The Effect of Lane Change Maneuvers on a Simplified Car-following Theory

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***Abstract*—This paper investigates the linearity of empirically observed spacing-speed relation for various drivers in the context of car-following theory and how lane change maneuvers perturb the relation. It is shown that the impacts of lane change maneuvers are not balanced, the response time to an exiting vehicle is much longer than the response time to an entering vehicle. This accommodation imbalance will propagate upstream and as discussed herein, it appears to be a source of speed and flow fluctuations (or oscillations) within a queue.**

The present work is motivated by Newell's simplified car-following theory, namely that during congested periods the trajectory of a given vehicle is essentially identical to the preceding vehicle's trajectory except for a translation in space and time. One of the basic assumptions in Newell's presentation is that spacing and speed are linearly related. While other researchers have found macroscopic evidence supporting Newell's theory, they have also found that it fails in the presence of frequent lane change maneuvers. This paper takes a microscopic approach, employing vehicle trajectory data. This paper provides support for Newell's assumed linear relation between spacing and speed over a large range of speeds when vehicles are not impacted by lane change maneuvers. It also offers a possible explanation for the degraded performance of Newell's theory in the presence of heavy lane change maneuvers. Although the focus is on Newell's simplified car-following theory, the empirical results of this study have similar implications for many other car-following theories as well.

***Index Terms*—Car Following, Traffic Flow Theory, Road Transportation, Lane Change Maneuvers**

I. Introduction

A recent paper by Newell [1] proposed a car-following theory based on a very simple rule: namely that during congested periods on a homogeneous highway, the time-space trajectory of a given vehicle is essentially identical to the preceding vehicle's trajectory except for a translation in space and time. Based on this theory, a driver follows the preceding vehicle by choosing the same speed and an appropriate spacing. This logic differs from preceding car-following theories[1] such as stimulus-response models (e.g., [3]-[4]) where drivers observe stimuli and react to these stimuli after accounting for a finite reaction time; safe distance models (e.g., [5]) where drivers choose speed in such a way that the vehicle can be stopped safely if the preceding vehicle acted unpredictably; and psychophysical models (e.g., [6]) where drivers react based on whether they can perceive any change in relative speed with the preceding vehicle. Compared to these preceding car-following models, Newell's model requires fewer parameters and is easier to implement. Newell has shown in his paper that this simplified model could be interpreted as a special case of existing models such as Chandler et al [3] and is consistent with the Lighthill Whitham and Richards [7]-[8] theory if a triangular flow-density relation is used. Separately, others have found empirical evidence to support a triangular relationship, e.g., [9]-[10].

The theory has drawn the attention of several researchers, e.g., [11]-[14]. Although [11] showed that Newell's model cannot predict the detailed microscopic behavior of following cars as accurately as more elaborate models, the simplicity of the model and ability to replicate macroscopic phenomena make Newell's model appealing. Empirical support for Newell's simplified car-following theory from a macroscopic point of view can be found in [12]-[13], but [12] also found that Newell's theory breaks down in the presence of frequent Lane Change Maneuvers, which leads to the premise of this paper, to investigate how lane change maneuvers impact the relation between spacing and speed for car-following models in general and to investigate Newell's basic assumption that spacing and speed are linearly related.

Using vehicle trajectory data we are able to study the relation between spacing and speed directly. The present study reveals clearly how lane change maneuvers affected this linear relation in different cases. Aside from the disturbances arising from lane change maneuvers, the assumption of the linear relation between spacing and speed appears to be reasonable over a large range of traffic speed for the vehicles on the study segment.

Meanwhile, there have been numerous studies into lane change maneuvers, both from the perspective of the driver (e.g., [15]-[16]) and from the perspective of the traffic flow

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[1] A more in depth review can be found in [2].

(e.g., [17]-[20]). At the crux of our research is the time it takes following drivers to accommodate to a lane change maneuver. Such reaction or relaxation times have been explicitly incorporated in car-following models (e.g., [21]-[22]) and models of lane change maneuvers (e.g., [20], [23]). The present research empirically measures the accommodation times and investigates the impacts.

This paper is organized as follows. First, Newell's car-following theory and related studies are reviewed in Section II. Next, the study site and the trajectory data used in this paper are presented in Section III. Then the effect of lane change maneuvers on the linear spacing-speed relation is presented in Section IV. The trajectory data are then used in Section V to investigate the linearity of the spacing-speed relation. Finally, in Section VI the paper closes with a summary and conclusions.

II. Background

Following Newell [1], suppose that on a homogeneous freeway segment, vehicle i is following vehicle $i-1$ and vehicle $i+1$ is following vehicle i, as shown in Fig. 1(a). Vehicle $i-1$ changes speed from v to v' and its trajectory can be approximated as being piecewise linear. Based on Newell's theory, during congested periods the trajectory of vehicle i replicates that of vehicle $i-1$ with space shift d_i and time shift τ_i.

Newell began his development of the simplified car-following theory from the assumed positive linear relation between spacing s and vehicle speed v, which is defined to be the distance between the front ends of two consecutive vehicles. Consistent with empirical data and earlier car-following theories, drivers tend to choose a larger spacing when vehicle speed is increased. However, when vehicle $i-1$ travels at a speed which is higher than the preferred speed, $\hat{V}_i$, of driver i, driver i will maintain $\hat{V}_i$, cease car-following, and the two vehicles will separate. The complete relation is represented by the piecewise linear curve in Fig. 1(b). For vehicle i the slope of this curve in the car-following region is μ_i and the intercept is s_i^0.

Based on this linear relation, if driver i travels at speed v ($v < \hat{V}_i$), they will choose spacing $s_i = s_i^0 + v\mu_i$ and for speed v' ($v' < \hat{V}_i$), choose spacing $s_i' = s_i^0 + v'\mu_i$. However from Fig. 1(a), it is evident that

$$s_i = d_i + v\tau_i \tag{1.a}$$

$$s_i' = d_i + v'\tau_i \tag{1.b}$$

Comparing these equations yields $d_i = s_i^0$ and $\tau_i = \mu_i$ if the linear relation holds. As a result, the wave velocity between vehicle $i-1$ and vehicle i, $v_w^{i-1,i}$, is

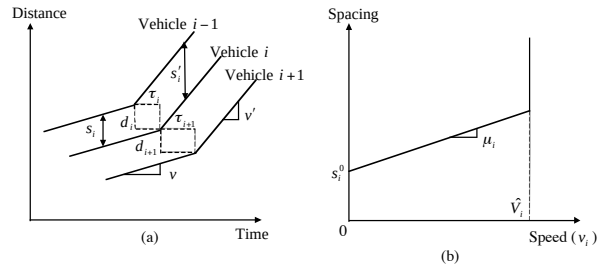

Fig. 1. (a) Piecewise linear approximation to vehicle trajectories; (b) Relation between spacing and speed for an individual vehicle (adapted from [1]).

$$v_w^{i-1,i} = \frac{d_i}{\tau_i} = \frac{s_i^0}{\mu_i} \tag{2}$$

In other words, (2) implies that the wave velocity is only determined by the parameters used to describe the linear relation between spacing and speed. A given driver might tend to maintain such a preferred relation over their trip and the parameters (s_i^0, μ_i, $\hat{V}_i$) are likely to be different for different drivers. Newell conjectured that [s_i^0, μ_i] will vary as if sampled independently from some joint probability distribution. So, the wave propagates like a random walk with increment of [s_i^0, μ_i], which depends on the driving behavior of driver i and is independent of vehicle speed.

