Modeling and Control of SRM in Electromechanic Brake System

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Agenda

I. Project Goal and Objectives
II. Publications
III. Modeling and Parameters Identification of Switched Reluctance Motor (SRM)
IV. Sensorless Control of SRM
V. Four-quadrant Torque and Force Control
I. Project Goal and Objective (1)

Overall project goal:

Develop and implement a low-cost drive system consisting of a sensorless switched reluctance motor (SRM) with power converter and controller. This drive system shall be operated as part of an electro-mechanical brake system for the purpose of converting a clamping force command into a physical clamping force.
Project Goal and Objective (2)

- Objectives:
  1. Four-quadrant operation
  2. Desired clamping force response
  3. Torque ripple minimization
  4. Elimination of rotor position sensor
  5. Elimination of clamping force sensor
  6. High performance, low cost
  7. Fault tolerance
II. Publications


III. Modeling and Parameters Identification of SRM

- Inductance Model of SRM
- Parameter Identification from Standstill Test Data
- Parameter Identification from Operating Data
Standstill SRM Model

\[ R = R(i) \]
\[ L = L(\theta, i) \]
Flux Linkage As a Function of Phase Current and Rotor Position
Phase Inductance of SRM (1)

- Using Fourier series to represent phase inductance

\[
L(\theta, i) = \sum_{k=0}^{m} L_k(i) \cos kN_r \theta
\]

\[
L_{\theta}(i) = \sum_{n=0}^{k} a_{\theta,n} i^n
\]
Phase Inductance of SRM (2)

- Three-term Inductance Model (8/6 SRM)

\[ L(\theta, i) = L_0(i) + L_1(i) \cos 6\theta + L_2(i) \cos 12\theta \]

\[
\begin{bmatrix}
L_{0^\circ} \\
L_{15^\circ} \\
L_{30^\circ}
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 \\
1 & \cos(6 \times 15^\circ) & \cos(12 \times 15^\circ) \\
1 & \cos(6 \times 30^\circ) & \cos(12 \times 30^\circ)
\end{bmatrix} \begin{bmatrix}
L_0 \\
L_1 \\
L_2
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 \\
1 & 0 & -1 \\
1 & -1 & 1
\end{bmatrix} \begin{bmatrix}
L_0 \\
L_1 \\
L_2
\end{bmatrix}
\]

So

\[
\begin{bmatrix}
L_0 \\
L_1 \\
L_2
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 \\
1 & 0 & -1 \\
1 & -1 & 1
\end{bmatrix}^{-1} \begin{bmatrix}
L_{0^\circ} \\
L_{15^\circ} \\
L_{30^\circ}
\end{bmatrix} = \begin{bmatrix}
1/4 & 1/2 & 1/4 \\
1/2 & 0 & -1/2 \\
1/4 & -1/2 & 1/4
\end{bmatrix} \begin{bmatrix}
L_{0^\circ} \\
L_{15^\circ} \\
L_{30^\circ}
\end{bmatrix}
\]

where \( L_{0^\circ} = L_{0^\circ}(i) \) \( L_{15^\circ} = L_{15^\circ}(i) \) \( L_{30^\circ} = \text{const} \)
Phase Inductance of SRM (3)

- Four-term Inductance Model (8/6 SRM)

\[ L(\theta, i) = L_0(i) + L_1(i) \cos6\theta + L_2(i) \cos12\theta + L_3(i) \cos18\theta \]

\[
\begin{bmatrix}
L_{00} \\
L_{10} \\
L_{20} \\
L_{30}
\end{bmatrix} = 
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & \cos(6*10^0) & \cos(12*10^0) & \cos(18*10^0) \\
1 & \cos(6*20^0) & \cos(12*20^0) & \cos(18*20^0) \\
1 & \cos(6*30^0) & \cos(12*30^0) & \cos(18*30^0)
\end{bmatrix}
\begin{bmatrix}
L_0 \\
L_1 \\
L_2 \\
L_3
\end{bmatrix}
\]

