Modeling and Control of a Fuel Cell Based Z-source Converter for Distributed Generation Systems

Jin-Woo Jung, Ph. D. Student

Advisor: Prof. Ali Keyhani

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Mechatronic Systems Laboratory
Department of Electrical and Computer Engineering
The Ohio State University
### Journal Papers


Publications (2)

- Conference Papers (1)


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Conference Papers (2)


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II. Circuit Analysis/System Modeling/PWM Implementation
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   C. Space Vector PWM Implementation

III. Control System Design
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   C. Discrete-time PI DC-link Voltage Controller

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I. INTRODUCTION

Why are fuel cell systems increasingly used in industry?

- **Emerging power generation technologies**
  - Wind turbines and photovoltaic cells (renewable technologies)
    - Full-grown technologies
    - No emission
    - Climate constraints: wind and sunshine
    - Efficiency: wind turbines (20 – 40%), photovoltaic cells (5 – 15%)
  - Fuel cells
    - Regardless of climate conditions
    - Hydrogen and oxygen
    - Products: electricity, heat, and water
    - Nearly zero emission
    - Efficiency: electricity (up to 60%), co-generation (up to 85%)
I. INTRODUCTION

Operating Principle of Fuel Cell Systems

- Operation of the fuel cells

![Diagram of Fuel Cell Generation System]

Fig. 1 Fuel cell generation system.
I. INTRODUCTION

Features of Fuel Cell Systems

- **Types of fuel cells**
  - Phosphoric Acid (PAFC), Solid Oxide (SOFC), Molten Carbonate (MCFC), Proton-Exchange-Membrane (PEMFC), Alkaline, Zinc-Air (ZA) fuel cells, etc.

- **Applications of fuel cells**
  - Residential (domestic utility), Transportation (Fuel cell vehicle), Portable power (laptop, cell phone), Stationary (buildings, hospitals, etc), Distributed power for remote location, etc.

- **Characteristics of fuel cells**
  - Environmental-friendly (nearly zero emission)
  - Modular electric generation
  - High efficiency (co-generation)
  - Slow dynamic response during initial start-up and load change
  - Boosted for most applications due to a low DC output voltage
  - Varying output voltage according to the load (current)
I. INTRODUCTION

Overview of Fuel Cell Systems (1)

- Fuel Cells for stationary or distributed power applications (1)

<table>
<thead>
<tr>
<th>Types</th>
<th>PAFC</th>
<th>SOFC</th>
<th>MCFC</th>
<th>PEMFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>100 – 200kW</td>
<td>1kW – 10MW</td>
<td>250kW – 10MW</td>
<td>3 – 250kW</td>
</tr>
<tr>
<td>Fuel</td>
<td>Natural gas, landfill gas, digester gas, propane</td>
<td>Natural gas, hydrogen, landfill gas, fuel oil</td>
<td>Natural gas, hydrogen, landfill gas, propane</td>
<td>Natural gas, hydrogen, propane, diesel</td>
</tr>
<tr>
<td>Operating Temperature</td>
<td>400°F</td>
<td>1,800°F</td>
<td>1,200°F</td>
<td>200°F</td>
</tr>
<tr>
<td>Installed Cost ($/kW)</td>
<td>3,000 – 3,500</td>
<td>1,300 – 2,000</td>
<td>800 – 2,000</td>
<td>4,000</td>
</tr>
<tr>
<td>Cooling Medium</td>
<td>Boiling Water</td>
<td>Excess Air</td>
<td>Excess Air</td>
<td>Water</td>
</tr>
<tr>
<td>Environmental – friendly (Nearly zero emission)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Commercial Availability</td>
<td>Yes</td>
<td>R&amp;D</td>
<td>R&amp;D</td>
<td>R&amp;D</td>
</tr>
</tbody>
</table>
### I. INTRODUCTION

**Overview of Fuel Cell Systems (2)**

- **Fuel Cells for stationary or distributed power applications (2)**

<table>
<thead>
<tr>
<th>Types</th>
<th>PAFC</th>
<th>SOFC</th>
<th>MCFC</th>
<th>PEMFC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cogeneration</strong></td>
<td>Yes (hot water)</td>
<td>Yes (hot water, LP or HP steam)</td>
<td>Yes (hot water, LP or HP steam)</td>
<td>Yes (80°C water)</td>
</tr>
<tr>
<td><strong>Efficiency (Electricity)</strong></td>
<td>36– 42%</td>
<td>45 – 60%</td>
<td>45 – 55%</td>
<td>30 – 40%</td>
</tr>
<tr>
<td><strong>Efficiency (Cogeneration)</strong></td>
<td>Up to 85%</td>
<td>Up to 85%</td>
<td>Up to 85%</td>
<td>Up to 85%</td>
</tr>
<tr>
<td><strong>Commercial Status</strong></td>
<td>Some commercially available</td>
<td>Likely commercialization 2004</td>
<td>Likely commercialization 2004</td>
<td>Likely commercialization 2003/2004</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

What is the research focus?