The linear relation between spacing and speed is a basic assumption in Newell's theory and it leads both to a linear car-following model and to a linear relation between flow, q, and density, k. However, if this linear relation does not hold, from (1), the wave velocity between vehicle $i-1$ and vehicle i becomes,

$$v_w^{i-1,i} = \frac{d_i}{\tau_i} = \frac{s_i' v - s_i v'}{s_i - s_i'} \tag{3}$$

Where $s_i = f(v)$ and $f(\bullet)$ is a non-linear function. So, $v_w^{i-1,i} = g(v, v', f(\bullet))$, that is the wave velocity depends on v, v' and the driving behaviors, which is what a non-linear flow density curve would tell us.

The exact shape and nature of the spacing-speed relations has been the subject of many studies and debate continues as to whether the curves are linear or not (see, e.g., [2]), some studies have found a linear relation between spacing and speed, e.g., [24], but others found a non-linear relation, e.g., [25]. After reviewing arguments for a non-linear relation in his paper, Newell concluded that there seemed to be little evidence of the non-linear effects. Two subsequent papers have validated Newell's theory (with the linear assumption) under certain conditions by studying wave velocities through the traffic stream. Mauch and Cassidy [12] measured wave trip time on a freeway segment (T) and the number of vehicles through which the wave propagated (N). Based on Newell's theory, $[T, N]$ can be described by a bivariate normal distribution. Two types of segments, with frequent and

infrequent lane changing maneuvers, were studied both in moderately dense queues and very dense incident-induced queues. Independent of the traffic state, Mauch and Cassidy found the results for all segments marked by infrequent lane changing maneuvers show that $[T, N]$ on different segments were drawn from a common bivariate normal distribution. On segments marked by frequent lane change maneuvers, the results for the moderately dense queues fail to support Newell's theory but the results during a very dense, incident-induced queue remained consistent with Newell's theory. Mauch and Cassidy explained the former observation with the oscillations induced by lane changing maneuvers and left detailed explanation of the later observation for future research.

Ahn et al. [13] measured the total time (T) and distance (D) covered by a wave arising from long queues discharging at signalized intersections. Vehicle trajectories were constructed but they were only studied in an aggregate manner. The location where traffic reached specific speeds was mapped from one trajectory to the next to reveal the propagation of waves. Four waves were studied, which traced vehicle speed of 0 km/h, 6.5 km/h, 13 km/h and 19.5 km/h. The paper showed that $[s_i^0, \mu_i]$ were independent across drivers and $[T, D]$ for all four observed waves came from the same bivariate normal distribution, which can be characterized by $[s_i^0, \mu_i]$. So, $[s_i^0, \mu_i]$ varied as if they were sampled independently from some joint probability distribution, as was conjectured by Newell [1].

Both papers provided empirical support for Newell's simplified car-following theory under certain conditions by confirming that $[s_i^0, \mu_i]$ for different drivers are independent samples from one common distribution. However, the effect of lane change maneuvers on Newell's simplified car-following theory was not discussed in detail in either paper. Mauch and Cassidy [12] applied Newell's theory on freeway segments with frequent lane change maneuvers, but they did not study the details as to why Newell's theory was valid for some segments but invalid for others. Lane change maneuvers were not included in Ahn's study [13], all cycles in which lane changing was observed were excluded from consideration. More recently, two works have found evidence that lane change maneuvers could at least partially explain speed and flow fluctuations within a queue [19]-[20].

This present study starts from the microscopic point of view and utilizes vehicle trajectory data to plot the spacing-speed diagram. Representative examples are shown to illustrate the effect of lane change maneuvers on the relation between spacing and speed for four different types of vehicles, namely, entering vehicles (vehicles changing lanes to the study lane from another lane), exiting vehicles (vehicles changing lanes from the study lane to another lane), vehicles following entering vehicles in the study lane, and vehicles following exiting vehicles in the study lane. The effect of lane change maneuvers is then quantified across a large number of vehicles. Newell's assumption of linear relation between spacing and speed is also investigated via the correlations between spacing and speed.

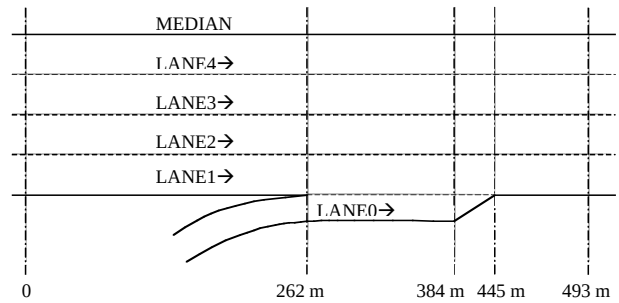

Fig. 2. Schematic of I-405 section in Los Angeles, not to scale (adapted from [26]).

III. Data Description

The present study employs a vehicle trajectory data set collected by Turner-Fairbank Highway Research Center (TFHRC) in June 1983 from I-405 southbound at Santa Monica Blvd., Los Angeles, CA [26]-[27]. A schematic diagram of the site is shown in Fig. 2. The site was filmed from a circling aircraft flying at a slow speed. Data were reduced at one frame per second for one hour of the film. Each record contains the information of a vehicle at an instant, which includes vehicle ID, type code, length, speed, lateral and longitudinal position, color code and lane number. Traffic was free flow for the first several minutes, after which a surge in ramp volume caused the average speed in lane 1 to drop to about 30 km/h and this state continued for the remainder of the data set. This queuing in lane 1 extend upstream of the section approximately 30 minutes into the film. No incidents affected flow in the section. The second half hour data in lane 1 was selected for our study since it is focused on the car-following phenomena in congested traffic conditions. Because of the ramp merging activity in the segment after 262 m it is common to observe multiple successive lane changing maneuvers, making it difficult to isolate the impacts of any single lane change maneuver from other lane change maneuvers. Furthermore, the merging activity may differ from normal discretionary lane change maneuvers, as the vehicles entering from the on-ramp must exit the auxiliary lane before passing 445 m and these vehicles will generally be accelerating as they merge. To avoid these complications, only the segment between 0 to 244 m (800 feet) is studied. During the second half hour, 796 vehicles entered this 244 m long segment in lane 1, among which 118 vehicles left lane 1 to lane 2 (15 percent) and 31 vehicles entered lane 1 from lane 2 (4 percent).