So

\[
\begin{bmatrix}
L_0 \\
L_1 \\
L_2 \\
L_3
\end{bmatrix} = 
\begin{bmatrix}
1/6 & 1/3 & 1/3 & 1/6 \\
1/3 & 1/3 & -1/3 & -1/3 \\
1/3 & -1/3 & -1/3 & 1/3 \\
1/6 & -1/3 & 1/3 & -1/6
\end{bmatrix}
\begin{bmatrix}
L_{00} \\
L_{10} \\
L_{20} \\
L_{30}
\end{bmatrix}
\]
Voltage Equation

\[ V = R \cdot i + \frac{d(Li)}{dt} = R \cdot i + L \frac{di}{dt} + i \frac{dL}{dt} \]

\[ = R \cdot i + L \frac{di}{dt} + i \left( \frac{\partial L}{\partial \theta} \omega + \frac{\partial L}{\partial i} \frac{di}{dt} \right) \]

where

\[ \frac{\partial L}{\partial i} = \sum_{k=0}^{m} \frac{\partial L_k(i)}{\partial i} \cos kN_r \theta \]

\[ \frac{\partial L}{\partial \theta} = -\sum_{k=0}^{m} L_k(i)kN_r \sin kN_r \theta \]
Torque Computation

\[ T = \frac{\partial W_c(\theta, i)}{\partial \theta} = \frac{\partial \{ \int [L(\theta, i)i]di \}}{\partial \theta} = \frac{\partial \{ \sum_{k=0}^{m} [L_k(i) \cos(kN_r \theta)i]di \}}{\partial \theta} \]

\[ = -\sum_{k=0}^{m} \{kN_r \sin(kN_r \theta) \int [L_k(i)i]di\} \]
Identification of Inductance from Standstill Test Data (1)

**Obtaining** $L_\theta$

- Finite element analysis
- Standstill test
- Online test

**Basic idea of standstill test**

- Move the rotor to a specific position ($\theta$) and block it
- Inject a voltage pulse to the phase winding
- Measure the current generated in the phase winding
- Select a model structure and use maximum likelihood estimation (MLE) to estimate phase parameters
Identification of Inductance from Standstill Test Data (2)

- **System Model Structures**

\[
\dot{X}(k+1) = A(\theta_s) \cdot X(k) + B(\theta_s) \cdot u(k) + w(k)
\]

\[
Y(k+1) = C(\theta_s) \cdot X(k+1) + v(k)
\]

\[
X = [i_1] \quad Y = [i] \quad u = [V] \quad \theta_s = [R, L]
\]

w: process noise \quad v: measurement noise
Identification of Inductance from Standstill Test Data (3)

- Maximum Likelihood Estimation

![Diagram showing a phase winding model, input $u$, output $y$, and estimated parameter $R, L$.]
Experimental Setup with dSPACE DS1103
Pictures of Test-bed (1)

Complete Experimental System
Pictures of Test-bed (2)

8/6 SR motor with ROC 412 Single-turn Rotary Encoder
Pictures of Test-bed (3)

Flexible Power Converters
Pictures of Test-bed (4)

PC running dSPACE ControlDesk and Matlab
Standstill Test Results (1)

- Standstill Test Voltage and Current Waveforms
Standstill Test Results (2)

Inductance at aligned position

![Inductance vs. theta plot](image)

- Test data
- Curve-fitting
**Standstill Test Results (3)**

- **Inductance at unaligned position**

  ![Graph showing inductance at theta = 30 deg](image)

- Test data
- Curve-fitting
Standstill Test Results (4)

- Inductance at midway position

![Graph showing inductance at theta = 15 deg]
Standstill Test Results (5)

- Inductance under different currents at different rotor positions
Standstill Test Results (6)

- Flux linkage

![Image of flux linkage diagram](image-url)
SRM Model for Online Operation (1)

- Model Structure

\[
\begin{bmatrix}
L & -L_d \\
0 & L_d
\end{bmatrix}
\begin{bmatrix}
\dot{i}_1 \\
\dot{i}_2
\end{bmatrix}
= \begin{bmatrix}
0 & R_d \\
-R & -R - R_d
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1
\end{bmatrix}V
\]
SRM Model for Online Operation (2)