- **Research Focus**

  - Control of Power Electronic Interface System (i.e., Control of Power Converter)
    
    ⇒ Inexpensive, reliable, small-sized, and light-weighted

    - **Main factors:** size of L-C filter, number of power devices and sensors

    ⇒ Good performance:

    - Nearly zero steady-state voltage (RMS) error
    - Low Total Harmonic Distortion (THD)
    - Fast/no-overshoot current response
    - Good voltage regulation

⇒ Power generation applications (three-phase AC 208V (L-L) /60 Hz)
I. INTRODUCTION

Conventional Topology (1)

- **Conventional:** a DC-DC boost converter and a DC/AC inverter
  - DSP controller: two controllers (DC/DC and DC/AC power converters)
  - Power devices:
    - DC to DC boost converter: four power switches and four diodes
    - DC to AC inverter: six power switches
  - Sensors: DC input, DC output, and AC output

![Diagram of conventional system configuration](image)

*Fig. 2 Conventional system configuration.*
I. INTRODUCTION

Conventional Topology (2)

- Control Block Diagram

Fig. 3 Control block diagram of conventional system.
I. INTRODUCTION

Conventional Z-source Topology

- **Conventional Z-source converter:**
  - Impedance source (L-C) and a DC/AC inverter
  - Boosted by shoot-through zero vectors (both switches turned-on)
  - Open-loop control under only linear/heavy load
  - Dynamic load applications like motor
  - Fuel cell modeled by a DC voltage source (battery)

![Conventional Z-source converter diagram]

*Fig. 4 Conventional Z-source converter.*
I. INTRODUCTION

Proposed Z-source Topology

- Proposed Z-source converter:
  - Static load applications with fixed peak voltage/frequency (i.e., three-phase AC 208V and 60 Hz)
  - Dynamic response of fuel cell considered
  - System modeling/modified SVPWM implementation/closed-loop control system design
  - Good performance under both linear load and nonlinear load
  - Wide range of load, i.e., light load to a full load

Fig. 5 Proposed Z-source converter.
II. Circuit Analysis/System Modeling
/Space Vector PWM Implementation

Equivalent Circuit of the Fuel Cell (1)

- Slow Dynamics of Reformer and Stacks

- Voltage-current characteristic of a cell

![Diagram of fuel cell system and voltage-current characteristic]

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II. Circuit Analysis/System Modeling /Space Vector PWM Implementation

Equivalent Circuit of the Fuel Cell (2)

- **Selected Equivalent Circuit Model of the Fuel Cell**
  - Dynamic modeling of reformer and stacks: R-C circuit
  - Voltage-current characteristic of a cell: Region of Ohmic Polarization

![Equivalent Circuit of the Fuel Cell](image)

Fig. 6 Equivalent circuit of the fuel cell.
II. Circuit Analysis/System Modeling /Space Vector PWM Implementation

Z-source converter configuration (1)

- **Z-source converter:**
  - a fuel cell, a diode, L-C impedance, a DC/AC inverter, a DSP controller, L/C filter, and a load
  - **Diode:** to prevent a reverse current that can damage the fuel cell

![Diagram of Z-source converter configuration](image)

**Fig. 7 Total system configuration with Z-source converter.**

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Z-source converter configuration (2)

Fig. 8 Total system diagram of a fuel cell based Z-source converter.
II. Circuit Analysis/System Modeling
/Space Vector PWM Implementation

A. Circuit Analysis (1)

- **Two Operation Modes:**
  - Mode 1 (Fig. 9 (a))
    - **Shoot-through switching mode:** both switches in a leg are simultaneously turned-on
  - Mode 2 (Fig. 9 (b))
    - **Non-shoot-through switching mode:** basic space vectors \((V_0, V_1, V_2, V_3, V_4, V_5, V_6, V_7)\)

(a) In the shoot-through switching mode.  
(b) In the non-shoot-through switching mode.

**Fig. 9 Equivalent circuit of two operation modes.**
II. Circuit Analysis/System Modeling
/Space Vector PWM Implementation

A. Circuit Analysis (2)

- **Assumption:** $L_1 = L_2$, $C_1 = C_2$

  $$V_{C1} = V_{C2} \text{ and } v_{L1} = v_{L2}.$$  

- **Case 1:** one of shoot-through zero vectors (Fig. 9 (a))

  $$v_{L1} = V_{C1}, v_f = 2V_{C1}, \text{ and } v_i = 0.$$  

- **Case 2:** one of non-shoot-through switching vectors (Fig. 9 (b))

  loop 1: $v_{L1} = V_{in} - V_{C1}$ and loop 2: $v_{i} = V_{C1} - v_{L1} = 2V_{C1} - V_{in}$,

  where, $V_{in}$ is the output voltage of the fuel cell.