Such trajectory data are uncommon due to the difficulty both in collecting unobstructed views of a non-trivial length of freeway, and more importantly the extensive labor needed to reduce the data with sufficient precision. Until recently the TFHRC data sets were the only publicly available vehicle trajectory data sets containing a large number of vehicles. Since this research was conducted, the Next Generation SIMulation (NGSIM) data have become available [28], providing several additional hours of such data and we will

employ NGSIM data in future studies. Instrumented vehicles offer another approach to collecting vehicle trajectory data, incorporating positioning (e.g., GPS) and ranging (e.g., LIDAR) to measure conditions around a probe (e.g., [29]-[30]).

The overall positional error of TFHRC data is estimated to be within 1.5m in the longitudinal direction and 1m in the lateral direction [27]. However, the precision of the longitudinal location is subject primarily to the distance from the control point pairs: the farther the distance is, the lower the precision. This error is systematic, depending on position within the field of view, so neighboring locations will have similar error. Since spacing is calculated as the difference in longitudinal position of two adjacent vehicles and they would have similar distance to the control points, the error in spacing is subject primarily to errors by the operator in placing the cross hairs of the cursor over the true position of the vehicle in the digitization process, which are typically 0.6m or less [27]. Based on error propagation theory [31], the estimated error in spacing is 0.9m and in speed is 3 km/h. Although not presented here, we have explicitly examined the reasonableness and consistency of TFHRC data. The precision level is comparable to the recently collected NGSIM data set, which has an accuracy of 1.2m in the longitudinal direction and 0.6m in the lateral direction [28].

IV. The Effect of Lane Change Maneuvers on The Linear Spacing-Speed Relation

Since the location of each vehicle at each second is known from the trajectory data, it is trivial to calculate the spacing and speed for each vehicle and plot the relation as shown in Fig. 1(b). Such trajectory plots are studied in this section to find the impacts of lane change maneuvers. From the discrete nature of the maneuver, a vehicle abruptly enters or leaves a lane, one would expect that lane change maneuvers would disturb the spacing-speed relation and cause noise in the spacing-speed plots.

A. Spacing-Speed Relation for Vehicles Following an Exiting or Entering Vehicle

Naturally a given driver responds to vehicles ahead of them (and thus, downstream traffic conditions) when car-following. During a lane change maneuver these stimuli include the new and old lead vehicles, sudden changes in spacing due to the maneuver, as well as the overall evolution of the traffic state in the absence of the lane change maneuver. If traffic is decelerating prior to a lane change maneuver, it will be easier for a following vehicle to close a gap (by decelerating less or even accelerating), but by the same token, it becomes that much harder to open a gap under these conditions. When traffic is accelerating the reverse is true, it becomes harder to close a gap but easier to open one. In any event, closing or opening a gap will cause the driver to deviate from their normal spacing-speed relation.

Suppose that vehicle i in Fig. 1(a) leaves the lane, vehicle $i+1$ would then be a vehicle following an exiting vehicle. Upon departure of vehicle i, vehicle $i+1$ will abruptly have a larger spacing and it may take several seconds for this driver to adjust to their desired spacing for the given speed. In this accommodation period the observed spacing and speed might deviate far from the driver's preferred relation and the period should be treated separately from normal car-following. An example based on vehicle 7922 (TFHRC ID number) is shown in Fig. 3. The trajectory plot of Fig. 3(b) shows that vehicle 7922 was following vehicle 7917 at the beginning of the segment and at time 3260 sec, vehicle 7917 left lane 1. The 6 observations denoted with triangles in the spacing-speed plot (Fig. 3(a)) were observed during the 6 sec immediately after the departure of vehicle 7917. They are defined as *affected points* since these observations are affected by the exiting vehicle and reflect a transient deviation from the normal relation between spacing and speed observed for this vehicle away from the lane change maneuver.[2] Fig. 3(c-d) show the time series of vehicle speed and headway, respectively, where headway is defined as the time needed for the study vehicle to travel from its location at time t to the location where its preceding vehicle is located at time t.

The evolution of the spacing-speed curve in Fig. 3(a) shows clearly that after vehicle 7917 left lane 1, vehicle 7922 increased its speed relative to the new leading vehicle to take advantage of the large spacing left by vehicle 7917 and after 6 sec, it appears to return to the driver's original spacing-speed relation. So, Fig. 3(a) provides evidence that drivers remember their driving preference after the interruption from lane change maneuvers, similar to what was found on a macroscopic scale in [32][3]. From Fig. 3(a), it is clear that prior to vehicle 7917 exiting the lane, vehicle 7922 was slowing in response to traffic conditions. Fig. 3(c) shows this deceleration was reversed for a few seconds immediately after the lane change maneuver while the driver started closing the resulting large headway evident in Fig. 3(d). But vehicle 7922 resumed the deceleration before fully returning to the same headway held prior to the lane change maneuver. This vehicle then accelerated for the last 10 sec it was in the study segment. So the data points in Fig. 3(a) are observed from both a deceleration process and an acceleration process, with speed ranging between 10 and 30 km/h. Except for the affected points, to the eye the data in Fig. 3(a) seem to fit in one linear relation.

The effect of lane change maneuvers can be quantified by Lane Change Accommodation Time (LCAT), which is defined as the time period in which a vehicle deviates from its preferred linear spacing-speed relation (e.g., as in Fig. 3(a)) due to the disturbance of lane change maneuvers. The value of LCAT is equal to the number of affected points. In the abovementioned example, the LCAT for vehicle 7922 is 6 sec. The longer the LCAT is, the longer time that the lane change

[2] Note that the affected points are defined based strictly on deviation from the trend arising from the points away from the lane change maneuver. The number of points and duration varies from vehicle to vehicle and lane change maneuver to lane change maneuver.

[3] In fact this driver memory is evident in all four types of drivers (entering or exiting, making the maneuver or following the maneuver), e.g., Figs. 4, 5 and 6 all show similar evidence.

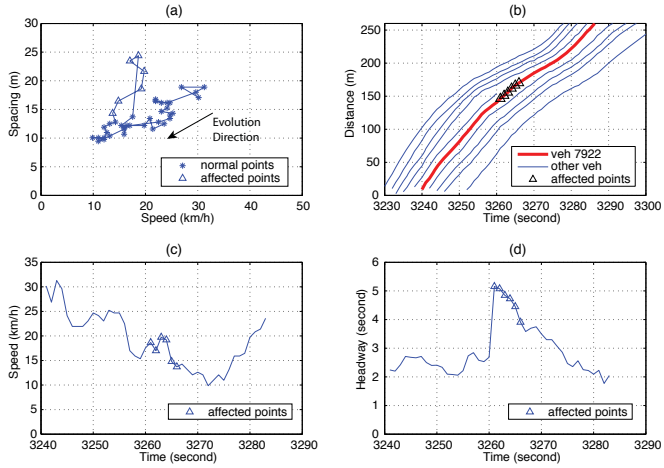

Fig. 3. a) Spacing-speed plot of vehicle 7922 (following exiting vehicle 7917), (b) The corresponding vehicle trajectories, (c) Time series of speed of vehicle 7922, and (d) Time series of headway of vehicle 7922.

