Homework set #7, due 5/17/04
6-4, 6-6, 6-10, 6-24, 6-36, 7-7, 7-13, 7-14, 7-23, 7-28, 7-29

First Order circuits- RC & RL

RC - can you reduce the given circuit to a single resistor and a single capacitor?

\[
\begin{align*}
\text{Resistors and sources} & \quad \begin{array}{c}
i(t) \\
+ \\
\end{array} & \quad \begin{array}{c}
v(t) \\
- \\
\end{array} \\
\quad C & \equiv \\
\text{Capacitor constraint:} \quad \begin{array}{c}
i(t) = C \frac{dv(t)}{dt} \\\n\end{array}
\end{align*}
\]

Choose an equivalent circuit with independent variable \(i(t)\) for everything else => Thevenin

\[
R_T i(t) + v(t) = v_T(t)
\]

Combining constraints:

\[
R_T C \frac{dv(t)}{dt} + v(t) = v_T(t)
\]

Other things to touch on: the state variable, the fact that it is a 1st order LDE w/ constant coefficients, what’s the input and output
RL - can you reduce the given circuit to a single resistor and a single inductor?

Inductor constraint:

\[ v(t) = L \frac{di(t)}{dt} \]

Choose an equivalent circuit with independent variable \( v(t) \) for everything else => Norton

\[ G_N v(t) + i(t) = i_N(t) \]

Combining constraints:

\[ G_N L \frac{di(t)}{dt} + i(t) = i_N(t) \]

Other things to touch on: the state variable, the fact that it is a 1st order LDE w/ constant coefficients, what’s the input and output
Duality...
How do you solve these beasts? Three factors contribute to our solution:

1) The input(s) driving the circuit (e.g., $v_T(t)$)
2) The values of the circuit parameters (e.g., $R_T$ and $C$)
3) The initial stored energy (initial condition) (e.g., $v(0)$)

(1) and (2) apply to any linear circuit, but (3) is new

To address (3) we must find the "zero-input" or "natural" response

$$R_T C \frac{dv(t)}{dt} + v(t) = 0$$

This is a homogeneous equation, so $v(t)$ and its derivative must have the same form. Only one family of functions satisfy this condition:

$$v(t) = Ke^{st}$$
\[ v(t) = V_0 e^{-t/R_TC}, \quad t \geq 0 \]

Can you identify the characteristic equation and time constant? What about repeating the exercise for the RL circuit?
Try this on for size

Given $L=40\text{mH}$ and $I_0=-50\text{mA}$, solve for $v_L(t)$ and $v_1(t)$.
Given $I_o = 100\,mA$, solve for $v(t)$
Given $V_0=3V$, solve for $i(t)$
But what if I can’t find the equivalent circuit???