THEVENIN'S EQUIVALENT OF PHOTOVOLTAIC SOURCE MODELS [1]

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1 Abstract

The model of a photovoltaic (PV) source is needed for Maximum Power Point Tracking (MPPT) and power grid studies. The single diode model can fairly emulate PV's characteristic. The only nonlinear element in this model is the single diode. This report presents Thevenin's equivalent model for a PV source by piecewise linearization of the diode characteristic. The variation of the parameters with the change in temperature and irradiance is also studied. It is shown that Thevenin's equivalent model of PV produces a voltage-current characteristic which represents the PV source operation fairly well.

A PV source has a non-linear voltage-current (V-I) characteristic, which can be modeled using current sources, diode(s), and resistors. Single-diode and double-diode models are widely used to simulate PV characteristics. The single-diode model emulates the PV characteristics fairly accurately. The manufacturer provides information about the electrical characteristics of PV by specifying certain points in its V-I characteristics, which are called remarkable points [2].

This report uses the single-diode model to develop a Thevenin's equivalent model of PV. It first discusses the parameter estimation of a single-diode model for a given temperature and irradiance and then it discusses developing Thevenin's equivalent model by using those parameters.

The single diode model for PV consists of a current source representing the photo-generated current, a diode, and two resistances (series and parallel).

2 Linearization of Single Diode Model

The only non-linear element in the model is the diode. The voltage and current (V-I) in a diode are related by an exponential relationship as given by Shockley and is given in (1) [3]:

$$I_D = I_o \left\{ \exp\left(\frac{v_D}{n_s.V_t} \right) - 1 \right\} \tag{1}$$

where V_D and I_D are the diode voltage and current, respectively.

The piecewise linearization is used to linearize the diode as shown in Figure 1. In this technique, the function is divided into a number of small regions. In each region, a straight line is used to closely approximate the actual nonlinear function, as shown in Figure 1. It is assumed that the non-linear function can be approximated by the straight line in that region.

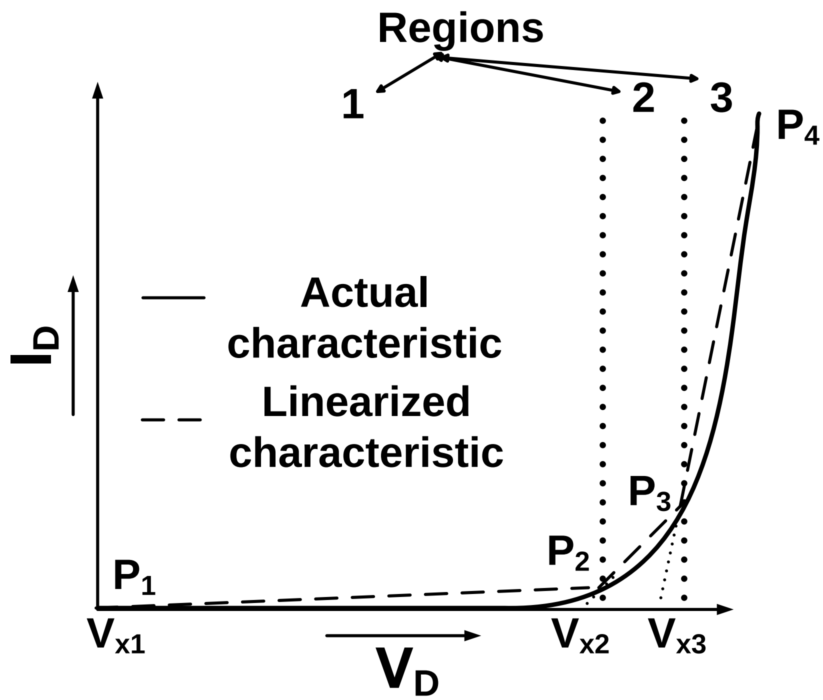

Figure 1. Voltage current characteristic of diode showing actual characteristic and the linear approximation [3].

In Figure 1, the diode characteristic has been approximated by dividing its characteristic into three regions, in each of which a diode is represented by a straight line. In terms of circuit, each of these lines can be approximated by a voltage source V_x and a resistance R_D. The voltage source V_x is actually the voltage axis intercept of the straight lines represented by V_{x1}, V_{x2}, and V_{x3} in Figure 1for regions 1, 2, and 3 respectively. The resistance R_{D1}, R_{D2}, and R_{D3} are the inverse of the slope of the lines in each region. It goes without saying that as the number of regions is increased, the piecewise linearization approximates the actual diode characteristic more closely, decreasing the error caused by the approximation.