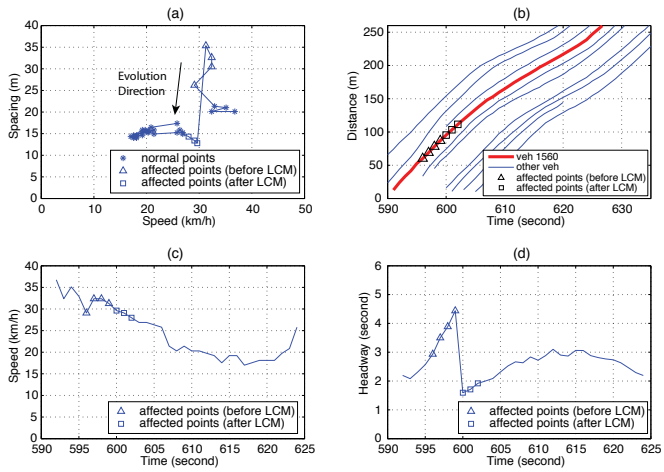

Fig. 4. (a) Spacing-speed plot of vehicle 1560 (following entering vehicle 1557), (b) The corresponding vehicle trajectories, (c) Time series of speed of vehicle 1560, and (d) Time series of headway of vehicle 1560.

maneuver will disrupt the assumptions of Newell's theory. During the LCAT the linear relation between spacing and speed does not hold. The Pearson correlation coefficient can be used to measure the strength and direction of the linear relation between spacing and speed. The larger the coefficient is, the stronger the two variables are linearly related [33]-[34]. For this example the Pearson correlation coefficient between spacing and speed is 0.87 when excluding the affected points but decreases to 0.54 when including the affected points.

This analysis can be repeated for the case that vehicle i in Fig. 1(a) enters the lane, i.e., vehicle $i+1$ follows an entering vehicle. Such an example employing vehicle 1560 is shown in Fig. 4. Vehicle 1557 entered lane 1 before vehicle 1560 at time 600 sec. Vehicle 1560 decelerated to create a large spacing 4 sec before this lane change maneuver was recorded in the data set, as shown with the triangles in Fig. 4, presumably in response to vehicle 1557 either signaling for the maneuver or beginning to enter lane 1 without signaling. Vehicle 1560 continued adjusting its spacing and speed and finally returned to its original spacing-speed relation 3 sec after vehicle 1557 changed lanes (denoted with squares in Fig. 4). The triangles and squares in Fig. 4 are also defined as affected points and LCAT is 7 sec in this example. After removing the 7 affected points, the Pearson correlation coefficient between spacing and speed is increased from 0.61 to 0.88.

This study was repeated for all of the other lane change maneuvers within the study segment and time window. To identify the affected points, a large jump in spacing was sought coincident to the given lane-change maneuver. For vehicles following an exiting vehicle the instant of the largest positive deviation is taken as the first affected point, e.g., as evidenced by the positive jump in time headway at 3261 sec in Fig. 3(d). Statistics are not generated for a vehicle that encountered two or more lane change maneuvers to avoid the possibility that the observed response to one lane change maneuver is unknowingly influenced by another. For those vehicles impacted by a single lane change maneuver, the first data point after the set of affected points is taken as a candidate and evaluated. The linearity of all non-affected points in the spacing-speed plane (e.g., Fig. 3(a)) is subjectively judged both with and without this candidate point.[4] If the linear relation is stronger without the candidate point, the point is labeled affected and then the next point is taken as the candidate and evaluated. The process continues until the inclusion of the candidate data point does not degrade the linear relation, at which point the candidate is excluded from the set of affected points and the process stops. The underlying linear relationship is established from the trends in the data points far away from the lane change maneuver. On rare occasions, as discussed in Sections IV.C and V, the observed ranges of speed and of spacing were too small to exhibit any clear relationships, in which case no statistics were generated for such a vehicle.

The affected points when accommodating an entering vehicle are found in a manner very similar to accommodating an exiting vehicle with the following exceptions. First, the largest negative spike in spacing is found, rather than a positive spike. Subsequent points are added as above. Once the process stops adding points after the maneuver, an additional step is used to check for any advance accommodation before the maneuver. Stepping backwards in time from the large negative spike in headway, the same recursive process is applied to find any unusually large spacing before the maneuver. If any such points are found, they too are added to the set of affected points.

One would expect the accommodation to an exiting vehicle is discretionary and to an entering vehicle is mandatory.[5] In other words, if vehicle i forces its way into lane A from lane B, vehicle $i+1$ in lane A has to decelerate immediately to avoid a collision, either before or after the maneuver. However, vehicle $i+1$ in lane B does not have to accelerate immediately to take the advantage of large spacing. So, LCAT

[4] Although the procedures involve subjective judgment, formal use of the Pearson correlation coefficient would lead to similar results.

[5] A driver following an entering vehicle may elect to begin accommodation prior to the maneuver, e.g., the triangles in Fig. 4, but the act of accommodation is mandatory to avoid collision. In these cases, the accommodation time before the maneuver is included in the LCAT.

for vehicles following exiting vehicles should in general be larger than that for vehicles following entering vehicles. This point will be examined in Section IV-C below.

B. Spacing-Speed Relation for Vehicles Changing Lanes

The impact of a lane change maneuver is not limited to the spacing-speed relation of following vehicles, the maneuver will also likely affect the spacing-speed relation for the vehicle changing lanes. When a driver undertakes a lane change maneuver, they might be more aggressive or conservative following vehicles in either lane since a key task is to accelerate or decelerate to find an acceptable gap in the target lane. After completing the lane change maneuver, the driver would then presumably focus strictly on car-following in the new lane, but it might take several seconds to return to their target spacing-speed relation if the initial spacing deviates from the relation. We refer to the lane changing vehicle's observations just before and after the lane change as *end points* and the end points are expected to deviate from the driver's typical preferred spacing-speed relation. An example of entering vehicle is shown in Fig. 5 and an example of exiting vehicle is shown in Fig. 6.

In Fig. 5, vehicle 8586 entered lane 1 from lane 2 at time 3524 sec and then followed vehicle 8583. The 6 observations denoted with circles in the spacing-speed plot were observed within 6 sec after the lane change maneuver. The data points show that vehicle 8586 had relatively high speed and low spacing immediately after entering lane 1. After that, the observations show the pattern similar to Fig. 1(b). There are two observations far from the preferred relation at time about 3543 sec (denoted with "+" in Fig. 5) which appear to be due to an acceleration wave moving upstream, i.e., the overall evolution of the traffic state in the absence of the lane change maneuver. After removing just the 6 end points, the Pearson correlation coefficient between spacing and speed is increased from 0.05 to 0.69.

In Fig. 6, vehicle 5471 left lane 1 at time 2268 sec, 3 sec before this lane change maneuver, vehicle 5471 accelerated and the observations deviated from the rest of the observed spacing-speed relation. After removing the 3 end points, the Pearson correlation coefficient between spacing and speed is increased from 0.48 to 0.69. As expected in Fig. 6, the peak in headway leads the trough in speed because the instantaneous headway calculation incorporates the impact of the future speed measurements.