- **Average voltage of the inductors**

  $$v_{L1} = \frac{1}{T_z} \int_{0}^{T_z} v_{L1} \, dt = \frac{T_a \cdot V_{C1} + T_b \cdot (V_{in} - V_{C1})}{T_z} = 0 \Rightarrow V_{C1} = \frac{T_b}{T_b - T_a} \cdot V_{in} = \frac{1 - \frac{T_a}{T_z}}{1 - 2 \cdot \frac{T_a}{T_z}} \cdot V_{in}$$

  where, $T_z = T_a + T_b$, $T_z$: switching period, $T_a$: total duration of shoot-through zero vectors over $T_z$ and $T_b$: total duration of non-shoot-through switching vectors over $T_z$.  

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II. Circuit Analysis/System Modeling

/Space Vector PWM Implementation

A. Circuit Analysis (3)

- Average DC-link voltage:
  \[ V_i = \bar{v}_i = \frac{1}{T_z} \int_0^{T_z} v_i dt = T_a \cdot 0 + T_b \cdot (2V_{CL} - V_{in}) = \frac{T_b}{T_b - T_a} V_{in} = V_{CL} \]

- Peak DC-link voltage:
  \[ V_{p_{DC}} = 2V_{CL} - V_{in} = \frac{T_z}{T_b - T_a} V_{in} = K \cdot V_{in} \]

  where, \( K \) is called a boost factor, \( K = \frac{T_z}{T_b - T_a} = \frac{1}{1 - 2 \cdot \frac{T_a}{T_z}} \geq 1, \quad 0 \leq \frac{T_a}{T_z} < 0.5 \)

- Peak phase voltage of inverter output
  \[ V_{a_p} = M \cdot \frac{V_{p_{DC}}}{2} = M \cdot K \cdot \frac{V_{in}}{2} \quad \Rightarrow \text{Control Factors: } M \text{ and } K \]

  where, \( M \) denotes the modulation index, \( T_z = T_a + T_b \quad \Rightarrow \quad 1 = \frac{T_a}{T_z} + \frac{T_b}{T_z} (= M) \)

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II. Circuit Analysis/System Modeling
/Space Vector PWM Implementation

B. System Modeling (1)

- Simplified system circuit model

![Simplified system circuit model of Z-source converter.](image)

Note that a fuel cell, a diode (D), and impedance components (L₁,₂ and C₁,₂) are replaced with a DC-link source (V_{dc}). That is, V_{dc} is equal to V_{C₁} or V_{C₂}.

Fig. 10 Simplified system circuit model of Z-source converter.
II. Circuit Analysis/System Modeling /Space Vector PWM Implementation

B. System Modeling (2)

- **Current/voltage equations from the L-C filter (KVL and KCL)**

\[
\begin{align*}
\frac{dV_{LAB}}{dt} &= \frac{1}{3C_f}i_{AB} - \frac{1}{3C_f}(i_{LA} - i_{LB}) \\
\frac{dV_{LBC}}{dt} &= \frac{1}{3C_f}i_{BC} - \frac{1}{3C_f}(i_{LB} - i_{LC}) \\
\frac{dV_{LCA}}{dt} &= \frac{1}{3C_f}i_{CA} - \frac{1}{3C_f}(i_{LC} - i_{LA}) \\
\frac{di_{AB}}{dt} &= -\frac{1}{L_f}V_{LAB} + \frac{1}{L_f}V_{iAB} \\
\frac{di_{BC}}{dt} &= -\frac{1}{L_f}V_{LBC} + \frac{1}{L_f}V_{iBC} \\
\frac{di_{CA}}{dt} &= -\frac{1}{L_f}V_{LCA} + \frac{1}{L_f}V_{iCA}
\end{align*}
\]

\[
\frac{dV_L}{dt} = \frac{1}{3C_f}I_i - \frac{1}{3C_f}T_iI_L \\
\frac{dI_i}{dt} = -\frac{1}{L_f}V_L + \frac{1}{L_f}V_i
\]

where, state variables: \( V_L = [V_{LAB} V_{LBC} V_{LCA}]^T \) and \( I_i = [i_{iAB} i_{iBC} i_{iCA}]^T = [i_{iA} - i_{iB} i_{iB} - i_{iC} i_{iC} - i_{iA}]^T \),
control input (u): \( V_i = [V_{LAB} V_{iBC} V_{iCA}]^T \),
disturbance (d): \( I_L = [i_{iA} i_{iB} i_{iC}]^T \).