The PV model, where the diode is linearized, is shown in Figure 2. The value of R_D and V_x will depend on the region of operation.

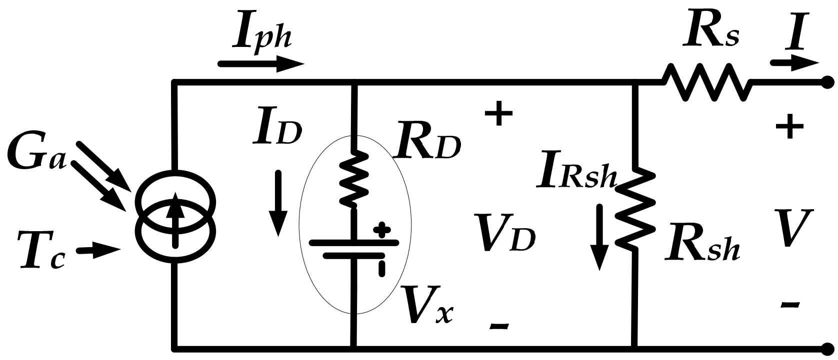

Figure 2. PV model with diode linearized

The linearized model of Figure 2 can now be represented by Thevenin's equivalent voltage and resistance given by (2) and (3):

$$V_{Th,i} = V_{x,i} + R_{D,i} \cdot \frac{I_{ph}.R_{sh} - V_{x,i}}{R_{sh} + R_{D,i}} \tag{2}$$

$$R_{Th,i} = R_s + \frac{R_{sh}.R_{D,i}}{R_{sh} + R_{D,i}} \tag{3}$$

where, $V_{Th,i}$ and $R_{Th,i}$ are Thevenin's equivalent voltage and resistance of the model of Figure 2 at region i (*i = 1, 2, ...number of regions*). Thevenin's equivalent circuit of the PV looking back from its output terminals is shown in Figure 3.

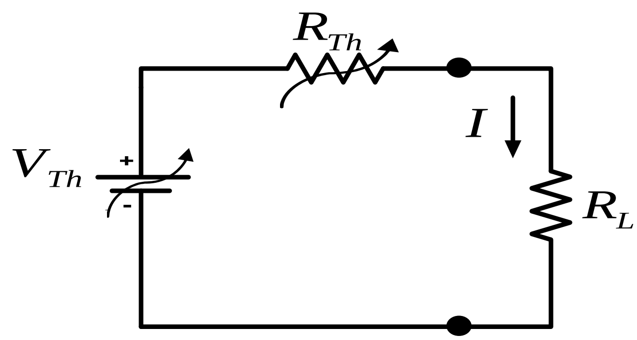

Figure 3. Thevenin's equivalent circuit of PV

The piecewise linearization is an approximation technique, which has inherent error at points which do not coincide with the actual function. It is only those points given by the boundaries of the regions where the approximation exactly matches with the original function. Hence, at the boundaries of the regions, the error is zero.

The PV source is controlled to operate at maximum power point. Hence, it is desired that the approximation is error free at the point of operation. Therefore, one of the boundaries is chosen at MPP. Other boundaries can be either distributed evenly over the voltage range of operation or chosen, such that the error is minimal.

3 Voltage-Current Characteristic of PV

Once the five parameters of the PV are determined, the diode characteristic defined by A and I_o is also known. From the maximum power point of PV, the corresponding point on the diode voltage-current characteristic is also determined. One of the points for linearization is chosen at this point on the diode characteristic, so that when maximum power point tracking (MPPT) is applied, PV operates on the exact MPP instead of the approximated point. The other points are chosen on either side of the PV characteristics. More points are chosen on the right side of the PV characteristics as it is more curved and lesser on the left, as the characteristic is almost linear.

Since the relationship between the output voltage and current of PV is implicit, a numerical solution is needed to obtain the value of current at a given voltage. Several computational and simulation methods of voltage current characteristics are found in the literature [4-7]. But with the linearization of the diode characteristic and obtaining Thevenin's equivalent circuit of PV, the numerical solution is avoided.

4 Simulation Results

Thevenin's equivalent model of PV is simulated at various temperatures and irradiances to see the effect of linearizing the diode characteristic of PV. The error in the approximation is plotted to show its accuracy. Here, the linearization has been done with ten points of linearization between zero and open circuit voltage. This means that there are nine regions with different Thevenin's voltages and resistances.

Figure 4 shows the voltage-current and voltage-power characteristic of PV for both the actual case and Thevenin's equivalent at STC along with the percent error in the plot for Thevenin's equivalent model. Table 1 lists the Thevenin's equivalent voltage and resistance at STC and for different values of currents.

