Measuring Wave Velocities on Highways during Congestion using Cross Spectral Analysis

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Abstract— **Previous research has shown that disturbances in the traffic state (flow, occupancy, and velocity) propagate upstream with nearly constant velocity through congested freeway traffic. The velocities of these backward propagating waves can be estimated by comparing the time series of the state observed at successive detector stations on the same congested freeway. The earlier research examined a relatively small number of disturbances using labor-intensive techniques. In order to characterize the disturbances more comprehensively, this paper develops an automated method to extract their velocity in the presence of noise. Time delay estimation (TDE), a common technique in digital signal processing, was used to solve the problem. Out of the TDE algorithms, cross-spectral analysis was chosen to analyze one month of data from the Berkeley Highway Laboratory (BHL) and the results are presented.**

I. Introduction

There are backward moving slow-and-go waves propagating through the traffic with flow, density and vehicle velocities when traffic is congested. The dynamic traffic evolution model, LWR model ([6], [7]), can be used to predict the existence of waves and how fast waves propagate given flow and density relationship on a homogeneous highway (q-k curve). Thus, the shape of the q-k curve is important for modeling and forecasting. Out of convenience, reference [8] chose to simplify the relationships through the use of a triangular q-k curve. Subsequent empirical analysis suggests that this simplification may be representative of typical q-k curves [1-4]. Under the prediction of LWR model and the simplification of triangular curve, backward waves should propagate with constant velocities.

Empirical research has begun to verify the existence and velocities of backward moving waves. Reference [1] showed that the backward moving waves could be found by comparing the cumulative curves of vehicle counts from two neighboring loop detector stations. They used data from a 2 km segment of northbound Interstate 880, just east of San Francisco, California, and found that the wave velocities during congestion periods were nearly constant (about 17-20 km/hr) over the homogeneous freeway segment, independent of the flow. They concluded that the results implied that "traffic on homogeneous freeway segments might better be modeled using linear (triangular-shaped) relations." Reference [2] observed traffic data over multiple days from 10 km stretch of the Queen Elizabeth Way (Ontario, Canada) and found that the waves propagating upstream in congestion had nearly constant velocities about 22-24 km/hr, independent of the location and the flow. They measured the wave velocities by plotting the cumulative curves of vehicle counts across all the lanes from several successive detector stations with vertical displacements in proportion to the physical distance separating the successive detector stations. Deviating from the earlier techniques of manually extracting the wave speeds, reference [3] applied cross correlation to 2-second vehicle counts (N-curve) across two lanes for a single sample period and found the wave velocity about 19.4 km/hr.

These wave velocities can be used as inputs to estimate link travel times [4]. Furthermore, they could verify the shape of the flow and density relationship, e.g., if traffic is best modeled using a triangular-shaped q-k curve instead of non-linear (parabolic-like) relations [1]. But none of the earlier studies examined a large number of disturbances. To provide a more comprehensive analysis of wave velocities, this research seeks to extract the velocities automatically.

II. Automated Data Extraction

Our first attempt extract wave velocities automatically is

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to simply replicate the cross correlation method on one month (August, 2003) of 2-second N-curves from seven successive detector stations in the westbound of the Berkeley Highway Laboratory (BHL) [10] along Interstate-80, north of Oakland, CA (Fig. 1). There are 5 lanes in each direction on this segment of freeway; the innermost lane is reserved for HOVs (High occupancy vehicles). Because these lanes generally exhibit relatively little congestion compared to the general flow lanes, they were excluded from the analysis of backward moving waves. Detector stations are spaced about 1/2 km apart from each other and at each detector station there is a dual-loop detector in each lane.

The event data from each detector are recorded at 60Hz and then aggregated to 2-second average velocity and flow measurements. Because this study examines backward moving waves that arise in the congested traffic, a preprocessing step is used to find these periods. Specifically, the criterion to define a "congestion period" is that for the given lane, the 5-minute aggregated velocity should be slower than 48 km/hr (30 mi/hr) for at least one hour for both the upstream station and the downstream station on a link. Fig. 2 shows the cumulative distribution (cdf) of wave velocities extracted via cross correlation applied to the 2-second N-curves on an individual lane basis. Lane 5 has few samples under our definition of congestion, so for clarity, the results for lane 5 are not shown. Over all links and in all lanes, the cross correlation method yielded mixed results. Many of the observations are consistent with the earlier research, but many more of the measurements are infeasible.