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II. Circuit Analysis/System Modeling
Space Vector PWM Implementation

B. System Modeling (3)

- Coordinate Transformation: three variables $\Rightarrow$ two variables

\[ \mathbf{f}_{dq0} = \mathbf{K}_s \mathbf{f}_{abc} \]

where, \( \mathbf{f}_{dq0} = [f_d, f_q, f_0]^T \), \( \mathbf{f}_{abc} = [f_a, f_b, f_c]^T \),
and \( f \) denotes either a voltage or a current variable.

\[ \mathbf{K}_s = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \]

Fig. 11 Relationship between “abc” reference frame and stationary “dq” reference frame.
II. Circuit Analysis/System Modeling
/Space Vector PWM Implementation

B. System Modeling (4)

- State equations in the stationary dq reference frame

\[
\frac{dV_{Ldq}}{dt} = \frac{1}{3Cf} I_{idq} - \frac{1}{3Cf} T_{idq} I_{Ldq} \\
\frac{dI_{idq}}{dt} = -\frac{1}{Lf} V_{Ldq} + \frac{1}{Lf} V_{idq} \\
\text{where, } T_{idq} = [K_s T_s K_s^{-1}]_{row, column, 1, 2} = \frac{3}{2} \begin{bmatrix} 1 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 1 \end{bmatrix}
\]

- Continuous-time state space equation of the given plant model

\[
\dot{X}(t) = AX(t) + Bu(t) + Ed(t)
\]

where,

\[
A = \begin{bmatrix}
0_{2 \times 2} & \frac{1}{3Cf} I_{2 \times 2} \\
-\frac{1}{Lf} I_{2 \times 2} & 0_{2 \times 2}
\end{bmatrix}_{4 \times 4}, \\
B = \begin{bmatrix}
0_{2 \times 2} \\
\frac{1}{Lf} I_{2 \times 2}
\end{bmatrix}_{4 \times 2}, \\
E = -\frac{1}{3Cf} T_{idq},
\]

\[
X = \begin{bmatrix}
V_{Ldq} \\
I_{idq}
\end{bmatrix}_{4 \times 1}, \\
u = \begin{bmatrix}
V_{idq}
\end{bmatrix}_{2 \times 1} = \begin{bmatrix}
V_{id} \\
V_{iq}
\end{bmatrix}, \\
d = \begin{bmatrix}
I_{Ldq}
\end{bmatrix}_{2 \times 1} = \begin{bmatrix}
i_{Ld} \\
i_{Lq}
\end{bmatrix}
\]

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**C. Space Vector PWM Implementation (1)**

**Conventional Space Vector PWM:**

\[
\begin{align*}
T_1 & = T_z \cdot a \cdot \frac{\sin(\pi / 3 - \alpha)}{\sin(\pi / 3)} \\
T_2 & = T_z \cdot a \cdot \frac{\sin(\alpha)}{\sin(\pi / 3)} \\
T_0 & = T_z - (T_1 + T_2)
\end{align*}
\]

\(a = \frac{\left| V_{\text{ref}} \right|}{\frac{2}{3} V_{\text{dc}}}\)

(\text{where, } 0 \leq \alpha \leq 60^\circ)

\[
\begin{align*}
T_0 & = \int_0^{T_z} V_{\text{ref}} \, dt = \int_0^{T_1} V_1 \, dt + \int_{T_1}^{T_1+T_2} V_2 \, dt + \int_{T_1+T_2}^{T_z} V_0 \, dt \\
& = T_1 \cdot V_1 + T_2 \cdot V_2
\end{align*}
\]

\[
T_z \cdot V_{\text{ref}} = (T_1 \cdot V_1 + T_2 \cdot V_2)
\]

\[
\Rightarrow T_z \cdot V_{\text{ref}} = \left[ \cos(\alpha) \right] = T_1 \cdot \frac{2}{3} \cdot V_{\text{dc}} \cdot \left[ 1 \right] + T_2 \cdot \frac{2}{3} \cdot V_{\text{dc}} \cdot \left[ \frac{\cos(\pi / 3)}{\sin(\pi / 3)} \right]
\]

Fig. 12 Basic space vectors and switching patterns.  Mechatronics Lab.
II. Circuit Analysis/System Modeling
/Space Vector PWM Implementation

C. Space Vector PWM Implementation (2)

- Modified Space Vector PWM Implementation

![Diagram](image)

(a) Sector 1 (0°~60°)

(b) Sector 2 (60°~120°)

*shoot-through zero vectors (T = T_a/3)

Fig. 13 Modified SVPWM implementation.
II. Circuit Analysis/System Modeling /Space Vector PWM Implementation