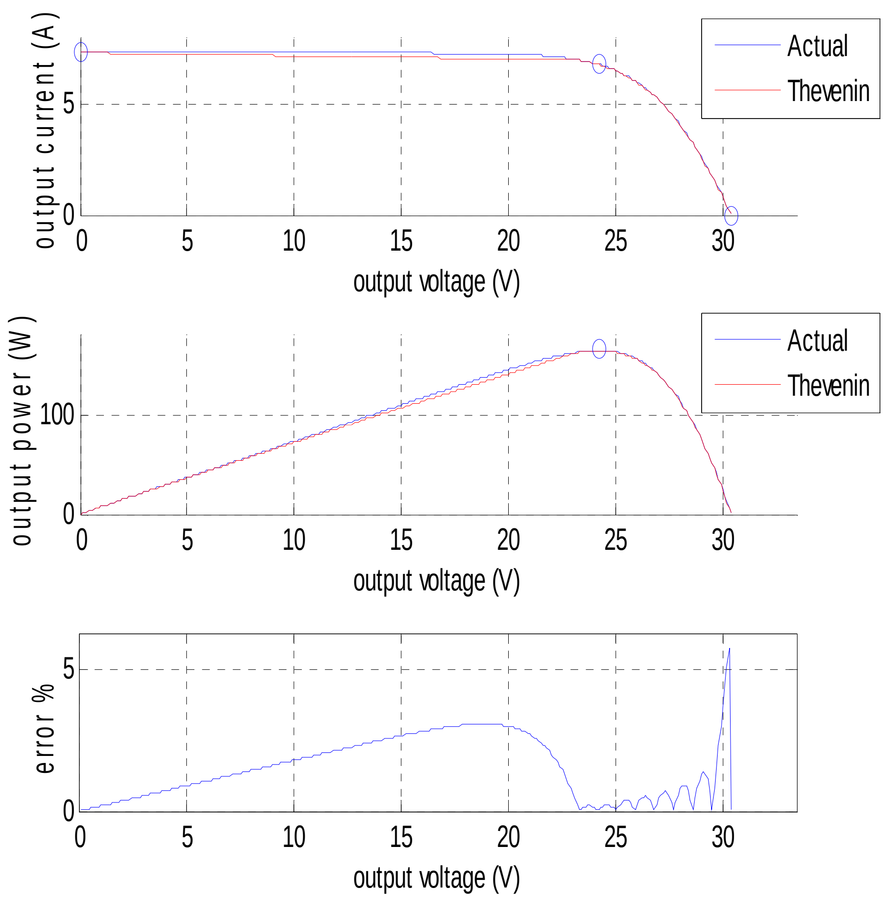

Figure 4. Plot of Thevenin's equivalent circuit in comparison with the actual response at STC (25° C and 1000 Wm^{-2})

$T = 25° C, G = 1000 Wm^{-2}$		
Output current, A	Thevenin's voltage, V	Thevenin's resistance, Ω
0 to 0.9624	*42.1000*	*1.2469*
0.9624 to 1.7514	*42.3638*	*1.5210*
1.7514 to 2.3709	*43.0926*	*1.9371*
2.3709 to 2.8361	*44.6160*	*2.5797*
2.8361 to 3.1709	*47.4633*	*3.5836*
3.1709 to 3.4033	*52.4730*	*5.1635*
3.4033 to 3.5600	*60.9636*	*7.6583*
3.5600 to 3.6634	*74.9968*	*11.6002*
3.6634 to 3.8700	*608.9235*	*157.3446*

Table 1. Thevenin's voltage and resistance for 25° C and 1000 W/m^2

Figure 5 shows the voltage-current and voltage-power characteristic of PV for both the actual case and Thevenin's equivalent at 25° C and 200 Wm^{-2} along with the percent error in the plot for Thevenin's equivalent model. Table 2 lists the Thevenin's equivalent voltage and resistance at the above conditions and for different values of currents.

Figure 6 shows the voltage-current and voltage-power characteristic of PV for both the actual case and Thevenin's equivalent at 100° C and 1000 Wm^{-2}, along with the percent error in the plot for Thevenin's equivalent model. Table 3 lists the Thevenin's equivalent voltage and resistance at the mentioned conditions and for different values of currents.