- State Space Representation

\[
\dot{X} = AX + BU \\
Y = CX + DU
\]

\[
U = [V] \\
X = [i_1 \ i_2] \\
Y = i_1 + i_2
\]

\[
A = \begin{bmatrix} L & -L_d \\ 0 & L_d \end{bmatrix}^{-1} \begin{bmatrix} 0 & R_d \\ -R & -R - R_d \end{bmatrix} \\
B = \begin{bmatrix} L & -L_d \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
C = [1 \ 1] \\
D = 0
\]
SRM Model for Online Operation (3)

- Torque Computation

\[
T = \frac{\partial W_c(\theta, i_1)}{\partial \theta} = \frac{\partial \{ \int [L(\theta, i_1)i_1]di_1 \}}{\partial \theta} \\
= \frac{\partial \{ \sum_{k=0}^{m} [L_k(i_1)\cos(kN_r\theta)i_1]di_1 \}}{\partial \theta} \\
= -\sum_{k=1}^{m} \{kN_r \sin(kN_r\theta)\int [L_k(i_1)i_1]di_1 \}
SRM Model for Online Operation (4)

- 2-Layer Recurrent Neural Network
SRM Model for Online Operation (5)

Application of Neural Network to Estimate Rd and Ld

After the neural network is trained with simulation data (using parameters obtained from standstill test). It can be used to estimate exciting current during on-line operation. When $i_1$ is estimated, the damper current can be computed as

$$i_2 = i - i_1$$

and the damper voltage can be computed as

$$V_2 = V - i \cdot R$$

then the damper resistance $R_d$ and inductance $L_d$ can be identified using output error or maximum likelihood estimation.
Model Validation (1)

Model validation with operating data

Current Response in Phase A - cov = 1.1962

Current Response in Phase B - cov = 0.7887

Current Response in Phase C - cov = 0.5873

Current Response in Phase D - cov = 0.6806
Model Validation (2)

- Model validation with operating data

![Current Response in Phase A](image-url)

- measured current (A)
- estimated current (A)
- measures voltage (V)
IV. Sensorless Control of SRM Drives

- Inductance-based analytical model of SRM
- Sliding mode observer based sensorless controller
- Sensorless control at near zero speeds
- Sensorless control at standstill
- Conclusions
Sensorless Control of SRM

Why Sensorless
- To enhance reliability, and reduce size and cost of the system
- To cope with harsh ambient conditions dictated by the application

What is an optimal Sensorless scheme
- No extra hardware/sensors
- Good resolution/accuracy over the entire speed range
- 4-Q capability in Motoring/generating operation

What are typical problems facing Sensorless technology in SRM drives
- Poor dynamics during startup, Starting hesitation
- Limited speed range
- Failure at super high speeds
Sliding Mode Observer (1)

- Sliding mode observer based controller

Note: The controller uses estimated \( \hat{\theta} \) and \( \hat{\omega} \) in this case
Sliding Mode Observer (2)

System differential equations (I)

\[ V_j = R \cdot i_j + \frac{d\lambda_j}{dt} = R \cdot i_j + L_j \frac{di_j}{dt} + i_j \left( \frac{\partial L_j}{\partial \theta} \omega + \frac{\partial L_j}{\partial i_j} \frac{di_j}{dt} \right) \]

\[
\frac{di_j}{dt} = \frac{1}{L_j + i_j \frac{\partial L_j}{\partial i_j}} \left[ V_j - R \cdot i_j - \frac{\partial L_j}{\partial \theta} \omega \cdot i_j \right]
\]

Or, to simplify,

\[
\frac{di_j}{dt} = f_1(i_j, \theta, \omega) + g(i_j, \theta, \omega)V_j \quad (1)
\]

where

\[
f_1(i_j, \theta, \omega) = -\frac{R \cdot i_j - \frac{\partial L_j}{\partial \theta} \omega \cdot i_j}{L_j + i_j \frac{\partial L_j}{\partial i_j}}
\]

\[
g(i_j, \theta, \omega) = \frac{1}{L_j + i_j \frac{\partial L_j}{\partial i_j}}
\]
System differential equations (II)

\[
\frac{d\theta}{dt} = \omega 
\]

\[
\frac{d\omega}{dt} = \frac{1}{J} [T - T_i] = f_2(X, \theta, \omega) 
\]

where \( f_2(X, \theta, \omega) = \frac{1}{J} [T - T_i] = \frac{1}{J} \sum_{j=1}^{n} T_j - T_i \) and \( X = [i_1, i_2, ...] \)