As with Section IV-A this study was repeated for all of the other lane change maneuvers within the study segment and time window. The end points for an entering vehicle are found as follows. The first point after a lane change maneuver is considered an end point. The next point is considered a candidate point and the linearity in the spacing-speed plane with and without the candidate is evaluated as in the previous section. The process continues recursively until the candidate data point does not degrade the linear relation, at which point the candidate is excluded from the set of affected points and the process stops. The selection of end points for an exiting vehicle follows the similar procedure except that the selection starts from the last observation and steps backward in time, rather than forward, to find the next candidate point.

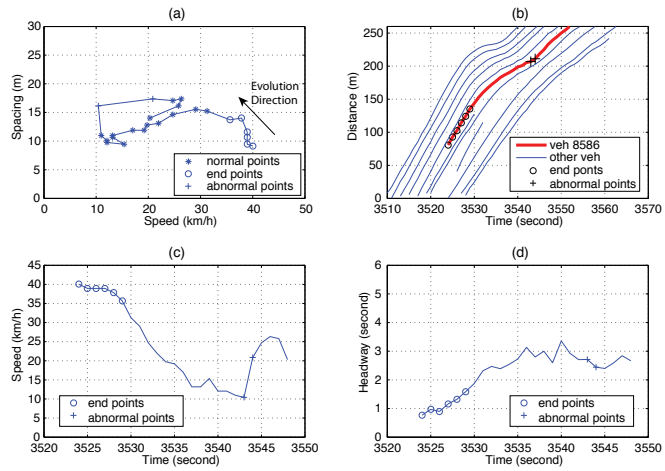

Fig. 5. (a) Spacing-speed plot of the entering vehicle 8586, (b) The corresponding vehicle trajectories, (c) Time series of speed of vehicle 8586, and (d) Time series of headway of vehicle 8586.

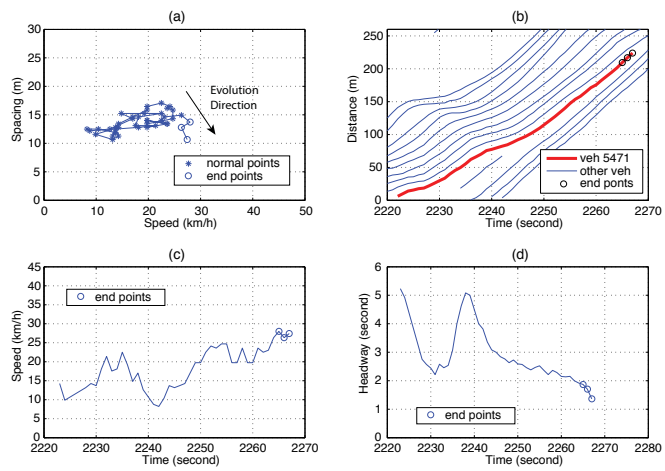

Fig. 6. (a) Spacing-speed plot of the exiting vehicle 5471, (b) The corresponding vehicle trajectories, (c) Time series of speed of vehicle 5471, and (d) Time series of headway of vehicle 5471.

C. Spacing-Speed Relation for Many Lane Change Maneuvers and the Resulting Implications

To control for the fact that any single LCAT will be subject to prevailing traffic conditions, the driver's responsiveness, and so forth, the LCAT are found for as many lane change maneuvers as possible and the distributions are used to overcome many of these externalities in the individual LCAT measurements. After extending the analysis to all 118 vehicles that left lane 1 to lane 2 and 31 vehicles that entered lane 1 from lane 2, we were able to extract relations similar to those shown in Figs. 3-6 for approximately 62 percent of the vehicles. The remaining vehicles were excluded either due to the fact that multiple successive maneuvers overlapped or because the range exhibited in the spacing-speed plane was too small (under 16 km/h) to distinguish between noise and the impacts of lane change maneuvers. Fig. 7 shows the Cumulative Distribution Function (CDF) of the LCAT from the four cases. The sample size of each group is shown in parentheses in the legend, ranging between 18 and 72 vehicles

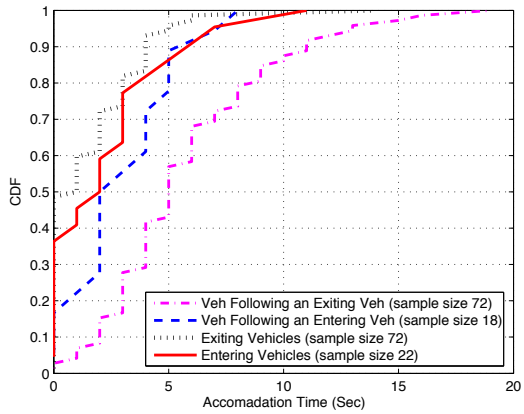

Fig. 7. CDF of Lane Change Accommodation Time from four cases.

per curve. One should also note that the accommodation by an entering vehicle and following vehicle are concurrent rather than successive, e.g., vehicle 1560 following entering vehicle 1557 in Fig. 4 both accommodated the lane change maneuver over the same time period.

Vehicles following an exiting vehicle tend to have a larger accommodation time than those behind an entering vehicle (discretionary versus mandatory accommodation). For this data set the median LCAT for entering vehicles and vehicles following entering vehicles is 3 sec faster than the median LCAT for vehicles following exiting vehicles. This imbalance serves to exacerbate the congested traffic state upstream. When a vehicle changes lanes, on average the new lane completely accommodates the vehicle before the "hole" it left behind in the old lane is filled. If Newell's simplified car-following model holds, after a lane change maneuver all subsequent vehicles in the new lane will have to make the abrupt deceleration while all subsequent vehicles in the old lane will make the more gradual acceleration. As such, this accommodation imbalance will propagate upstream and may be part of the mechanism underlying the instability of congested traffic as manifest by fluctuating traffic speed within a queue.

To appreciate the implications consider a hypothetical example where there are two lanes, all vehicles are identical, initially both lanes are in the same traffic state: (q_E, k_E, v_E), and the only disturbance is a single lane change maneuver. Thereby removing any change of state resulting from the overall evolution of traffic in the absence of the lane change maneuver. For simplicity the exited and entered lanes are denoted with subscript X and N, respectively. Fig. 8(a) shows the initial state E on the flow-density curve for one lane. Fig. 8(b) shows several vehicle trajectories with solid lines in the entered lane, both upstream and downstream of the lane change maneuver, denoted with a star. Fig. 8(c) repeats the exercise for the exited lane over the same time-space region. In both cases all vehicles following the lane change maneuver have to change speed, resulting in a disturbance in each lane that propagates upstream with wave velocity v_w (as highlighted with dashed lines). This new traffic state is denoted with an X or an N in Fig. 8(a).[6] Far downstream of the lane change maneuver, vehicles that came after the maneuver in the exited lane have gained one headway, h_E, and the corresponding vehicles in the entered lane have lost h_E, where $h_E = \frac{1}{q_E}$.