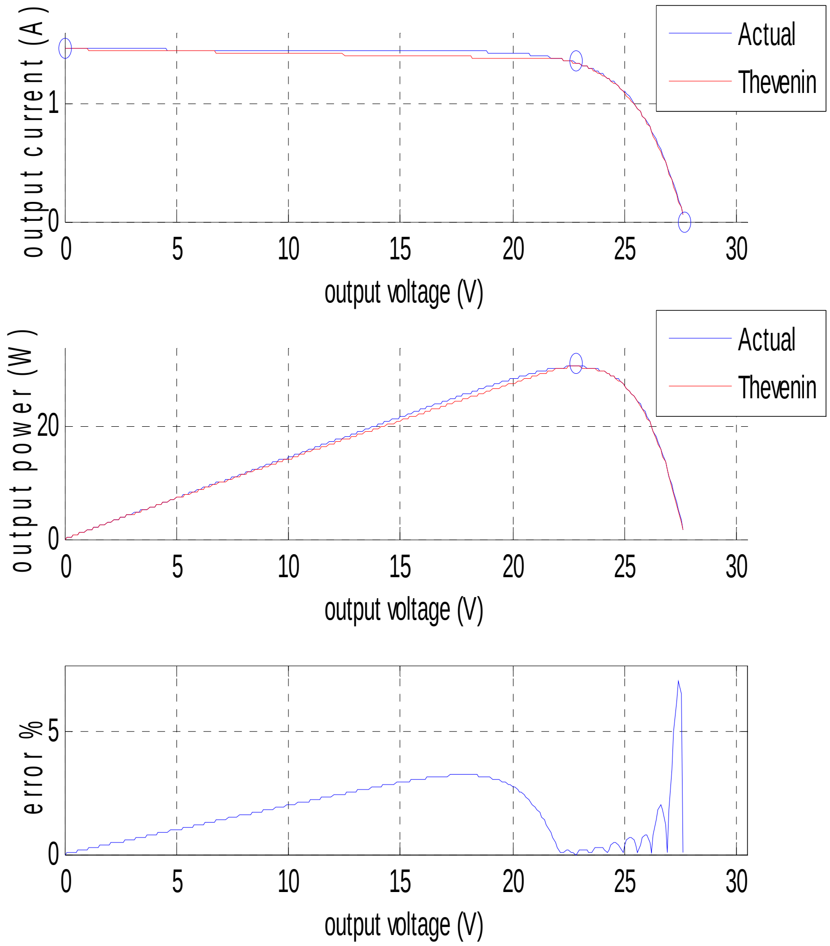

Figure 5. Plot of Thevenin's equivalent circuit in comparison with the actual response at 25° C and 200 Wm^{-2}

T = 25° C, G = 200 Wm^{-2}		
Output current, A	Thevenin's voltage, V	Thevenin's resistance, Ω
0 to 0.2320	*37.8812*	*4.6605*
0.2320 to 0.3879	*38.2880*	*6.4143*
0.3879 to 0.4896	*39.2314*	*8.8464*
0.4896 to 0.5708	*40.9328*	*12.3213*
0.5708 to 0.6275	*43.9640*	*17.6319*
0.6275 to 0.6669	*48.8218*	*25.3733*
0.6669 to 0.6942	*56.3153*	*36.6094*
0.6942 to 0.7132	*67.5644*	*52.8133*
0.7132 to 0.7740	*380.3989*	*491.4715*

Table 2. Thevenin's voltage and resistance for 25° C and 200 W/m^2

It is seen from the plots shown in Figure 4 through Figure 6 that Thevenin's equivalent model closely approximates the response of the single-diode model. The error is zero at the points of linearization. Since MPP is one of the points of linearization, the error is also zero at that point. PV is controlled to operate at MPP; therefore, Thevenin's model leads to an operation at the exact MPP. The error between two points of linearization increases and reaches a maximum of 7% for 25° C and 200 Wm^{-2} at a point close to open circuit. This is a point where PV will seldom operate. The error near the MPP where PV is most likely to operate with MPP is negligible. Table 1 through Table 3 tabulate Thevenin's equivalent voltage and impedance for different environmental conditions and as a function of output current.