C. Space Vector PWM Implementation (3)

- Switching time calculation at each sector

<table>
<thead>
<tr>
<th>Sector</th>
<th>Conventional (Upper)</th>
<th>Z-source (Upper)</th>
<th>Conventional (Lower)</th>
<th>Z-source (Lower)</th>
</tr>
</thead>
</table>
| 1      | $S_1 = T_1 + T_2 + T_0 / 2$  
$S_3 = T_2 + T_0 / 2$  
$S_5 = T_0 / 2$ | $S_1 = T_1 + T_2 + T_0 / 2 + T$  
$S_3 = T_2 + T_0 / 2$  
$S_5 = T_0 / 2 - T$ | $S_2 = T_0 / 2$  
$S_6 = T_1 + T_0 / 2$  
$S_2 = T_1 + T_2 + T_0 / 2$ | $S_2 = T_0 / 2$  
$S_6 = T_1 + T_0 / 2 + T$  
$S_2 = T_1 + T_2 + T_0 / 2 + 2T$ |
| 2      | $S_1 = T_1 + T_0 / 2$  
$S_3 = T_1 + T_2 + T_0 / 2$  
$S_5 = T_0 / 2$ | $S_1 = T_1 + T_0 / 2$  
$S_3 = T_1 + T_2 + T_0 / 2 + T$  
$S_5 = T_0 / 2 - T$ | $S_2 = T_2 + T_0 / 2$  
$S_6 = T_0 / 2$  
$S_2 = T_1 + T_2 + T_0 / 2$ | $S_2 = T_2 + T_0 / 2 + T$  
$S_6 = T_0 / 2$  
$S_2 = T_1 + T_2 + T_0 / 2 + 2T$ |
| 3      | $S_1 = T_0 / 2$  
$S_3 = T_1 + T_2 + T_0 / 2$  
$S_5 = T_2 + T_0 / 2$ | $S_1 = T_0 / 2 - T$  
$S_3 = T_1 + T_2 + T_0 / 2 + T$  
$S_5 = T_0 / 2 - T$ | $S_2 = T_1 + T_2 + T_0 / 2$  
$S_6 = T_0 / 2$  
$S_2 = T_1 + T_0 / 2$ | $S_2 = T_1 + T_2 + T_0 / 2 + 2T$  
$S_6 = T_0 / 2$  
$S_2 = T_1 + T_0 / 2 + T$ |
| 4      | $S_1 = T_0 / 2$  
$S_3 = T_1 + T_0 / 2$  
$S_5 = T_1 + T_2 + T_0 / 2$ | $S_1 = T_0 / 2 - T$  
$S_3 = T_1 + T_0 / 2$  
$S_5 = T_1 + T_2 + T_0 / 2 + T$ | $S_2 = T_1 + T_2 + T_0 / 2$  
$S_6 = T_0 / 2$  
$S_2 = T_0 / 2$ | $S_2 = T_1 + T_2 + T_0 / 2 + 2T$  
$S_6 = T_0 / 2$  
$S_2 = T_0 / 2 + T$ |
| 5      | $S_1 = T_2 + T_0 / 2$  
$S_3 = T_0 / 2$  
$S_5 = T_1 + T_2 + T_0 / 2$ | $S_1 = T_2 + T_0 / 2$  
$S_3 = T_0 / 2 - T$  
$S_5 = T_1 + T_2 + T_0 / 2 + T$ | $S_2 = T_1 + T_0 / 2$  
$S_6 = T_1 + T_2 + T_0 / 2$  
$S_2 = T_0 / 2$ | $S_2 = T_1 + T_0 / 2 + T$  
$S_6 = T_1 + T_2 + T_0 / 2 + 2T$  
$S_2 = T_0 / 2$ |
| 6      | $S_1 = T_1 + T_2 + T_0 / 2$  
$S_3 = T_0 / 2$  
$S_5 = T_1 + T_0 / 2$ | $S_1 = T_1 + T_2 + T_0 / 2 + T$  
$S_3 = T_0 / 2 - T$  
$S_5 = T_1 + T_0 / 2$ | $S_2 = T_0 / 2$  
$S_6 = T_1 + T_2 + T_0 / 2$  
$S_2 = T_2 + T_0 / 2$ | $S_2 = T_0 / 2$  
$S_6 = T_1 + T_2 + T_0 / 2 + 2T$  
$S_2 = T_2 + T_0 / 2 + T$ |

Note that when the shoot-through duration (T) is equal to zero, the switching time of each power switch for the Z-source converter is exactly the same as that for the conventional one.
III. Control System Design

Entire Control-loop Structure

where, Three controllers
1. Discrete-time Optimal Voltage Controller
2. Discrete-time Sliding Mode Current Controller (DSMC)
3. Discrete-time PI DC-link Voltage Controller

