}[m] \lambda_{A2}[m] \dots)^{1/d_{fad}}$ .

One can easily see that the first term in the right hand side (RHS) of (5) depends on the CCI only through  $N_1$  and hence, does not depend on the CCI *dynamics*. In fact, this term depends on the code design only through the two parameters  $d_{fad}$  and  $\mu$ . These parameters are the fading diversity and coding

<sup>3</sup>The pairwise probability of error assumes that the codebook only contains  $\mathbf{c}$  and  $\mathbf{e}$  [1].

gain, respectively, as defined in [3]. Therefore, the optimization of this term is *independent* of the presence or absence of CCI. In other words, any two codes with the same pairwise error probability profile in the absence of CCI, will be *equivalent* with respect to this term. This motivates the question: given two codes with the same performance in the absence of CCI, which one of them will perform better in the presence of CCI? Using the pairwise probability of error analysis, one can see that this question reduces to the following simpler one: for a given  $d_{fad}$ , what is the best distribution of  $d_{fad}^{(m)}$  that minimizes (5)?

To answer this question, we first note that every term in the product term in (5) belongs to the region  $[\rho, 1]$ , where the  $m^{th}$  term is equal to one if  $d_{fad}^{(m)} = 0$  and equal to  $\rho$  when  $d_{fad}^{(m)} \rightarrow \infty$ . This means that for  $N_1 \gg N_0$ , it is sufficient to have  $d_{fad}^{(m)} \neq 0$  to ensure that the corresponding term is sufficiently close to its minimum. This observation motivates the following design criterion.

- *Baseband Rank Criterion for CCI Diversity*: Maximize the CCI diversity advantage

$$d_{cci} = \sum_{m=1}^M I[\text{rank}(f(\mathbf{c}[m]) - f(\mathbf{e}[m]))]$$

over all pairs of distinct code words  $\mathbf{c}, \mathbf{e} \in C$ , where  $I(x)$  is the indicator function (i.e.,  $I(x) = 1$  for  $x > 0$  and  $I(x) = 0$  for  $x \leq 0$ ).

Intuitively, this design criterion aims at distributing the energy in the error events as uniformly as possible among the independent blocks, and hence, maximizing the temporal spread of the fading diversity across the independent blocks. This way, we maximize the probability that a significant fraction of the error event energy is not corrupted by the CCI. One can further strengthen this argument by considering the erasure channel (EC) (i.e.,  $N_0 = 0, N_1 \rightarrow \infty$ ). In this case, it is straightforward to show that the pairwise probability of error is upper bounded by  $\rho^{d_{cci}}$ . This argument leads us to infer that the CCI diversity advantage represents the code robustness to high CCI noise powers. Therefore, proper code design in this scenario should aim for jointly maximizing  $d_{fad}$  and  $d_{cci}$ . It is also interesting to observe that, unlike the fading diversity advantage which can be increased by exploiting the spatial dimension, the CCI diversity advantage depends only on the temporal properties of the code. This is due to the CCI model adopted here,

where we assume that the **same** interfering power affects signals assigned to **different transmit antennas** in the same fading block.

## 4 Algebraic Code Constructions

In this section, we present a systematic approach for constructing linear binary STCs that optimize the CCI diversity advantage. By combining this approach with the one proposed for optimizing the fading diversity advantage in [3], one can construct codes that jointly optimize both criteria. We first develop our results for arbitrary linear block codes and then specialize the framework to the case of convolutional codes. The main results are stated for the binary phase shift keying (BPSK) constellation (i.e.,  $f(c) = (-1)^c$  for  $c = 0, 1$ ). One can now *lift* the proposed codes to the appropriate number ring (for example, the quaternary number ring  $\mathbb{Z}_4$  for QPSK modulation) to obtain codes that achieve the same levels of fading and CCI diversity for higher order QAM constellations. For brevity, we do not discuss the necessary details here. The interested reader can consult [2, 5] for more details.