In an attempt to improve the performance of the automated data extraction, we used time delay estimation (TDE) from signal processing. More specifically, we employed cross-spectral analysis to estimate the wave velocities, as discussed in the next section.

III. Time Delay Estimation Model

Under LWR, the waves determine how the local velocity measurements change. Assume the time series of velocity is the unknown transmitted signal, thus the measurements of local velocities at two successive detector stations are the two observations of the unknown signal. The travel time of the waves could be regarded as the time delay between two observations of a random signal at two locations. The two observations can be represented in the mathematical model as below:

$$\begin{aligned} r_1(t) &= s(t) + n_1(t) \\ r_2(t) &= \alpha s(t-\tau_0) + n_2(t) \end{aligned} \tag{1}$$

Where $s(t)$ is the unknown transmitted signal, i.e. the time series of velocity as predicted by LWR model, $r_1(t)$ and $r_2(t)$ are the signals received or measured at two spatially separated receivers, $n_1(t)$ and $n_2(t)$ are the corrupting noise, α is unknown channel attenuation coefficient, and τ_0 is the time difference of arrival between $r_1(t)$ and $r_2(t)$ [9].

In our problem, we set $r_1(t)$ as the average velocity measured at a downstream detector station, and $r_2(t)$ as the corresponding velocity time series measured at an upstream detector station. Given a triangular q-k curve, and thus, constant wave velocities in congestion, τ_0 is the travel time of backward moving waves from a downstream station to an upstream station. Wave velocities are simply the quotient of the distance between two consecutive detector stations and the travel time τ_0.

There are many TDE algorithms in the literature. Generally, classic TDE algorithms could be separated into two groups: adaptive and nonadaptive. Among the nonadaptive algorithms, there are algorithms derived in time domain, such as cross-correlation and algorithms derived in frequency domain, such as the one used in this paper, cross-spectral analysis. [9]

IV. Algorithm

Let the upstream velocity series be $y(t)$ and downstream velocity series be $x(t)$, according to the TDE model specified in (1),

$$y(t) = \alpha x(t-\tau_0) + n(t) \tag{2}$$

Let $R_{xy}(\tau)$ represent the cross correlation of $x(t)$ and $y(t)$, and $R_{xx}(\tau)$ be the auto-correlation of $x(t)$. From equation (2) and the definitions of $R_{xy}(\tau)$ and $R_{xx}(\tau)$, we have:

$$\begin{aligned} R_{xy}(\tau) &= E[x(t)y(t+\tau)] \\ &= E[x(t)\{\alpha x(t+\tau-\tau_0) + n(t+\tau)\}] \\ &= \alpha E[x(t)x(t+\tau-\tau_0)] \\ &= \alpha R_{xx}(\tau-\tau_0) \end{aligned} \tag{3}$$

The Fourier transform of $R_{xy}(\tau)$ is the two-sided cross-spectral density function $S_{xy}(f)$, and the Fourier transform of $R_{xx}(\tau)$ is the two-sided auto spectral density function $S_{xx}(f)$, from equation (3) and the definition of $S_{xy}(f)$ and $S_{xx}(f)$, the relationship of $S_{xy}(f)$ and $S_{xx}(f)$ are derived as below:

$$
\begin{aligned}
S_{xy}(f) &= \int_{-\infty}^{+\infty} R_{xy}(\tau) e^{-j2\pi f\tau} d\tau \\
&= \int_{-\infty}^{+\infty} \alpha R_{xx}(\tau - \tau_0) e^{-j2\pi f\tau} d\tau \\
&= \alpha e^{-j2\pi f\tau_0} \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau \\
&= \alpha e^{-j2\pi f\tau_0} S_{xx}(f)
\end{aligned} \tag{4}
$$

Where $-\infty < f < +\infty$

Let the corresponding one-sided cross-spectral density function be $G_{xy}(f)$ and auto-spectral density function be $G_{xx}(f)$, from equation (4) and the definition of $G_{xy}(f)$ and $G_{xx}(f)$, the relationship between $G_{xx}(f)$ and $G_{xy}(f)$ are derived as below:

$$G_{xy}(f) = \alpha G_{xx}(f) e^{-j2\pi f\tau_0} \tag{5}$$

Where $0 < f < +\infty$

From equation (5), the magnitude and phase angle of $G_{xy}(f)$ are respectively,

$$
\begin{aligned}
\left|G_{xy}(f)\right| &= \alpha G_{xx}(f) \\
\theta_{xy}(f) &= 2\pi f\tau_0
\end{aligned} \tag{6}
$$