One observer
1. Asymptotic Observer for estimation of load currents

Fig. 14 Total control system block diagram.
III. Control System Design

A. Discrete-time Sliding Mode Current Controller (1)

- Block diagram of Current Controller

Fig. 15 Discrete-time current controller using DSMC.
III. Control System Design

A. Discrete-time Sliding Mode Current Controller (2)

- Asymptotic observer design for estimation of load currents
  - State equation for asymptotic observer
    \[
    \frac{d\hat{\mathbf{v}}_{ldq}}{dt} = \frac{1}{3C_f} \mathbf{i}_{ldq} - \frac{1}{3C_f} \mathbf{T}_{ldq} \cdot L_1 (\hat{\mathbf{v}}_{ldq} - \mathbf{v}_{ldq}) \\
    \hat{\mathbf{i}}_{ldq} = L_1 (\hat{\mathbf{v}}_{ldq} - \mathbf{v}_{ldq})
    \]
  - Continuous-time state-space representation for asymptotic observer
    \[
    \dot{\hat{\mathbf{x}}}_a (t) = \mathbf{A}_a \hat{\mathbf{x}}_a (t) + \mathbf{A}_b \mathbf{x}_b (t) - \mathbf{A}_a \mathbf{x}_a (t) \\
    \dot{\mathbf{d}}(t) = \hat{\mathbf{i}}_{ldq} (t) = L_1 (\hat{\mathbf{v}}_{ldq} - \mathbf{v}_{ldq}) = L_1 (\hat{\mathbf{x}}_a - \mathbf{x}_a)
    \]
    where, \( \hat{\mathbf{x}}_a = [\hat{\mathbf{v}}_{ldq}] \), \( \mathbf{x}_b = [\mathbf{i}_{ldq}] \), \( \mathbf{x}_a = [\mathbf{v}_{ldq}] \), \( \mathbf{A}_a = \left[-\frac{L_1}{3C_f} \mathbf{T}_{ldq}\right] \), \( \mathbf{A}_b = \left[\frac{1}{3C_f} I_{2 \times 2}\right] \).
    \( L_1 \) = a constant observer gain.
III. Control System Design

A. Discrete-time Sliding Mode Current Controller (3)

- Discrete-time state-space model for asymptotic observer

\[
\begin{aligned}
\dot{\hat{X}}_a (k+1) &= A^*_a \hat{X}_a (k) + A^*_b X_b (k) - A^{**} a X_a (k) \\
\dot{\hat{d}}(k) &= \hat{i}_{Ldq} (k) = L_1 (\hat{X}_a (k) - X_a (k)) \\
\end{aligned}
\]

where,

\[
A^*_a = e^{A_a T_z}, \quad A^*_b = \int_0^{T_z} e^{A_a (T_z - \tau)} A_b d\tau, \quad A^{**} = \int_0^{T_z} e^{A_a (T_z - \tau)} A_a d\tau
\]

- Block diagram of asymptotic observer

![Block diagram of asymptotic observer](image)

Fig. 16 Block diagram of asymptotic observer.
III. Control System Design

A. Discrete-time Sliding Mode Current Controller (4)

- **Continuous-time state space equation**

\[
\dot{X}(t) = AX(t) + Bu(t) + Ed(t)
\]

\[
y_1(t) = C_1X(t)
\]

\[
e_{idq}(t) = y_1(t) - y_{ref}(t)
\]

where,

\[
A = \begin{bmatrix}
0_{2 \times 2} & \frac{1}{3C_f}I_{2 \times 2} \\
-\frac{1}{L_f}I_{2 \times 2} & 0_{2 \times 2}
\end{bmatrix}_{4 \times 4}, \quad B = \begin{bmatrix}
0_{2 \times 2} \\
\frac{1}{L_f}I_{2 \times 2}
\end{bmatrix}_{4 \times 2}, \quad E = \begin{bmatrix}
-\frac{1}{3C_f}T_{idq} \\
0_{2 \times 2}
\end{bmatrix}_{4 \times 2}, \quad X = \begin{bmatrix}
V_{ldq} \\
I_{idq}
\end{bmatrix}_{4 \times 1}
\]

\[
u = \begin{bmatrix}V_{idq}
V_{iq}
\end{bmatrix}_{2 \times 1}, \quad \hat{d} = \begin{bmatrix}\hat{i}_{ld} \\
\hat{i}_{iq}
\end{bmatrix}, \quad y_1 = \begin{bmatrix}I_{id} \\
I_{iq}
\end{bmatrix}, \quad C_1 = \begin{bmatrix}0 & 0 & 1 & 0
0 & 0 & 0 & 1
\end{bmatrix}
\]

- **Discrete-time state space equation**

\[
X(k + 1) = A^*X(k) + B^*u(k) + E^*\hat{d}(k)
\]

\[
y_1(k) = C_1X(k)
\]

\[
e_{idq}(k) = y_1(k) - y_{ref}(k)
\]

where,

\[
A^* = e^{AT_z}, \quad B^* = \int_0^{T_z} e^{A(T_z - \tau)}Bd\tau, \quad E^* = \int_0^{T_z} e^{A(T_z - \tau)}Ed\tau
\]
III. Control System Design

A. Discrete-time Sliding Mode Current Controller (5)

**Sliding-mode manifold**

\[ s(k) = y_1(k) - y_{1_{\text{ref}}}(k) = C_1 X(k) - y_{1_{\text{ref}}}(k) \]