Under our model, the linear binary STC  $C$  is defined to consist of code words

$$\mathbf{c} = g(\underline{x}) = \begin{bmatrix} \underline{x}\mathbf{M}_{11} & \underline{x}\mathbf{M}_{12} & \cdots & \underline{x}\mathbf{M}_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{x}\mathbf{M}_{L_t1} & \underline{x}\mathbf{M}_{L_t2} & \cdots & \underline{x}\mathbf{M}_{L_tM} \end{bmatrix},$$

where the binary information vector  $\underline{x} \in \mathbb{F}^K$ ,  $\mathbb{F}$  is the binary field, the code generator matrices  $\mathbf{M}_{ij} \in \mathbb{F}^{K \times N/M}$  and the encoder  $g : \mathbb{F}^K \rightarrow \mathbb{F}^{L_t \times N}$ . We will employ the following matrices in our analysis:

$$\begin{aligned} \tilde{\mathbf{M}}_1 &= [\mathbf{M}_{11}, \mathbf{M}_{21}, \cdots, \mathbf{M}_{L_t1}] \\ &\vdots \\ \tilde{\mathbf{M}}_M &= [\mathbf{M}_{1M}, \mathbf{M}_{2M}, \cdots, \mathbf{M}_{L_tM}]. \end{aligned}$$

**Proposition 1 (Stacking Construction for CCI Diversity)** *Suppose that  $d_{cci}$  is the largest integer such that,  $\tilde{\mathbf{M}}_1, \tilde{\mathbf{M}}_2, \dots, \tilde{\mathbf{M}}_M$  satisfy the condition that  $\forall a_1, a_2, \dots, a_M \in \mathbb{F}, a_1 + a_2 + \dots + a_M = M - d_{cci} + 1$  :*

$$\tilde{\mathcal{M}} = \begin{bmatrix} a_1 \tilde{\mathbf{M}}_1 & a_2 \tilde{\mathbf{M}}_2 & \dots & a_M \tilde{\mathbf{M}}_M \end{bmatrix}$$

*is of full rank  $K$ , over the binary field  $\mathbb{F}$ . Then, the space-time code  $C$  achieves a CCI diversity of  $d_{cci}$ .*

*Proof:* ( $\Rightarrow$ ) Suppose there exists some  $a_1, a_2, \dots, a_M \in \mathbb{F}$ , satisfying  $a_1 + a_2 + \dots + a_M = M - d_{cci} + 1$ , such that  $\tilde{\mathcal{M}}$  is singular. This implies that there exists some  $\underline{x}_1, \underline{x}_2 \in \mathbb{F}^k$ ,  $\underline{x}_1 \neq \underline{x}_2$ , such that  $\underline{x}_1 \tilde{\mathcal{M}} = \underline{x}_2 \tilde{\mathcal{M}}$ . This implies that, by letting  $\mathbf{c} = g(\underline{x}_1)$  and  $\mathbf{e} = g(\underline{x}_2)$ , we get  $\mathbf{c} \neq \mathbf{e}$  and

$$f(\mathbf{c}[m]) = f(\mathbf{e}[m]), \quad \forall 1 \leq m \leq M \text{ and } a_m \neq 0. \quad (6)$$

Since  $a_1 + a_2 + \dots + a_M = M - d_{cci} + 1$  then there are  $M - d_{cci} + 1$  non-zero coefficients  $a_m$  that satisfy (6), and hence,  $C$  does not achieve a CCI diversity advantage of  $d_{cci}$ .

