

# Neural Network Estimation of Microgrid Maximum Solar Power

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**Abstract** — The integration of photovoltaic (PV) generating stations in the power grids requires the amount of power available from the PV to be estimated for power systems planning on yearly basis and operation control on daily basis. This paper proposes a neural network (NN) to estimate the optimal tilt angle at a given location and thus an estimate of the amount of energy available from the PV stations.

To utilize maximum solar energy from the sun, optimum tilt angle of PV panels must be estimated. The tilt angle can be adjusted by using costly hardware based servo systems. To avoid cost and complexity of servo system, the PV panels should be installed at optimum tilt angle which needs be adjusted during the year.

The energy received from the sun at a particular location depends on factors like the latitude of the location, the time of the year, the weather (clarity of the atmosphere) and the ground reflectivity. The irradiation which is the measure of amount of solar energy reaching a particular surface of unit area also depends on the angle at which the radiation is incident on the surface. Therefore, the angle which the PV panels make with the horizontal earth's surface is a major deciding factor. By varying this angle over time, maximum amount of solar energy can be tapped. For optimum solar energy generation, the tilt angle must be adjusted to its optimal value during the year. To estimate the optimal tilt angle, the geographical and meteorological data from the location must be known.

## I. INTRODUCTION

PHOTOVOLTAIC (PV) energy is obtained from the conversion of light energy absorbed by series and parallel connected PV cells to electrical energy [1-2]. To receive the maximum energy from the sun, the panels of PV microgrid must be able to receive the rays from the sun at 90 degrees. For power system planning, the output power of PV stations are needed for power flow analysis and grid design. For operation control, the PV station output power must be forecasted for daily power grid scheduling. The scheduling problem of smart power grid, where millions of PV stations are installed in the form of distributed generation, need forecasted output power of PV stations.

Although, for operation control, the measurement system for irradiance can be installed, however, for microgrid of distributed generation system they are costly. The costly servo tracking can be avoided, by determining the expected irradiance on a seasonal basis to rotate the PV panels to track the sun. To reduce cost, a mapping model of the optimum tilt angle of PV panels of microgrid can be computed, as shown in (1) by mapping it as a function of irradiation,  $G$ , latitude,  $\phi$ , and reflectivity,  $\rho$ , so that microgrid receives the maximum energy from the sun over a year, on daily, weekly, monthly and seasonal basis. This mapping model can be programed in controller of inverter for operation control.

$$\beta^{\text{opt}} = f(G, \phi, \rho) \quad (1)$$

In this paper, the estimation of the amount of irradiation energy received from the sun on the PV panel, calculated based on the optimal tilt angle of microgrid PV panels using neural network (NN) nonlinear mapping, is proposed. This estimated energy/ power is used for load flow studies and the grid design.

## II. PROBLEM DESCRIPTION

The rotation of the earth on its own axis causes apparent movement of the sun over a day from east to west. As the earth revolves around the sun over a year, the position of the sun in the sky also changes. Therefore, the rotation and revolution of the earth causes a variation of the energy received from the sun. The amount of energy received at a particular location on earth depends on the latitudinal position, the clarity of the atmosphere and the ground conditions. The angle at which the light from the sun reaches the ground depends on the time of the year. To avoid the cost and effort involved in adjusting the tilt angle on a continuous basis, it is proposed to adjust it only once in three months instead. With the data of the location available, the amount of solar energy received on a horizontal panel can be calculated.

For power systems planning it is of prime importance to estimate the amount of energy that will be available from a new PV installation. In this paper, the estimation of the amount of irradiation energy received from the sun on the PV panel, calculated based on the optimal tilt angle of microgrid PV panels on a quarterly basis using NN nonlinear mapping is proposed. This estimated energy or power is extremely important for load flow studies performed on the grid.

The sun can be considered as ideal source of radiation energy. The flux of radiation energy received on a unit area at outside of the Earth's atmosphere is designated as solar constant and it is estimated to be equal to  $1367 \text{ W/m}^2$  [3]. Irradiation is defined as total energy absorbed from radiation on a unit area.