**Equivalent control law**

\[ s(k + 1) = y_1(k + 1) - y_{1_{\text{ref}}}(k + 1) \]
\[ = C_1 A^* X(k) + C_1 B^* u(k) + C_1 E^* \dot{d}(k) - y_{1_{\text{ref}}}(k + 1) = 0 \]
\[ \therefore u_{eq}(k) = \left( C_1 B^* \right)^{-1} \left( I_{idq}^* (k) - C_1 A^* X(k) - C_1 E^* \dot{d}(k) \right) \]

**Control limit**

\[
 u(k) = \begin{cases} 
   u_{eq}(k) & \text{for } \|u_{eq}(k)\| \leq u_0 \\
   \frac{u_0}{\|u_{eq}(k)\|} u_{eq}(k) & \text{for } \|u_{eq}(k)\| > u_0 
\end{cases}
\]

where, \( u_0 = \frac{2}{\sqrt{3}} V_{dc} \)
III. Control System Design

B. Discrete-time Optimal Voltage Controller (1)

- Block diagram of Voltage Controller
  - Robust Servomechanism Controller (RSC)
    - Servo compensator: internal model principle
    - Stabilizing compensator: optimal control

Fig. 17 Discrete-time optimal voltage controller using RSC.
III. Control System Design

B. Discrete-time Optimal Voltage Controller (2)

- **Discrete-time state space plant model**

\[ X(k + 1) = A^*X(k) + B^*u(k) + E^*\hat{d}(k) \]

- **Augmented system model including DSMC**

\[
\begin{cases}
X(k + 1) = A_d X(k) + B_d u_1(k) + E_d \hat{d}(k) \\
y_d(k) = C_d X(k)
\end{cases}
\]

where,

\[ A_d = A^* - B^*(C_1B^*)^{-1}C_1A^* , \quad B_d = B^*(C_1B^*)^{-1} , \quad E_d = E^* - B^*(C_1B^*)^{-1}C_1E^* , \]

\[ C_d = [I_{2\times2} \ 0_{2\times2}] , \quad u_1(k) = I_{cmd, idq}(k) , \quad y_d = [V_{Ld} \ V_{Lq}] \]
III. Control System Design

B. Discrete-time Optimal Voltage Controller (3)

- Continuous-time servo-compensator
  - Sinusoidal Tracking/disturbance model

\[
\dot{\eta} = A_c \eta + B_c e_{vdq}, \quad e_{vdq} = V^*_{Ldq} - V_{Ldq}
\]

where, 
\[
A_c = \begin{bmatrix}
A_{c1} & 0_{4\times4} & 0_{4\times4} \\
0_{4\times4} & A_{c2} & 0_{4\times4} \\
0_{4\times4} & 0_{2\times2} & A_{c3}
\end{bmatrix}_{12\times12}, \quad B_c = \begin{bmatrix}
B_{c1} \\
B_{c2} \\
B_{c3}
\end{bmatrix}_{12\times2}
\]

\[
A_{ci} = \begin{bmatrix}
0_{2\times2} & I_{2\times2} \\
-\omega_i^2 \cdot I_{2\times2} & 0_{2\times2}
\end{bmatrix}_{4\times4}, \quad B_{ci} = \begin{bmatrix}
0_{2\times2} \\
I_{2\times2}
\end{bmatrix}_{4\times2}
\]

\(A_c\) is the given tracking/disturbance poles, \(\omega_i (i = 1, 2, 3), \omega_1 = \omega, \omega_2 = 5\cdot\omega, \omega_3 = 7\cdot\omega.\)

\(\omega = 2\pi f \text{ [rad/sec]}, f = 60 \text{ Hz}.\)

- Discrete-time servo-compensator

\[
\eta(k+1) = A^*_c \eta(k) + B^*_c e_{vdq}(k)
\]

where, 
\[
A^*_c = e^{A_c T_z}, \quad B^*_c = \int_0^{T_z} e^{A_c (T_z-\tau)} B_c d\tau
\]
III. Control System Design

B. Discrete-time Optimal Voltage Controller (4)

- Augmented system combining both the plant and the servo-compensator

\[
\dot{X}(k+1) = \hat{A}X(k) + \hat{B}u_1(k) + \hat{E}_1d(k) + \hat{E}_2y_{d\_\text{ref}}(k)
\]

where,

\[
\hat{X}(k) = \begin{bmatrix} X(k) \\ \eta(k) \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} A_d & 0 \\ -B_c^*C_d & A_c^* \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}, \quad \hat{E}_1 = \begin{bmatrix} E_d \\ 0 \end{bmatrix}, \quad \hat{E}_2 = \begin{bmatrix} 0 \\ B_c^* \end{bmatrix},
\]

\[
u_1(k) = I_{cmd,\text{idq}}(k), \quad \hat{d}(k) = I_{Ldq}(k), \quad y_{d\_\text{ref}}(k) = V_{Ldq}^*(k)
\]