( $\Leftarrow$ ) Assume that  $C$  achieves a CCI diversity advantage less than  $d_{cci}$ . Then there exist two code word matrices  $\mathbf{c}, \mathbf{e} \in C$ ,  $\mathbf{c} \neq \mathbf{e}$ , where at least  $M - d_{cci} + 1$  of the corresponding baseband differences in the different CCI blocks  $\{f(\mathbf{c}[m]) - f(\mathbf{e}[m]), 1 \leq m \leq M\}$  are zero matrices. Hence there exist  $\underline{x}_1, \underline{x}_2 \in \mathbb{F}^k$ , such that  $\mathbf{c} = g(\underline{x}_1)$  and  $\mathbf{e} = g(\underline{x}_2)$ , satisfying  $\underline{x}_1 \tilde{\mathcal{M}} = \underline{x}_2 \tilde{\mathcal{M}}$ , where the  $a_m$ 's in  $\tilde{\mathcal{M}}$  are chosen such that  $a_m = 1$  if  $f(\mathbf{c}[m]) = f(\mathbf{e}[m]), \forall 1 \leq m \leq M$ . Then the matrix  $\tilde{\mathcal{M}}$  is not full ranked.  $\square$

**Corollary 2**  *$C$  achieves full CCI diversity advantage  $M$ , if and only if the matrices  $\tilde{\mathbf{M}}_1, \tilde{\mathbf{M}}_2, \dots, \tilde{\mathbf{M}}_M$  have full rank  $K$  over the binary field  $\mathbb{F}$ .*

The fact that the CCI diversity depends on the properties of the set of matrices  $\{\tilde{\mathbf{M}}_i\}_{i=1}^M$  rather than  $\{\mathbf{M}_i\}_{i=1}^{ML_t}$  highlights the dependence of the CCI diversity on the temporal properties of the code. The following result establishes the necessary tradeoff between throughput and code robustness.

**Lemma 3** *The upper bound on the maximum achievable CCI diversity advantage by  $C$  with BPSK modulation, is given by  $\lfloor M(1 - r) \rfloor + 1$ , where  $r = \frac{K}{NL_t}$ .*

*Proof:* The result follows directly from the Singleton bound.  $\square$

We now specialize the conditions in Proposition 1 to the case of space-time convolutional codes. Consider a binary convolutional code of rate  $K/ML_t$ <sup>4</sup>, whose encoding action is summarized by the matrix equation

$$\begin{aligned}
\mathbf{Y}(D) &= \mathbf{X}(D)\mathbf{G}(D), \text{ where} \\
\mathbf{Y}(D) &= \begin{bmatrix} Y_1(D) & Y_2(D) & \cdots & Y_{ML_t}(D) \end{bmatrix}, \\
\mathbf{X}(D) &= \begin{bmatrix} X_1(D) & X_2(D) & \cdots & X_K(D) \end{bmatrix} \text{ and} \\
\mathbf{G}(D) &= \begin{bmatrix} G_{1,1}(D) & G_{1,2}(D) & \cdots & G_{1,ML_t}(D) \\ \vdots & \vdots & \ddots & \vdots \\ G_{K,1}(D) & G_{K,2}(D) & \cdots & G_{K,ML_t}(D) \end{bmatrix}. \tag{7}
\end{aligned}$$

It is understood here that  $\mathbf{X}(D)$  and  $\mathbf{Y}(D)$  correspond to the  $D$ -transform of the  $K$  binary input sequences and  $ML_t$  coded binary output sequences respectively. Now let  $C$  denote the STC in which the output sequence corresponding to  $Y_{(m-1)L_t+l}(D)$  is assigned to the  $l$ -th transmit antenna in the  $m$ -th fading block. In order to analyze the performance of this code we define the following matrices:

$$\begin{aligned}
\tilde{\mathbf{F}}_1(D) &= [\mathbf{F}_1(D), \cdots, \mathbf{F}_{L_t}(D)] \\
&\vdots \\
\tilde{\mathbf{F}}_M(D) &= [\mathbf{F}_{(M-1)L_t+1}(D), \cdots, \mathbf{F}_{ML_t}(D)], \text{ where} \\
\mathbf{F}_\ell(D) &= \begin{bmatrix} G_{1,\ell}(D) \\ \vdots \\ G_{K,\ell}(D) \end{bmatrix}.
\end{aligned}$$

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<sup>4</sup> $K$  now refers to the number of independent information streams rather than the number of information bits. Since  $K$  is arbitrary, any convolutional code whose rate is an integer multiple of  $1/ML_t$ , can be represented in this form.