The time delay τ_0 appears only in the phase angle function $\theta_{xy}(f)$ and $\theta_{xy}(f)$ is a linear function of f with a slope equal to $2\pi\tau_0$. Measurement of $\theta_{xy}(f)$ enables us to determine the time delay. [11]

To illustrate this theory, consider one sample that is two 68-minute-long velocity series $x(t)$ and $y(t)$ from each of the two stations when both stations are congested. The discrete time series $x(t)$ and $y(t)$ we used are 2048 points long since the sampling period is 2 seconds. After subtracting the best-fitted line (in least-square sense) from each of the time series, cross-spectral density estimates of $x(t)$ and $y(t)$ are computed by Fast Fourier Transforms (FFT). The raw cross-spectral density estimates are smoothed by band averaging and ensemble averaging to obtain the final estimate of $\hat{G}_{xy}(f_k)$. We choose those phase angles whose coherences are higher than the critical value at 0.1 significance level. After unwrapping the phase angles $\theta_{xy}(f_k)$ of $\hat{G}_{xy}(f_k)$, i.e., shift any absolute jumps greater than π to their 2π complement; linear regression gives the slope of $\theta_{xy}(f_k)$ and thus, the time delay τ_0. Fig. 3 illustrates the squared coherency spectrum and critical value at 0.1 significance level of $\hat{G}_{xy}(f_k)$ from one sample. Fig. 4 shows the corresponding phase angle estimates for those frequency components with coherence above the critical value. Finally, the corresponding unwrapped phase angles from Fig. 4 are used to fit a line and find its slope.

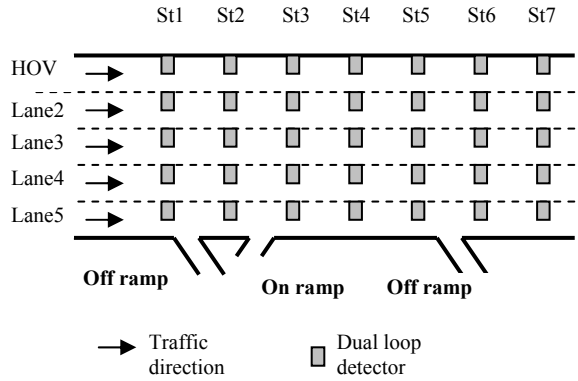

Fig. 1: The segment of Interstate-80 in Berkeley, CA, in this study.

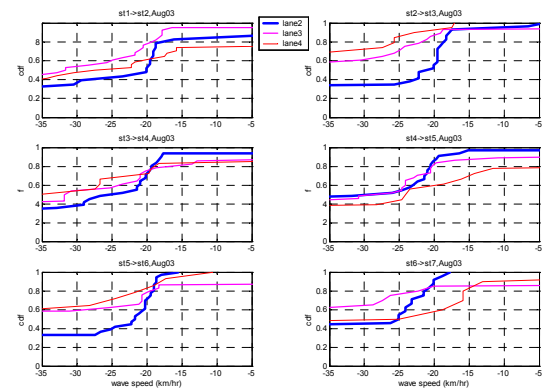

Fig. 2: CDF's of wave velocities from 6 station pairs along BHL westbound using cross-correlation.

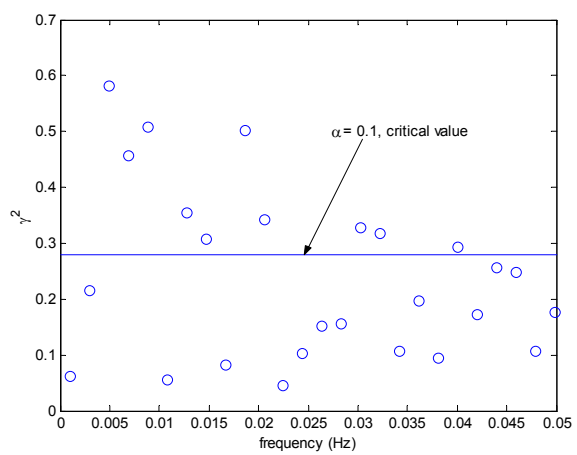

Fig. 3: Squared coherency spectrum and critical value at 0.1 significance level.