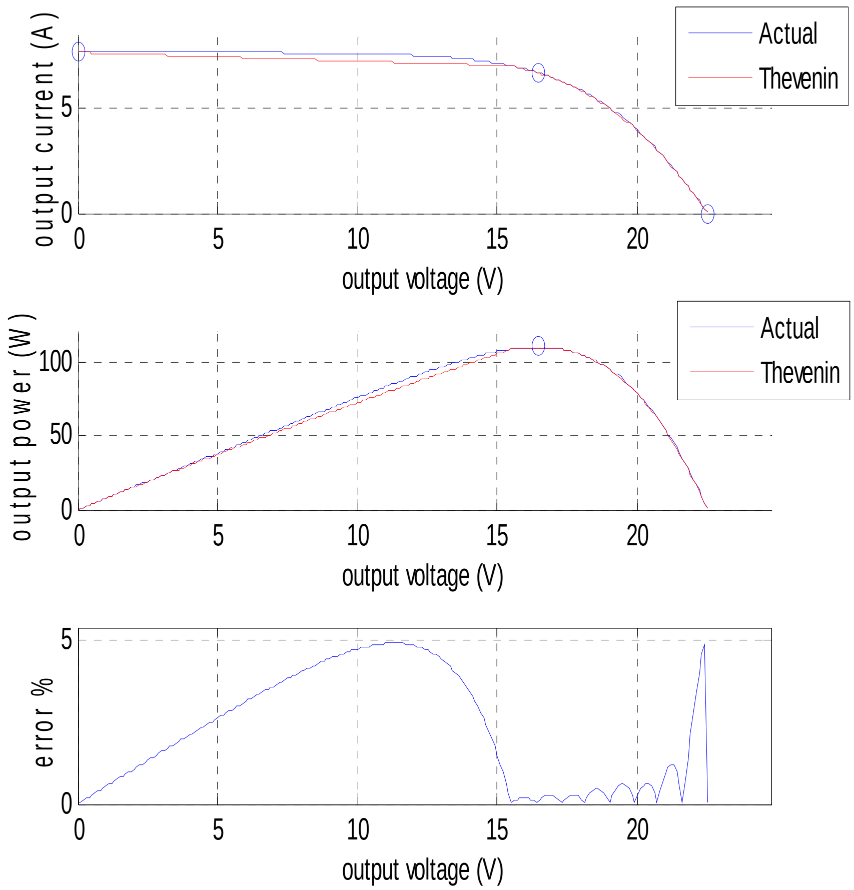

Figure 6. Plot of Thevenin's equivalent circuit in comparison with the actual response at 100° C and 1000 Wm^{-2}

T = 100° C, G = 1000 Wm^{-2}		
Output current, A	Thevenin's voltage, V	Thevenin's resistance, Ω
0 to 0.9388	*36.1001*	*1.3849*
0.9388 to 0.7151	*36.3720*	*1.6745*
0.7151 to 2.2936	*37.0576*	*2.0743*
2.2936 to 2.7843	*38.3764*	*2.6493*
2.7843 to 3.1286	*40.7044*	*3.4854*
3.1286 to 3.4051	*44.5086*	*4.7013*
3.4051 to 3.5902	*50.5825*	*6.4851*
3.5902 to 3.7332	*59.9197*	*9.0851*
3.7332 to 4.0587	*324.3096*	*79.9064*

Table 3. Thevenin's voltage and resistance for 100° C and 1000 W/m^2

The accuracy of the Thevenin's equivalent model is depends on the number of points chosen for linearization. The error in the Thevenin's equivalent model will decrease with an increase in the number points. There are several algorithms which can be employed to determine where exactly the points should be located. One of them locates the points based on the equal area of error. This means that the area under the error curve for all the regions should be the same. This ensures a good accuracy throughout the curve. However, this algorithm comes with higher degree of complication and requires iterative method of solving for the area of the error. Moreover, if this method is employed, the MPP may not be one of the points of linearization where the error is zero. Since the PV source is mostly operated at MPP, the error at that point is desired to be zero. In the simulation shown here, the points are uniformly distributed on the voltage axis, between the MPP and the open circuit voltage. Most points are selected on the right side of the MPP because the curve is more nonlinear in this region. Only one point is selected between MPP and zero voltage. From Figure 4 through Figure 6, it is seen that the error is mostly small on the left hand side even when there is only one point to the left of the MPP. Whereas, the error is relatively high for the regions to the right of MPP. If the PV is not made to operate at the MPP, it is desired that it operates at a point to the right of it so that the voltage regulation of the PV is good. Therefore, more points for linearization should be placed on the right side to reduce the error in linearization.

5 Summary

Thevenin's equivalent model derived from the single-diode model closely approximates PV characteristics as seen from the simulation results. This report presents to piecewise linearize the diode characteristic to develop Thevenin's equivalent circuit of a PV source. The simulation results show that the error in Thevenin's equivalent model is negligible at the maximum power point, a point where PV is controlled to operate at a given irradiance and temperature. From the simulation, it is concluded that the developed Thevenin's equivalent model can be used for simulation to study the behavior of PV. The advantage of using this model is that it avoids a numeric solution of the V-I characteristics of PV.