The following result establishes necessary and sufficient conditions for achieving a certain level of CCI diversity.

**Proposition 4** *The BPSK space-time binary convolutional code  $C$  will achieve  $d_{cci}$  levels of CCI diversity if and only if  $d_{cci}$  is the largest integer such that  $\tilde{\mathbf{F}}_1(D), \tilde{\mathbf{F}}_2(D), \dots, \tilde{\mathbf{F}}_M(D)$  have the property that  $\forall a_1, a_2, \dots, a_M \in \mathbb{F}, a_1 + a_2 + \dots + a_M = M - d_{cci} + 1$ :*

$$\tilde{\mathcal{F}}(D) = \begin{bmatrix} a_1 \tilde{\mathbf{F}}_1(D) & a_2 \tilde{\mathbf{F}}_2(D) & \dots & a_M \tilde{\mathbf{F}}_M(D) \end{bmatrix}$$

*is of full rank  $K$ .*

*Proof:* Follows the same lines as proposition 1. □

We use these results along with the sufficient conditions for fading diversity developed in [3] to search for good codes. We focus on rate  $1/L_t$  codes<sup>5</sup> because these codes can achieve full fading diversity in quasi-static fading channels [1]. Tables 1 and 2 enlist the rate  $1/L_t$  codes that achieve the best  $d_{cci}, d_{fad}$  and free distance  $d_{free}$  up to a particular  $M$ . These codes were found through an exhaustive search for two and three transmit antennas with bit multiplexing across different transmit antennas. In cases where a single code fails to achieve the best CCI and fading diversity advantages simultaneously, we enlist two codes; one that achieves the best  $d_{fad}, d_{free}$  and another that achieves the best  $d_{cci}, d_{free}$ .

## 5 Numerical Results

Except as noted to the contrary, we employ BPSK modulation in all our simulations. In our simulations, SNR is the signal-to-thermal noise ratio in the CCI free blocks, SINR is the signal-to-thermal plus CCI noise ratio in fading blocks impaired by CCI, and average SNR is the expected value of the signal-to-thermal plus CCI noise ratio.

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<sup>5</sup>With BPSK modulation these codes achieve a rate of one bit/channel use.

Fig. 1 compares the two 16-state codes with generator polynomials  $(36, 64)$  ( $d_{fad} = 4, d_{cci} = 4$ ) and  $(46, 72)$  ( $d_{fad} = 4, d_{cci} = 3$ ) in a system with six fading blocks per code word (i.e.,  $M = 6$ ), two transmit and one receive antenna. The superiority of the  $(36, 64)$  code at high CCI noise powers (i.e. low SINR) and low  $\rho$ 's is due to the higher CCI diversity advantage of this code. Both codes exhibit similar performance at low CCI noise powers, since both codes achieve the same fading diversity advantage. We also note that both codes reach an error ceil at low SINR's due to the fact that, under high SNR and high CCI noise power conditions, the channel approaches an erasure channel (EC) where the performance is completely determined by  $d_{cci}$ . Fig. 2 compares the two codes in an EC where the performance is completely determined by  $d_{cci}$ . Fig. 3 compares the performance of the two codes with a multi-state CCI model. Here  $N_i$  refers to the CCI power in the  $i$ th state and  $\rho_i$  the probability of being in this state. This figure supports our claim that the performance predictions offered by CCI diversity gains are not limited to our simple two-state model and carry over to more realistic models. Fig. 4 compares the performance of Tarokh, Seshadri and Calderbank (TSC) 16-state code [1] ( $d_{fad} = 2, d_{cci} = 2$ ) and the linear  $\mathbb{Z}_4$  code obtained by "lifting" the binary  $(6, 7)$  code with QPSK modulation [3] ( $d_{fad} = 3, d_{cci} = 2$ ). The superior performance of the new code at higher SINR's is due to its higher fading diversity advantage. At high CCI noise powers, both codes achieve similar performance, since both codes have the same CCI diversity advantage.