Equations (1)~(3) form the system differential equations.
Sliding Mode Observer (4)

Definition of sliding mode observer

\[
\frac{di_j}{dt} = f_1(\hat{\theta}, \hat{i}_j, \hat{\omega}) + g(\hat{\theta}, \hat{i}_j, \hat{\omega}) \cdot V_j \tag{4}
\]

\[
\frac{d\hat{\theta}}{dt} = \hat{\omega} + L_1 \text{sign}(\sum_{j=1}^{m} (i_j - \hat{i}_j)) \tag{5}
\]

\[
\frac{d\hat{\omega}}{dt} = f_2(\hat{X}, \hat{\theta}, \hat{\omega}) + L_2 \text{sign}(\sum_{i=1}^{m} (i_j - \hat{i}_j)) \tag{6}
\]

where $\hat{\theta}, \hat{i}, \hat{\omega}$ are the estimations of $\theta, i, \omega$. 
Simulation results of sliding mode observer based controller 1000RPM, $e_\theta < 2^\circ$, $e_\omega < 0.5$RPM
Sliding Mode Observer (8)

Simulation results of sliding mode observer based controller 20RPM, $e_\theta < 2^\circ \ e_\omega < 2$RPM

When speed is below 20 RPM, the error becomes significant.
Sensorless Control at Near Zero Speeds (1)

- Inductance model based sensorless method

\[ V_j = R \cdot i_j + \left( L_j + i_j \frac{\partial L_j}{\partial i_j} \right) \frac{di_j}{dt} + i_j \frac{\partial L_j}{\partial \theta} \omega \]

The last item in above equation can be omitted at near zero speeds. So we get

\[ L_j + i_j \frac{\partial L_j}{\partial i_j} \approx \frac{(V_j - R \cdot i_j)}{\frac{di_j}{dt}} \quad (7) \]
Sensorless Control at Near Zero Speeds (2)

- Determination of turn-on and turn-off position

  - When a phase is on, compute the value of left side of equation (7) at two special angles: 
    \[ f(\theta_{on} + \pi/2N_r) \text{ and } f(\theta_{off}) \]

  - At any time, compute the right side of Eqn(12) approximately by:
    \[ g \approx (V_j - R \cdot i_j) \frac{\Delta t}{\Delta i} \]

  - If \( g \) matches \( f(\theta_{on} + \pi/2N_r) \), then turn on next phase; If \( g \) matches \( f(\theta_{off}) \), then turn off current phase.
Sensorless Control at Near Zero Speeds (4)

Simulation results (3 RPM)

<table>
<thead>
<tr>
<th>Angle</th>
<th>Desired (o)</th>
<th>Actual (o)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{onA}$</td>
<td>-30</td>
<td>-29.49</td>
<td>0.51</td>
</tr>
<tr>
<td>$\theta_{offA}$</td>
<td>-7</td>
<td>-6.14</td>
<td>0.86</td>
</tr>
<tr>
<td>$\theta_{onB}$</td>
<td>-15</td>
<td>-14.49</td>
<td>0.51</td>
</tr>
<tr>
<td>$\theta_{offB}$</td>
<td>8</td>
<td>8.86</td>
<td>0.86</td>
</tr>
<tr>
<td>$\theta_{onC}$</td>
<td>0</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td>$\theta_{offC}$</td>
<td>23</td>
<td>23.86</td>
<td>0.86</td>
</tr>
<tr>
<td>$\theta_{onD}$</td>
<td>15</td>
<td>15.51</td>
<td>0.51</td>
</tr>
<tr>
<td>$\theta_{offD}$</td>
<td>38</td>
<td>38.86</td>
<td>0.86</td>
</tr>
</tbody>
</table>

When speed is below 20 RPM, the error is acceptable.
Sensorless Control at Standstill (1)

- Determine rotor position at standstill by applying short voltage pulse to each phase.
  - Apply a short voltage pulse to phases sequentially.
  - Measure the peak current value generated in each phase.
  - Determine the switching signals by comparing peak current values.
Sensorless Control at Standstill (2)

- Current waveform at different rotor position

At different rotor positions, the phase currents have different peak values when excited by a voltage pulse.
Sensorless Control at Standstill (3)

- Peak currents at different rotor positions

By comparing the peak values of phase currents, a specific rotor position region can be determined.