Looking closer at the entered lane, each vehicle trajectory will take v_N for $LCAT_N$, and then return to v_E, as shown in Fig. 8(d). But as evident in this plot, the disturbance to state N will persist at any given location upstream of the lane change maneuver for a duration t_N,

$$t_N = LCAT_N \cdot \left(1 + \frac{v_N}{v_w}\right) \tag{4.a}$$

Similarly, in the exited lane, the corresponding disturbance to state X will persist at a given location for a duration t_X,

$$t_X = LCAT_X \cdot \left(1 + \frac{v_X}{v_w}\right) \tag{4.b}$$

Figs. 8(e)-(f) show the traffic state evolution corresponding respectively to Figs. 8(b)-(c). For delay to increase by h_E in the entered lane,

$$q_N = q_E - \frac{1}{t_N} \tag{5.a}$$

and decrease by h_E in the exited lane,

$$q_X = q_E + \frac{1}{t_X} \tag{5.b}$$

Since vehicles in the exited lane speed up to close the gap and in the entered lane slow down to make a gap,

$$v_X > v_N \tag{6}$$

and as noted above,

$$LCAT_X > LCAT_N \tag{7}$$

thus,

$$t_X > t_N \tag{8}$$

and,

$$q_X - q_E < q_E - q_N \tag{9}$$

Taking the two lanes together, in Figs. 8(g)-(h), the net flows across the two lanes are,

$$q_0 = 2 \cdot q_E \tag{10.a}$$

$$q_2 = q_N + q_X < q_0 \tag{10.b}$$

$$q_3 = q_E + q_X > q_0 \tag{10.c}$$

So while the disturbances pass, the instantaneous net flow initially drops for t_N and then increases above the steady state flow, q_0, for t_X-t_N. The average flow over t_X, however, remains q_0.

In fact, one would still expect the traffic state fluctuation shown in Fig. 8(h) even if mean($LCAT_X$) = mean($LCAT_N$) because the inequality in (8) still follows from (4) and (6). If mean(t_X) = mean(t_N), on the other hand, the particular values

[6] Relaxing the piecewise linear trajectories, v_N and v_X would denote the average speed during the respective LCAT.

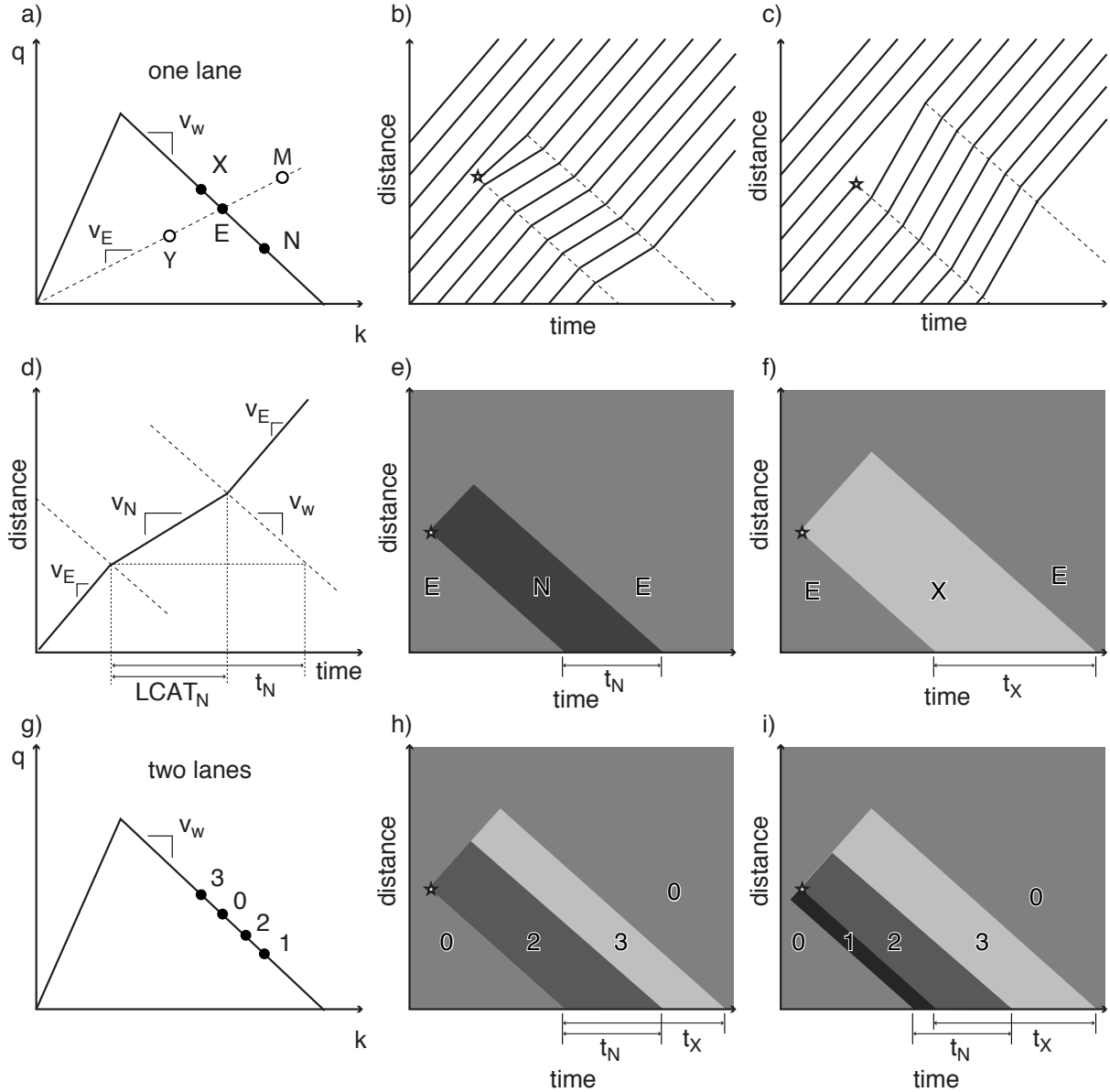

Fig.8. Two lane freeway with a single lane change maneuver (a) flow density relationship for either lane, (b) vehicle trajectories in the entered lane, (c) vehicle trajectories in the exited lane, (d) detail of B showing the duration of the disturbance, (e) state diagram from B, (f) state diagram from C, (g) flow density relationship for the two lanes combined, (h) state diagram for the two lanes combined, (i) state diagram for the two lanes combined when the entered lane begins accommodating before the exited lane.

of t_N and t_X for a given lane change maneuver are sampled from their respective distributions and individual samples should not in general be equal even if the means of the original distributions are equal.

Of course it is possible that the driver in the entered lane, immediately behind the vehicle that changes lanes, responded to a signal by the vehicle changing lanes and started to open a gap before the maneuver. In which case, the combined state diagram would look like Fig. 8(i), where,

$$q_1 = q_N + q_E < q_2 \tag{11}$$

So in this case the instantaneous net flow initially drops further than the previous example and then the recovery is split across two events instead of one, but the average flow across the two lanes at a given location measured from the start of the first disturbance in the entered lane to the end of the second disturbance in the exited lane remains q_0. In this case a traffic state fluctuation is guaranteed across the two lanes no matter what the two LCAT values are, because one lane begins responding to the maneuver before the other.