## 6 Conclusions

In this paper, we considered the design of space-time codes for communication systems operated in the presence of multi-path fading and CCI. We developed the base-band design criteria that determine the CCI diversity, fading diversity, and coding gain achieved by space-time codes operated in these environments. These criteria allow for constructing space-time codes that achieve the optimal tradeoff between spectral efficiency and power efficiency. We showed that the CCI diversity advantage is a metric associated with

the degree of temporal spread of the “fading diversity” advantage. A systematic approach for code design together with representative simulation results for the proposed codes have been presented to validate the effectiveness of the proposed approach.

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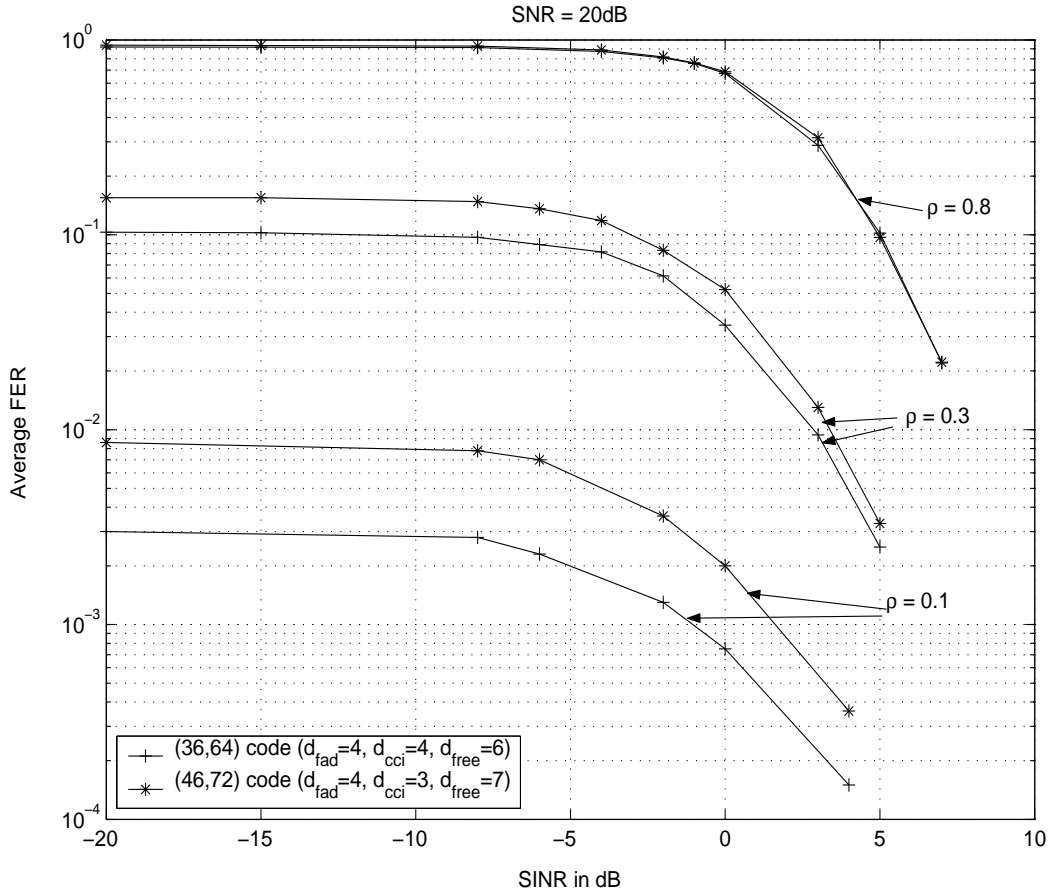


Figure 1: Performance Comparison of 16-state BPSK STC (64, 36) having the best  $d_{cci}$  with (46, 72) code having the best  $d_{free}$ , when  $SNR = 20\text{dB}$ ,  $L_t = 2$ ,  $L_r = 1$  and  $M = 6$ .