References

[1] A. Chatterjee, A. Keyhani, “Thevenin’s Equivalent of Photovoltaic Source Models for MPPT and Power Grid Studies,” *IEEE Power & Energy Society General Meeting*, pp. 1 – 7, 24-29 July, 2011, Detroit, Michigan

[2] M.G. Villalva, J.R. Gazoli, and E. R. Filho, “Comprehensive Approach to Modeling and Simulation of Photovoltaic Arrays,” *IEEE Trans. Power Electronics*, vol. 24, no. 5, pp. 1198 – 1208, May 2009.

[3] Ray-Lee Lin, and Yi-Fan Chen “Equivalent Circuit Model of Light-Emitting-Diode for System Analyses of Lighting Drivers,” *Industry Applications Society Annual Meeting 2009*, pp. 1 – 5

[4] R. C. Campbell, “A Circuit-based Photovoltaic Array Model for Power System Studies,” *IEEE Power Symposium 2007*, pp. 97 – 101

[5] M. Veerachary, “PSIM circuit-oriented simulator model for the nonlinear photovoltaic sources,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 42, no. 2, pp. 735 – 740, Apr. 2006.

[6] M. C. Glass, “Improved solar array power point model with SPICE realization,” in *Proc. 31st Intersoc. Energy Convers. Eng. Conf. (IECEC),* Aug. 1996, vol. 1, pp. 286 – 291.

[7] I. H. Atlas and A. M. Sharaf, “A photovoltaic array simulation model for matlab-simulink GUI environment,” in *Proc. Int. Conf. Clean Elect. Power (ICCEP)*, 2007, pp. 341 – 345