These mechanics are important, because while Lighthill Whitham and Richards [7]-[8] theory (LWR) can predict the propagation of waves in the traffic stream once they have formed, the theory offers no explanation of how these waves originate or grow. So upstream of a bottleneck with constant capacity, LWR would predict a constant flow and speed at any location within the queue. But it is widely known that the traffic state typically fluctuates within a queue, with stop-and-go traffic being an extreme example. The present work offers a potential explanation for the otherwise apparent discrepancy between LWR and empirical observations within a queue. The present work seeks only to document the existence of this

accommodation imbalance. The non-trivial task of investigating the detailed impacts of the imbalance is left to future research. Finally, the implications of this LCAT imbalance transcend Newell's model and should be applicable to most car-following models.

D. Transient States During Lane Change Accommodation

The state diagrams in Fig. 8 do not show the transient states encountered by the vehicles immediately following the lane change maneuver or the vehicles changing lanes. Initially these drivers maintain their speed, v_E, when they first encounter a discontinuity in spacing. If one were to extrapolate this transient microscopic state to the macroscopic flow density plane in Fig. 8(a), the traffic state is briefly disturbed from state E in both lanes to state Y in the exited lane and state M in the entered lane, these lead drivers then follow some path through the flow density plane to bring them to states X and N, respectively. Only these two lead vehicles and the vehicle that changes lanes are exposed to a perturbation. According to LWR the following vehicles experience the state diagrams as shown in the rest of Fig. 8, i.e., in each lane, the lane change maneuver creates a new prototype trajectory for all who follow. While the specific paths through the flow density plane for the three perturbed vehicles follow from the affected points, e.g., as shown in Figs. 3(a) and 4(a), the details of this path are not as important as the simple fact that while these vehicles are accommodating the lane change maneuver they are not on the static flow density curve. Of course each time such a perturbation occurs, there is an increased chance that a signal emanating from downstream will be distorted or even disappear, thereby distorting the traffic state evolution from its otherwise normal progression in the absence of a lane change maneuver. The longer LCAT is, the longer the lead drivers are off the curve. The more lane change maneuvers, the more drivers that fall into such a set. As the duration and frequency of lane change maneuvers increases, consistent with Mauch and Cassidy's findings [12], one would expect that any static flow density curve that holds for a low frequency of lane change maneuvers will eventually break down as more drivers experience perturbations off of the curve at any given instant.

E. Lane Change Accommodation as a Function of Traffic Density

As was shown, LCAT differs for entering and exiting maneuvers. This paper seeks only to show that there is a systematic difference, not to provide exact values of the LCAT for the various cases. Other factors likely influence LCAT as well, e.g., driving behaviors and traffic conditions. The small size of the present data set precludes a detailed study of these parameters, but a preliminary analysis can be conducted by comparing LCAT from vehicles following an exiting vehicle in lane 1, lane 2 and lane 3 over the study segment and time window. The average density is tabulated in the first row of Table I, which shows that density drops by almost a third from lane 1 to lane 3, while the last row compares the median LCAT for vehicles following an exiting vehicle[7]. Lane 3 has a larger LCAT than lane 2 and lane 2 larger than lane 1. This fact suggests that LCAT from vehicles following an exiting vehicle tends to be smaller as the traffic becomes more congested. In other words, drivers tend to fill the gap left by an exiting vehicle and return to their preferred spacing-speed relation quicker in a very dense queue compared to a moderately dense queue. If these preliminary observations across different levels of congestion remain when extended to a larger data set, evidence such as shown in Table I should help explain the phenomena observed by Mauch and Cassidy [12], they observed that in the presence of frequent lane change maneuvers Newell's theory seemed inappropriate for a moderately dense queue but reasonable for a very dense, incident-induced queue. Their observation might be due to the difference of LCAT as a function of density. A more detailed examination of LCAT as a function of traffic density is left to future research, it is mentioned here to make clear that such factors have not been eliminated from the distributions in Fig. 7 and to provide motivation for future research to do so.

TABLE I
Average density and speed of traffic over the second half hour

	Lane 1	Lane 2	Lane 3
Average density (veh/km)	51	44	34
Average speed (km/h)	28	35	49
Number of exiting vehicles	72	36	27
Median LCAT following an exit (sec)	5	7	8

V. Linear Spacing-Speed Relation

Figs. 3-6 show that the spacing-speed relation is non-linear during a lane change maneuver and associated LCAT. Although there are many lane change maneuvers in the study area, the majority of the observed vehicles did not change lanes or follow immediately behind a vehicle that did. The Pearson correlation coefficient between spacing and speed is calculated for all observed vehicles in lane 1 over the study segment and time window. To ensure that all observations are in the car-following mode, all observations with speed greater than 80 km/h (50 mph) are removed, i.e., 80 km/h is taken as a conservative value of $\hat{V}_i$ in Fig. 1(b). At very low speed, normal acceleration can lead to large relative changes in speed. Newell [1] anticipated this problem and deliberately avoided low speed conditions, likewise in this study all observations with speed smaller than 8 km/h (5 mph) are also removed. As a result, the speed range in our study is 72 km/h (45 mph).

In this study, 796 vehicles were observed in lane 1. However, after removing the observations outside of the 8-80 km/h range, the spacing-speed plots for 27 vehicles have 10 or fewer observations and 236 vehicles had speeds spanning less than 16 km/h (10 mph). In either case, the data do not show sufficient range to suppress measurement noise in the spacing-speed relation and these 263 vehicles are excluded from further

[7] These trends are consistent with the complete CDF, which is omitted due to space constraints.

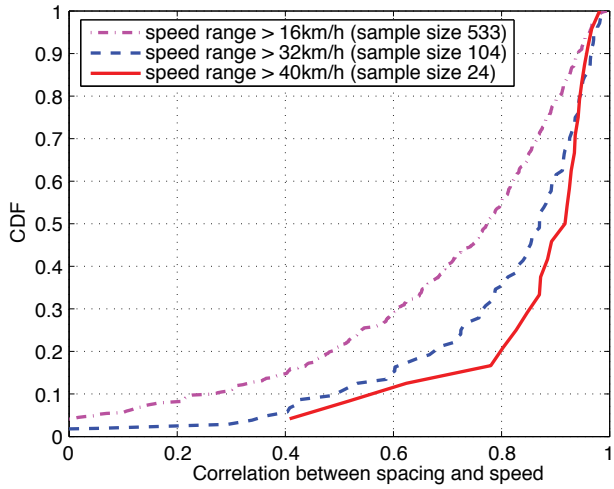

Fig. 9. CDF of correlation between spacing and speed for vehicles with different speed range.