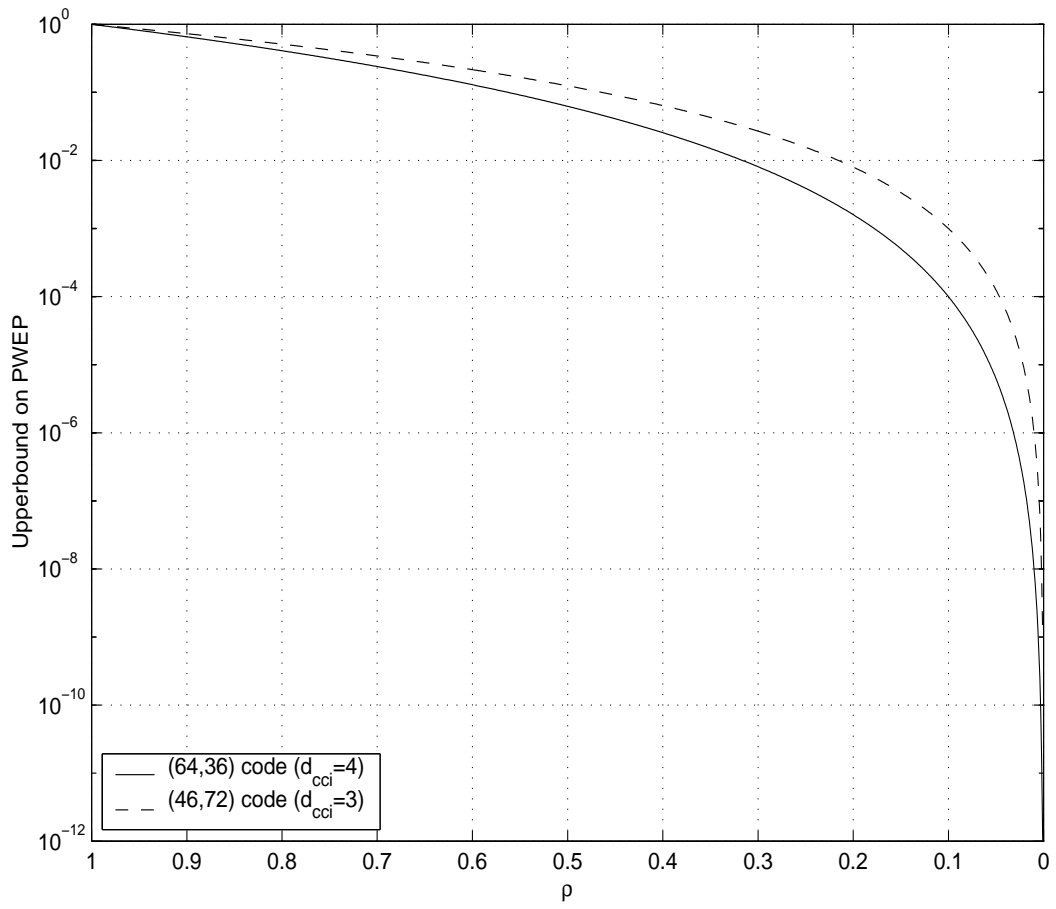


Figure 2: Performance of the (64, 36) and the (46, 72) STCs in an EC with  $L_t = 2$  and  $M = 6$ .