APPENDIX

```
%% Linearized diode model
function [V
I]=PV_completeVI_Lin_Plot(Isc,Voc,Impp,Vmpp,ns,T,Iph,Io,A,Rs,Rsh)
%% Parameters
points = 10;         % number of pieces
q = 1.60217646e-19;  % charge of electron
k = 1.3806503e-23;   % Boltzmann's constant
Vt = A*k*T/q;
tolerance = 1e-3;

V = 0:0.1:Voc;
I = zeros(size(V));
iteration = 0;
for i = 1:length(V) % I-V
    %mismatch_plot = I(i) - Iph + Io*(exp((V(i)+I(i)*Rs)/ns/Vt)-1) +
(V(i)+I(i)*Rs)/Rsh;
    for iteration =1:1000 % Newton Raphson
        J = 1 + Io*Rs/ns/Vt*exp((V(i)+I(i)*Rs)/ns/Vt) + Rs/Rsh;
        mismatch_plot = I(i) - Iph + Io*(exp((V(i)+I(i)*Rs)/ns/Vt)-1) +
(V(i)+I(i)*Rs)/Rsh;
        del_I = - mismatch_plot/J;
        I(i) = I(i) + del_I;
        if abs(mismatch_plot/I(i)) < tolerance
            break
        end
    end % end of Newton Raphson
    if iteration > 999
        fprintf('\n**PLOT DID NOT CONVERGE AFTER 1000 ITERATIONS**\n')
        return
    end

% Loacting points to linearize

    for j = -1:points-4
        if V(i) <= Vmpp + j*(Voc - Vmpp)/(points - 3);
            p(j+2) =i;
        end
    end


end % end of I-V

%% Linearization

x = [1 p length(V)];       % Points of linearization

for i = 1 : points-1
```

```
    V_d1(i) = V(x(i)) + I(x(i))*Rs;
    V_d2(i) = V(x(i+1)) + I(x(i+1))*Rs;
    I_d1(i) = Io*(exp(V_d1(i)/ns/Vt)-1);
    I_d2(i) = Io*(exp(V_d2(i)/ns/Vt)-1);
    Rd(i) = (V_d2(i) - V_d1(i))/(I_d2(i) - I_d1(i));
    V_d_x(i) = V_d1(i) - I_d1(i)*Rd(i);    % x-axis (Vd) intercept of
diode I-V
end


Vph = Iph * Rsh;

for i = 1:length(V)
    for j = 1:points-1 % point 1 to last but one point
        if (V(i) >= V(x(j)) && V(i) <= V(x(j+1)))
            %next_point = 0;
            rd = Rd(j);
            Vd_x = V_d_x(j);

            Vth = Vd_x + (Vph - Vd_x)/(Rsh + rd)*rd;
            Rth = Rs + Rsh*rd/(Rsh + rd);
            Il = (Vth - V(i))/Rth;
            %{
            Vd = V(i) + Il*Rs;
            if Vd > V(x(j+1))
                next_point = 1;
            end
            %}
        end
    end

I_lin(i) = Il;

end

rel_error = 100*(I - I_lin)./I;


P = V.*I;       % power calculation
P_lin = V.*I_lin;

Pmpp = Vmpp*Impp;
subplot(3,1,1)
hold on;
plot(V,I)
plot(V,I_lin,'r')
legend('Actual','Thevenin')
plot([0 Voc Vmpp], [Isc 0 Impp],'o');
grid on;
xlabel('output voltage (V)')
ylabel('output current (A)')
title('PV I-V characteristic plot')
```

```
axis([0 1.1*Voc 0 1.1*Isc])

subplot(3,1,2)
hold on;
plot(V,P)
plot(V,P_lin,'r')
legend('Actual','Thevenin')
plot(Vmpp,Pmpp,'o')
grid on;
xlabel('output voltage (V)')
ylabel('output power (W)')
title('PV P-V characteristic')
axis([0 1.1*Voc 0 1.1*Pmpp])

subplot(3,1,3)
plot(V,abs(rel_error))
grid on;
xlabel('output voltage (V)')
ylabel('error %')
title('absolute relative error in linearization')
axis([0 1.1*Voc 0 1.1*norm(rel_error,'inf')])

end
```

```
%% PV parameters at different environmental condition
function [Iph_ Io_ Isc_ Voc_ Vmpp_ Impp_] =
PV_env_change(Isc,Voc,Vmpp,Impp,ns,Tstc,Gstc,Ki,Kv,Kp,G,T,A,Rs,Rsh);
%% Constants
q = 1.60217646e-19;  % charge of electron
k = 1.3806503e-23;   % Boltzmann constant
w = 0.04;
%% Evaluation at changing environmental condition
Vt = A*k*T/q;
```

```
del_T = T - Tstc;
Isc_ = Isc + Ki*del_T;
Voc_ = Voc + Kv*del_T;

Io_ = (Isc_ - (Voc_ - Isc_*Rs)/Rsh)*exp(-Voc_/ns/Vt);
Iph_ = Io_*exp(Voc_/ns/Vt) + Voc_/Rsh;

Iph_ = Iph_*G/Gstc;
Isc_ = Isc_*G/Gstc;

%% Voc
for iteration = 1:10000
    Voc_old = Voc_;
    Voc_ = ns*Vt*log((Iph_*Rsh-Voc_)/Io_/Rsh);
    error = Voc_old - Voc_;
    if abs(error)/Voc_ < 1e-6
        break
    end
end

%% Vmpp & Impp
%Initialization
Vmpp_ = Voc_/Voc*Vmpp;
Impp_ = Isc_/Isc*Impp;

for iteration = 1:10000
    Vmpp_ = (1-w)*Vmpp_+w*(Voc_ - Impp_*Rs + ns*Vt*log(((Isc_-
Impp_)*(Rs+Rsh)-Vmpp_)/(Isc_*(Rs+Rsh)-Voc_)));
    Impp_ = (1-w)*Impp_+w*(Vmpp_*(1/Rsh + (Isc_*Rsh-
