Welcome to lecture 20

Like love, after the DC component fades away, the sinusoidal steady-state response continues on. Today we will move beyond simple infatuation between electrons and consider circuit relationships that have lasted for a long time. Carrying this analogy to its logical conclusion... or perhaps simply going too far, like any healthy relationship, you need phasors.

Euler’s relationship:

\[ e^{j\theta} = \cos \theta + j \sin \theta \]

Cast:

\[ j = \quad \text{why "} j \text{"?} \]
\[ \cos \theta = \]
\[ \sin \theta = \]

Bonus reading, Appendix-C, p. A12-A15
First calculus, then trigonometry and now imaginary numbers... man, this class is like a bad high school reunion!
Act I, they meet

\[ v(t) = V_A \cos(\omega t + \phi), \quad \forall t \]

Phasor representation of the sinusoid \( v(t) \):

\[ V = \]

![Phasor diagram](image-url)
Important points to note about phasors

- Phasors are written in boldface type like \( V \) or \( I \) to distinguish them from signal waveforms such as \( v(t) \) or \( i(t) \)
- A phasor is determined by amplitude and phase angle (get it, phase... phasor). **It does not contain any information about the frequency of the sinusoid!**

  e.g., AC current coming into your home
So you know how to get yourself into phasors, you don’t want to be stuck there forever. How do you get out?

A: just retrace your steps

\[ v(t) = \text{Re}\{V \cdot e^{j\omega t}\} = \]

So what does this mean to me?

\[ v(0) = \text{Re}\{V\} = V_A \cos(\phi) \]
\[ v(t) = \text{Re}\{V \cdot e^{j\omega t}\} = V_A \cos(\omega t + \phi) \]
But wait, there’s more, what if you have a party and all of your friends come over...

\[ v(t) = v_1(t) + v_2(t) + \ldots + v_N(t) \]

The police come and you want to get rid of the crowd quickly... take the derivative... or integrate with the cops.

\[ \frac{dv(t)}{dt} = \frac{d}{dt} \text{Re}\{ \mathbf{V} \cdot e^{j\omega t} \} \quad \int v(x)dx = \int \text{Re}\{ \mathbf{V} \cdot e^{j\omega x} \}dx \]
Construct the phasors for the following signals:

\[ v_1(t) = 10 \cos(1000t - 45) \]
\[ v_2(t) = 5 \sin(1000t - 60) \]
\[ v_3(t) = v_1(t) - v_2(t) \]
Using phasors, integrate the following signal:

\[ v(t) = 15 \cos(200t - 30) \]
Using phasors, find the forced response to the following and compare to the example in the last lecture.

\[ G_N L \frac{di(t)}{dt} + i(t) = I_A \cos \omega t \]