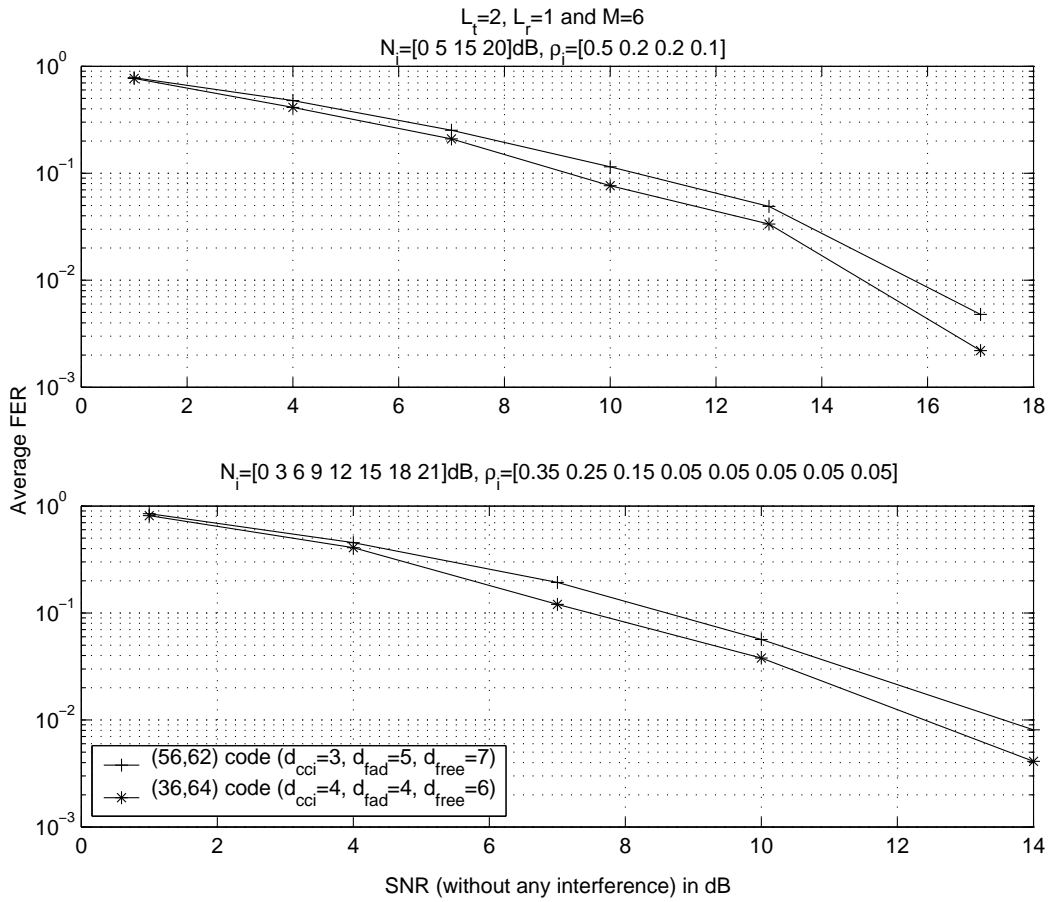


Figure 3: Performance Comparison with a multi-state CCI model with  $L_t = 2, M = 6$  and  $L_r = 1$ .