- Linear quadratic performance index

\[
J_\varepsilon = \sum_{k=0}^{\infty} \hat{X}^T(k)Q\hat{X}(k) + \varepsilon u^T(k)u(k)
\]

where, \(Q\) is a symmetrical positive-definite matrix and \(\varepsilon>0\) is a small number

- Control input

\[
u_1(k) = -K\hat{X}(k) = -[K_1 \quad K_2] \begin{bmatrix} X(k) \\ \eta(k) \end{bmatrix} = -K_1X(k) - K_2\eta(k)
\]
III. Control System Design

C. Discrete-time PI DC-link Voltage Controller (1)

- Discrete PI DC-link voltage controller equation

\[
\begin{align*}
e(k) &= V^*_{c2}(k) - V_{c2}(k) \\
T(k) &= T(k-1) + K_p \cdot (e(k) - e(k-1)) + K_i \cdot \frac{T_z}{2} \cdot (e(k) + e(k-1))
\end{align*}
\]

- Block diagram of DC-link Voltage Controller

Fig. 18 Discrete-time PI controller to regulate the DC-link average voltage.
III. Control System Design

C. Discrete-time PI DC-link Voltage Controller (2)

- Duration ($T_{cal} = T_a/3$) of shoot-through zero vectors

Desired capacitor voltage: $V_{C1} = V_{C2} = 340$ V

$$V_{C2} = \frac{T_b}{T_b - T_a} V_{in} = \frac{1}{1 - 2 \cdot \frac{T_a}{T_z}} V_{in} \Rightarrow T_a = \frac{(V_{C2} - V_{in})}{(2V_{C2} - V_{in})} = \frac{(340 - V_{in})}{(680 - V_{in})} \cdot T_z$$

$$T_{cal} = \frac{T_a}{3} \Rightarrow \begin{cases} T_{a_{\text{max}}} = 0.3814 \cdot T_z, & \text{where, } V_{in} = 130 \text{ V} \\ T_{a_{\text{min}}} = 0.1050 \cdot T_z, & \text{where, } V_{in} = 300 \text{ V} \end{cases}$$
### IV. Simulation Results

**System Parameters for Simulations**

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</table>

where,

⇒ Heavy load (10 kW): the output voltage of fuel cell is 130 V
⇒ Light load (0.5 kW): that of fuel cell is 300 V
⇒ Linear load: a resistor and an inductor
⇒ Nonlinear load: an three-phase inductor (2 mH), a three-phase diode bridge, a DC-link capacitor (800 $\mu$F), and a resistor (7 $\Omega$)
⇒ Load change: 5 kW (250V) to 10 kW (130V), vice versa
IV. Simulation Results

A. Linear Load (R)

Fig. 19 Simulation waveforms under a linear load.

(a) 130V/10kW (Heavy load)  (b) 300V/0.5kW (Light load)

(1: Fuel cell output voltage ($V_{in}$) and capacitor voltage ($V_{C2}$),
2: line to line voltages ($V_{LAB}$, $V_{LBC}$, $V_{LCA}$),
3: load phase currents ($i_{LA}$, $i_{LB}$, $i_{LC}$).)
IV. Simulation Results

B. Nonlinear Load (Rectifier load)

Fig. 20 Simulation waveforms under a nonlinear load (130V/7Ω).

(1: Fuel cell output voltage ($V_{in}$) and capacitor voltage ($V_{C2}$),
  2: line to line voltages ($V_{LAB}$, $V_{LBC}$, $V_{LCA}$),
  3: load phase currents ($i_{LA}$, $i_{LB}$, $i_{LC}$).)

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IV. Simulation Results

C. Load Change

(a) Load increase

(b) Load decrease

Fig. 21 Simulation waveforms under a load change at 70msec.

(1: Fuel cell output voltage ($V_{in}$) and capacitor voltage ($V_{C2}$),
2: line to line voltages ($V_{LAB}$, $V_{LBC}$, $V_{LCA}$),
3: load phase currents ($i_{LA}$, $i_{LB}$, $i_{LC}$).)
IV. Simulation Results

d. Six gating signals and estimated load current

(a) Gating signals for six power switches.  (b) Estimated load currents under heavy load.

Fig. 22 Six gating signals and estimated load current.

(S1 and S4: phase A, S3 and S6: phase B, S5 and S2: phase C,
Upper switches: S1, S3, and S5,
Lower switches: S4, S6, and S2.)
VI. Conclusions

- L-C Impedance components instead of a DC to DC boost converter
- Inexpensive, reliable, small-sized, and light-weighted
- Slow dynamic response of fuel cell
- Discrete-time state-space system model
- Modified PWM Technique using Shoot-through zero vectors
- Feedback controller design under both linear and nonlinear load

- Good performance:
  - Nearly zero steady-state voltage (RMS) error
  - Low Total Harmonic Distortion (THD)
  - Fast/no-overshoot current response
  - Good voltage regulation