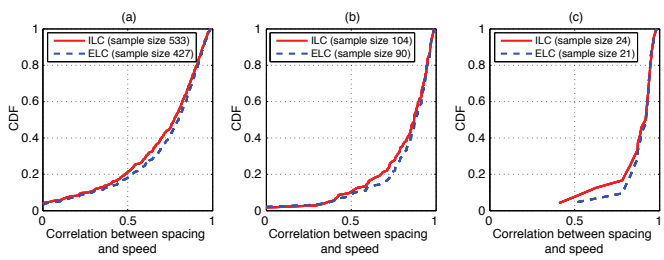

Fig. 10. CDF of correlation between spacing and speed Including Lane Change (ILC) affected vehicles and Excluding Lane Change (ELC) affected vehicles (a) for speed range greater than 16km/h, (b) for speed range greater than 32 km/h, and (c) for speed range greater than 40km/h.

analysis. It is not surprising to remove observations for so many vehicles because the 244 m long segment limited the duration a given vehicle was observed. The remaining set consists of 533 vehicles (66 percent).

While some of the vehicles are truncated at the 8 km/h lower threshold, none of the 533 vehicles approach the 80 km/h upper threshold. The largest observed speed range for a single vehicle is 50 km/h (31 mph), this particular vehicle's speed range is 8 km/h to 58 km/h. While 104 vehicles have speed range greater than 32 km/h (20 mph) and 24 vehicles have speed range greater than 40 km/h (25 mph). Fig. 9 shows the CDF of correlation between spacing and speed, under these three different speed range criteria. For the 533 vehicles with speed range greater than 16 km/h, about 45 percent of the vehicles show a correlation of spacing and speed higher than 0.80 and about 70 percent of the vehicles show a correlation higher than 0.60. These numbers increase to 80 percent and 95 percent, respectively, for the 24 vehicles with speed range greater than 40 km/h. Fig. 9 implies a trend that the longer the speed range is, the stronger the linear relation between spacing and speed. This observation might be explained by the fact that when the speed range is short, the impact of small deviations or measurement errors become more significant, and related to deviations, the impact of acceleration and deceleration can have a larger impact relative to the change in speed.

Note that this set includes all observations influenced by lane change maneuvers, i.e., the affected points in Figs. 3-4 and end points in Figs. 5-6 were not removed and account for some of the low correlations. If all vehicles that change lanes or are immediately behind a vehicle that changes lanes, (collectively called lane change affected vehicles), are removed, the linearity becomes stronger. Fig. 10 contrasts the CDF before and after removing the lane change affected vehicles. For example, Fig. 10(a) shows that the percentage of the vehicles with correlation higher than 0.80 increases from 45 percent to 50 percent after excluding lane change affected vehicles.

Although far from a perfect fit, Fig. 9 shows that for the range of speed and the short distance observed, the relation between spacing and speed is close to linear for most of the vehicles. A linear regression was applied to the observations from each of the 533 vehicles and one set of $[s_i^0, \mu_i]$ was obtained for each regression. The mean s_i^0 and μ_i are 6.9 m and 1.36 s. From (2), the average wave velocity through the segment is 18.3 km/h. These results are similar to those of [13], which were estimated with maximum likelihood. Furthermore, the estimated wave velocity is close to the congested wave velocity on freeways reported by other researchers: 17-20 km/h in [35], 19.4 km/h in [36], 15-20 km/h in [37], 15-25 km/h in [38], and it is just below 22-24 km/h in [12]. Two methods to estimate wave velocity based on speed observations at two discrete points in space have previously been employed by our group, namely, Cross Correlation Analysis [38] and Cross Spectral Analysis [37]. Both methods were applied on the subject data set and the estimated wave velocities are 17.3 km/h and 16.5 km/h, respectively, consistent with the average wave velocity via (2).

VI. Conclusions

Newell [1] proposed a simple car-following theory that uses fewer parameters than most other car-following models. An important assumption underlying this simplification is that the relation between spacing and speed for a single vehicle can be approximated by a piecewise linear curve. Two subsequent papers have validated Newell's theory on a macroscopic scale but found that Newell's theory fails in the presence of frequent lane change maneuvers.

This paper used microscopic vehicle trajectory data to examine the impacts of lane change maneuvers on the spacing-speed relation and to investigate the linearity of this curve. Using several examples it was shown that a given lane change maneuver not only perturbs the spacing-speed relation of vehicles immediately following the maneuver, but also perturbs the spacing-speed relation of the vehicle making the maneuver. Namely, the large spacing left by an exiting vehicle or the small spacing caused by an entering vehicle will induce a transient deviation from the normally preferred relation. Although the given driver eventually returns to their apparent desired spacing-speed relation, this recovery takes time. Additionally, when a vehicle changes lanes the driver might not follow their normal driving behavior and thus, cause further deviations from the spacing-speed relation. Obviously, the smaller the impact caused by lane change maneuvers, the

better the linear assumption captures car-following behavior and by extension, the better Newell's theory can describe macroscopic traffic behavior. LCAT was introduced to quantify the duration that lane change maneuvers perturb the spacing-speed relation. It is shown that LCAT from vehicles following an exiting vehicle is larger than those from entering vehicles, exiting vehicles and vehicles following an entering vehicle. At the very least, this imbalance serves to exacerbate the congested traffic state upstream of the maneuver. When a vehicle changes lanes, on average the new lane completely accommodates the vehicle before the "hole" it left behind in the old lane is filled. After a lane change maneuver all subsequent vehicles in the new lane will have to make the abrupt deceleration while all subsequent vehicles in the old lane will make the more gradual acceleration. As such, this accommodation imbalance will propagate upstream and appears to be a source of instability within queues, contributing to speed and flow fluctuations. As discussed herein, this imbalance offers a potential explanation for the otherwise apparent discrepancy between LWR and empirical observations of disturbances forming within a queue. The implications of this LCAT imbalance in explaining speed fluctuations transcend Newell's model and should be applicable to most car-following models.

Subject to the limited amount of vehicle trajectory data available, this paper provided preliminary evidence to suggest that the LCAT depends on traffic conditions, i.e., LCAT decreases with density. While more data will be necessary to fully investigate this latter point, if this finding holds, the discussion herein could explain the empirical results found by other researchers, namely that Newell's theory breaks down in the presence of frequent lane change maneuvers.

The Pearson correlation coefficients between spacing and speed for 533 vehicles were calculated. Although the covered range of speeds in this study is relatively small due to the raw data limitations, the results show that the assumption of the linear relation between spacing and speed is reasonable most of the time for the study sample. Extremely low speed conditions were excluded from the sample, as drivers clearly deviated from the linear relation when they were either stopping or starting. The calculated [s_i^0, μ_i] and estimated average wave velocity are consistent with earlier empirical studies. In other words, these results provide evidence that the linear relation between spacing and speed appears to capture the dominant preferences of drivers. However, dynamic events can cause significant deviations from this static curve, such as the lane change maneuvers in Figs. 3(a) and 4(a). It was shown that the linear relation improved when lane change affected vehicles were excluded. Provided the dynamic events are relatively infrequent, the linear relation would be expected to predominate average conditions.

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