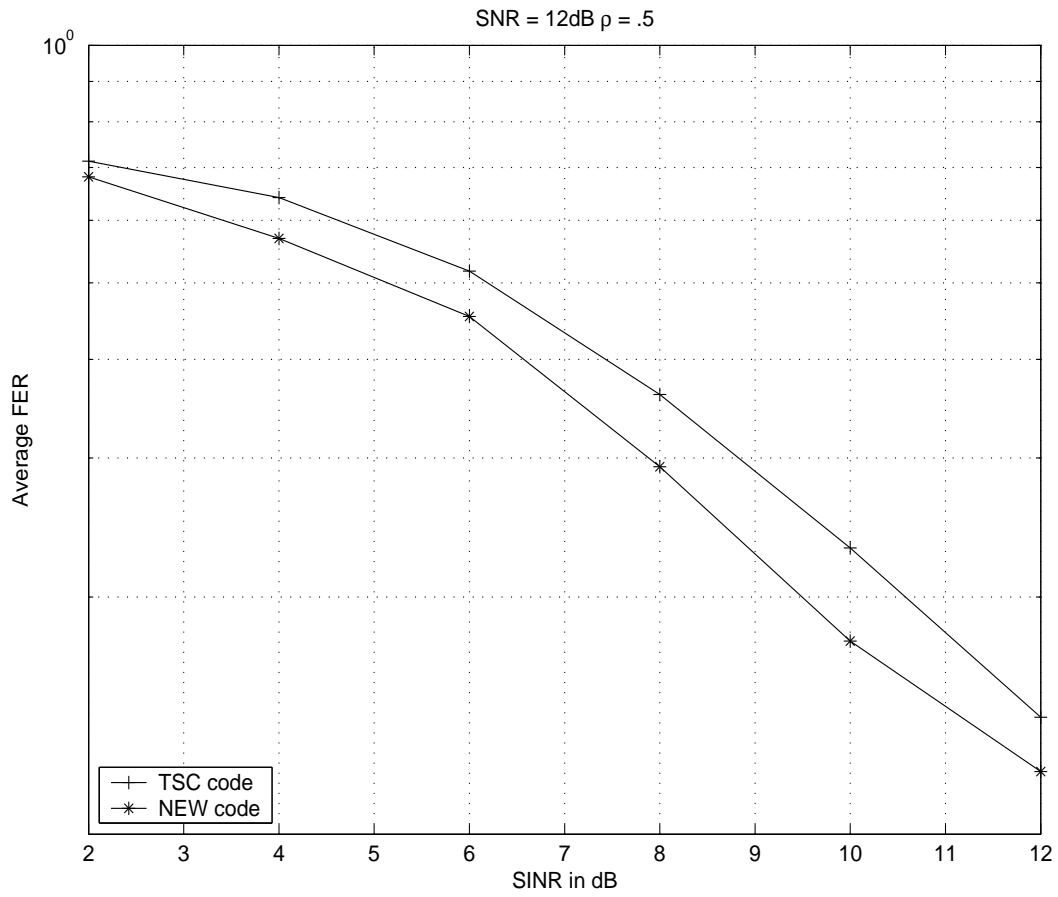


Figure 4: Performance Comparison of the New 16-state QPSK STC  $(1 + D + D^2, 1 + D + 2D^2)$  and the TSC STC when  $SNR = 12\text{dB}$ ,  $L_t = 2$ ,  $M = 4$  and  $L_r = 1$ .

$\nu$	Connection Polynomial	$d_{free}$	$(d_{cci}, d_{fad})$ for $M =$				
			2	3	4	5	6
2	(6,7)	4	(2, 3)	(2, 3)	(3, 3)	(3, 3)	(3, 3)
3	(24,70)	5	(2, 3)	(2, 4)	(3, 4)	(3, 4)	(4, 4)
4	(56,62) <sup>6</sup>	7	(2, 3)	(2, 4)	(3, 4)	(3, 4)	(3, 5)
4	(36,64) <sup>7</sup>	6	(2, 3)	(2, 4)	(3, 4)	(3, 4)	(4, 4)
5	(53,70) <sup>6</sup>	7	(2, 3)	(2, 4)	(3, 5)	(3, 5)	(3, 5)
5	(54,65) <sup>7</sup>	7	(2, 3)	(2, 4)	(3, 4)	(3, 5)	(4, 5)

Table 1: Rate  $\frac{1}{2}$  binary STCs for  $L_t = 2$ , with best  $d_{cci}$ ,  $d_{fad}$  and  $d_{free}$ .

$\nu$	Connection Polynomial	$d_{free}$	$(d_{cci}, d_{fad})$ for $M =$		
			1	2	3
3	(40,54,74)	8	(1, 3)	(2, 4)	(3, 4)
4	(26,62,76)	10	(1, 3)	(2, 5)	(3, 5)
5	(37,62,65)	12	(1, 3)	(2, 5)	(3, 6)

Table 2: Rate  $\frac{1}{3}$  binary STCs for  $L_t = 3$ , with best  $d_{cci}$ ,  $d_{fad}$  and  $d_{free}$ .

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<sup>6</sup>These codes achieve the best  $d_{fad}$  and  $d_{free}$ .

<sup>7</sup>These codes achieve the best  $d_{cci}$  and  $d_{free}$ .