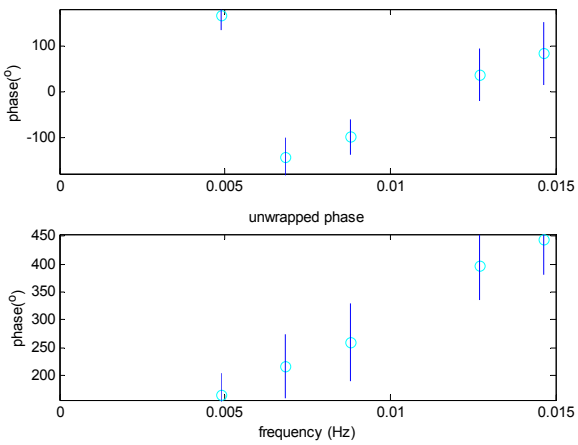

Fig. 4: Phase spectrum and corresponding unwrapped phase angles whose coherences are greater than the critical value at 0.1 significance level.

V. Results

Applying the above algorithm to the same month of data used in Section II, but to 2-second aggregated velocity measurements instead of N-curves, wave velocities for each one-hour sample during congestion in each lane over each successive station pair were calculated. The majority of the wave velocities are between –15 km/hr and –25 km/hr, consistent with [1-3]. More importantly, the percentage of wave velocities that fall between –15 km/hr and –25 km/hr using this algorithm are higher than those applying cross-correlation in Section II for all of the links (including lane 5 which is not shown). Fig. 5 shows the cumulative distribution of sample wave velocities on the links using our algorithm. The distributions in fig. 5 are tighter than those in fig. 2. After rescaling the CDF to show only those samples falling between –15 km/hr and –25 km/hr, fig. 6 shows their cumulative distributions.

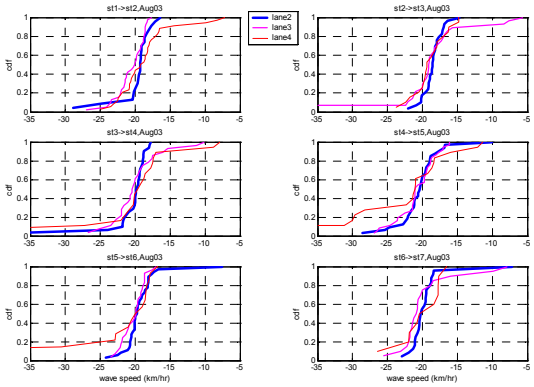

Fig. 5: CDF's of wave velocities from 6 station pairs along BHL westbound using our algorithm.

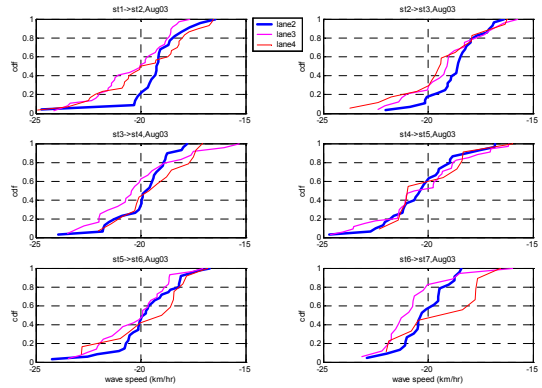

Fig. 6: CDF's of wave velocities between –15km/hr and –25km/hr.

VI. Conclusions

In this paper, we applied cross spectral analysis to estimate the time delay between local velocities measured at successive detector stations to calculate the velocity of waves propagating through the traffic stream. Our algorithm is more robust than earlier methods in terms of suppressing noise, and it can be used to extract wave velocities over a large amount of data. These wave velocities can further verify traffic flow theory and improve traffic measurements, e.g., [4].

Our results over one-month of data from the westbound lanes of the BHL show that wave velocities are nearly constant, which is consistent with earlier work. However, in the outside lane, waves disappear on one link and reappear on the next, a finding that is consistent with [12]. The reason why waves behave differently in these lanes is the subject of on-going research.

VII. References

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