Voc_+Isc_*Rs)*exp((Vmpp_+Impp_*Rs-Voc_)/ns/Vt)/ns/Vt/Rsh)/(1+Rs/Rsh +
Rs*(Isc_*Rsh-Voc_+Isc_*Rs)*exp((Vmpp_+Impp_*Rs-
Voc_)/ns/Vt)/ns/Vt/Rsh));

    parameter_new = [Impp_; Vmpp_];
    if iteration ~= 1
        mismatch = abs((parameter_new-parameter_old)./parameter_new);
        if (norm(mismatch,'inf') < 1e-6)
            break;
        end
    end

    parameter_old = [Impp_; Vmpp_];
end
end

%% Parameter estimation  of PV modules
function [Iph, Io, A, Rs, Rsh, iteration] =
PV_parameter_calculation(Isc, Voc, Impp, Vmpp, ns, T, Rs, Rsh,
tolerance)
%% Data sheet values
a_data = [Isc; Voc; Impp; Vmpp];
```

```matlab
%% Constants
q = 1.60217646e-19;  % charge of electron
k = 1.3806503e-23;   % Boltzmann constant

%% Gauss-Seidel Iteration
iteration_max = 200000;
for iteration = 1:iteration_max
    Vt = (Vmpp + Impp*Rs - Voc)/ns/...
        log((Impp*Rsh - Isc*Rsh + Vmpp + Impp*Rs - Isc*Rs)/(Voc -
Isc*Rs - Isc*Rsh));
    Rs = (Voc - Vmpp +ns*Vt*log((Vmpp*ns*Vt -
Impp*ns*Vt*(Rs+Rsh))/(Impp*Rs-Vmpp)/(Isc*Rsh -Voc +Isc*Rs)))/Impp;
    Rsh = (ns*Vt*Rsh + Rs*(Isc*Rsh - Voc + Isc*Rs)*exp((Isc*Rs-
Voc)/ns/Vt) + ns*Vt*Rs)/...
        (ns*Vt + (Isc*Rsh - Voc + Isc*Rs)*exp((Isc*Rs-Voc)/ns/Vt));
    parameter_new = [Vt; Rs; Rsh];
    if iteration ~= 1
        mismatch = abs((parameter_new-parameter_old)./parameter_new);
        if (norm(mismatch,'inf') < tolerance)
            break;
        end
    end

    parameter_old = [Vt; Rs; Rsh];

end

%% Calculating the remaining values
Io = (Isc*Rsh - Voc +Rs*Isc)/Rsh*exp(-Voc/ns/Vt);
Iph = Io*exp(Voc/ns/Vt) + Voc/Rsh;
A = q*Vt/k/T;               % quality factor of diode
Isc_cal = Io*exp(Voc/ns/Vt) + (Voc - Rs*Isc)/Rsh;
Voc_cal = Rsh*(Iph - Io*exp(Voc/ns/Vt));
Impp_cal = (Rsh*(Iph - Io*exp((Vmpp + Impp*Rs)/ns/Vt)) - Vmpp)/(Rs +
Rsh);
Vmpp_cal = Rsh*(Iph - Io*exp((Vmpp + Impp*Rs)/ns/Vt) - Impp) - Impp*Rs;

a_cal = [Isc_cal; Voc_cal; Impp_cal; Vmpp_cal];
mismatch = a_data - a_cal;

if iteration >= iteration_max
    fprintf('\n**PARAMETERS DID NOT CONVERGE AFTER %d ITERATIONS**\n',
iteration_max);
end
%% PV surce Thevenin Equivalent plot
% main file
% needs Data_XXX file, PV_env_change.m, PV_parameter_calculation.m,
PV_completeVI_Lin_Plot.m


clear all; clc;
plott = 1;      % 1 = make plots
```

```
array = 1;      % 0 = for module, 1 = for array

%% Environmental Conditions
T = 75;         % Junction temperature in degree C
T = 273 + T;    % Temperature converted to kelvin
G = 1000;       % Irradiance in W/m^2
%% Datasheet values
%Data_BPMSX120
Data_PVMF165EB3
tolerance = 1e-6;
%% Array sepcifications
Nss = 10;        % # of modules per string
Npp = 3;        % # of strings per array
%% Array value adjustment
if array
    Isc = Isc*Npp;
    Ki = Ki*Npp;
    Voc = Voc*Nss;
    Kv = Kv*Nss;
    Impp = Impp*Npp;
    Vmpp = Vmpp*Nss;
    ns = ns*Nss;
end
%% Initialization
Rs = 0;
Rsh = 1000;

%% Calculate parameters
[Iph, Io, A, Rs, Rsh, iteration] = PV_parameter_calculation(Isc, Voc,
Impp, Vmpp, ns, Tstc, Rs, Rsh, tolerance);

%% Parameters adjusted for change in environment
[Iph Io Isc Voc Vmpp Impp] =
PV_env_change(Isc,Voc,Vmpp,Impp,ns,Tstc,Gstc,Ki,Kv,Kp,G,T,A,Rs,Rsh);

%% Output the result
fprintf('PARAMETERS OF PV source at %2.2f C and %2.2f W/m^2\n', T-273,
G);
fprintf('\nOpen circuit voltage, Voc = %f V\n', Voc);
fprintf('Short circuit current, Isc = %f A\n', Isc);
fprintf('Voltage at MPP, Vmpp = %f V\n', Vmpp);
fprintf('Current at MPP, Impp = %f A\n', Impp);
fprintf('Photo generated current, Iph = %f A\n', Iph);
fprintf('Dark saturation current, Io = %f uA\n', Io*1e6);
fprintf('Diode quality factor, A = %f\n', A);
fprintf('Panel series resistance, Rs = %f ohm\n', Rs);
fprintf('Panel parallel resistance, Rsh = %f ohm\n', Rsh);
fprintf('Number of iterations = %d\n', iteration);
if(plott)
    [V I] =
PV_completeVI_Lin_Plot(Isc,Voc,Impp,Vmpp,ns,T,Iph,Io,A,Rs,Rsh);
end
```