![Graph showing peak currents at different rotor positions](image)

- Peak current when excited by 60µs pulse input

- Phase A
- Phase B
- Phase C
- Phase D
Sensorless Control at Standstill (4)

Determine switching signals by comparing peak values of phase currents:

<table>
<thead>
<tr>
<th>Peak Currents</th>
<th>Rotor Position</th>
<th>Phases to be excited</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_c &gt; I_d &gt; I_b &gt; I_a$</td>
<td>[0°, 7.5°]</td>
<td>B, C</td>
</tr>
<tr>
<td>$I_d &gt; I_c &gt; I_a &gt; I_b$</td>
<td>[7.5°, 15°]</td>
<td>C</td>
</tr>
<tr>
<td>$I_d &gt; I_a &gt; I_c &gt; I_b$</td>
<td>[15°, 22.5°]</td>
<td>C, D</td>
</tr>
<tr>
<td>$I_a &gt; I_d &gt; I_b &gt; I_c$</td>
<td>[22.5°, 30°]</td>
<td>D</td>
</tr>
<tr>
<td>$I_a &gt; I_b &gt; I_d &gt; I_c$</td>
<td>[30°, 37.5°]</td>
<td>D, A</td>
</tr>
<tr>
<td>$I_b &gt; I_a &gt; I_c &gt; I_d$</td>
<td>[37.5°, 45°]</td>
<td>A</td>
</tr>
<tr>
<td>$I_b &gt; I_c &gt; I_a &gt; I_d$</td>
<td>[45°, 52.5°]</td>
<td>A, B</td>
</tr>
<tr>
<td>$I_c &gt; I_b &gt; I_d &gt; I_a$</td>
<td>[52.5°, 60°]</td>
<td>B</td>
</tr>
</tbody>
</table>
Sensorless Control - Conclusions

- An inductance model of SRM can be used to design sensorless controller for full speed range.
- When speed is above 20RPM, a sliding mode observer based sensorless controller can yield satisfactory results.
- At near zero speeds is, the turn-on position for next phase and the turn-off position for current phase can be determined by matching inductances.
- At standstill, rotor position can be estimated by applying voltage pulses and comparing peak current values.
V. Four Quadrant Torque and Force Control of SRM for EMB
4-Q Torque and Force Control in EMB System Using SRM (1)

- **System structure**

- Four-quadrant operation
- Force control and torque ripple minimization
- Sensorless operation (no rotor position sensors)
4-Q Torque and Force Control in EMB System Using SRM (2)

- Two control loops + torque control
  - Outer loop: force control (PID)
  - Inner loop: current control (Hysteresis)
4-Q Torque and Force Control in EMB System Using SRM (3)

Simulink Model of the EMB System
4-Q Torque and Force Control in EMB System Using SRM (4)

- Torque Control and Torque Ripple Minimization
  - Because the phase torque in SRM is discontinuous, to generate a ripple-free resultant torque, there must be overlap between phases.
  - It is important to determine how the torque is distributed in phases during phase overlap.
  - Torque factor \( f(j, \theta) \) is defined here to represent the torque distribution among phases:

\[
T = \sum_{j=1}^{N} T_j = \sum_{j=1}^{N} f_j(\theta)T_{ref}
\]
4-Q Torque and Force Control in EMB System Using SRM (8)

- Choice of Torque Factor for 8/6 SRM:
Steps to determine reference current for each phase:

1. Compute the torque factors for all phases according to current rotor position and turning-on/turning-off angles;
2. Compute the desired torque $T_j$ for each phase by multiplying the $T_{ref}$ with corresponding torque factor;
3. Compute reference current $I_j$ for each phase to generate desired torque.
4-Q Torque and Force Control in EMB System Using SRM (13)

Simulation result – clamping force response
4-Q Torque and Force Control in EMB System Using SRM (14)

Simulation result – torque-speed curve
4-Q Torque and Force Control - Conclusions

- SRM in EMB needs 4-quadrant operation.
- Inductance model and pre-defined torque factor can be used for torque control and torque ripple minimization.
- Desired clamping force response can be obtained by applying force control + torque control + current control in EMB system.