The solar energy received at a location on earth depends on astronomical and geographical factors. The angle which the sun makes with the horizontal plane is the solar elevation angle,  $\alpha$ , which determines the amount of energy reaching the PV panel on the surface of the earth as depicted by Fig. 1. The plane tangential to the earth's surface at the location of the panel is the *horizontal plane*. A line perpendicular to the horizontal plane, at the location of the panel, is called the *zenith line*. The angle which the sun's rays make with the zenith line called *zenith angle*,  $\theta_z$ . The complementary angle to it, *solar elevation angle*,  $\alpha$ , is the angle between the horizontal plane and the sun. Due to the rotation of the earth, the sun appears to move from east to west and its position in the sky changes with time. The angle measured from the line joining the panel and the southern direction to the projection of the sun on the horizontal plane as shown in Fig. 1 is called *solar azimuthal angle* or *hour angle*,  $\omega$ . The hour angle of the sun at sun rise is shown in Fig. 1 as  $\omega_s$ . Instead of installing the panels horizontally, for capturing the maximum energy from the sun, the panels are tilted towards the equator (south for northern hemisphere). The angle which the plane of the panel makes with the horizontal is called *tilt angle*,  $\beta$ , as shown in Fig. 1.

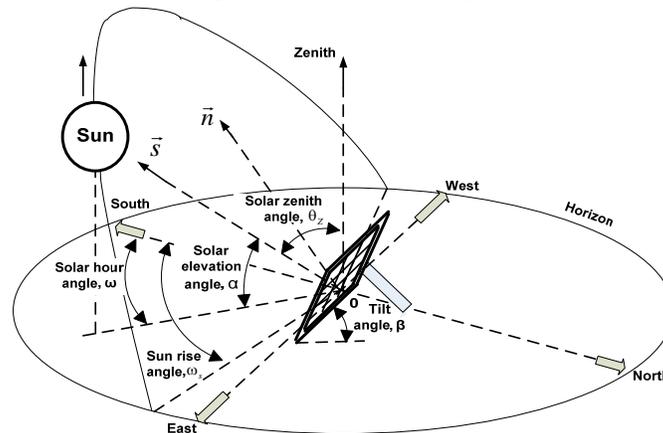


Fig. 1. The passage of the sun in the sky over a day

As the rays of the sun passes through the atmosphere, the atmosphere absorbs some of its energy. The amount of energy absorbed by the atmosphere depends on the distance the rays travel through the atmosphere. A factor called *air mass* (AM) is defined as the ratio of the distance the rays travel when the sun is at an angle to the distance travelled by the rays when the sun is perpendicularly overhead at the location. For small zenith angles, this ratio can be approximately found from (2). This relationship can be demonstrated from Fig. 2. Referring to Fig. 2, when the

sun is at position 1, it is perpendicular to the earth's surface. The distance travelled by the sun's rays is distance  $BA$ . When the sun moves to the position 2, the distance travelled by the rays is  $CA$ . From the definition of air mass is given by:

$$AM = \frac{\overline{CA}}{\overline{BA}} = \frac{1}{\cos \theta_z} \quad (2)$$

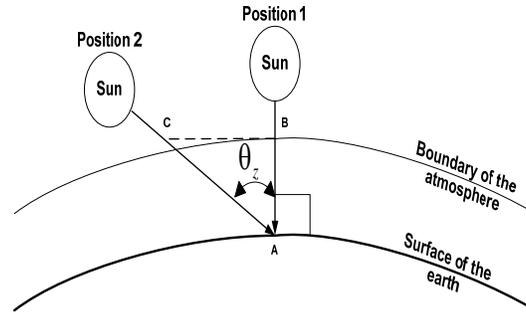


Fig. 2. Elaboration of the relationship of air mass and the zenith angle

Figure 3 shows orbit of the earth around the sun over one year. As the earth revolves around the sun, the angle of the sun appears to shift daily. This change is due to the variation in *declination angle*,  $\delta$  which is the angle between the line joining the centers of the sun and the earth, and the equatorial plane [4] as shown in Fig. 3(a).

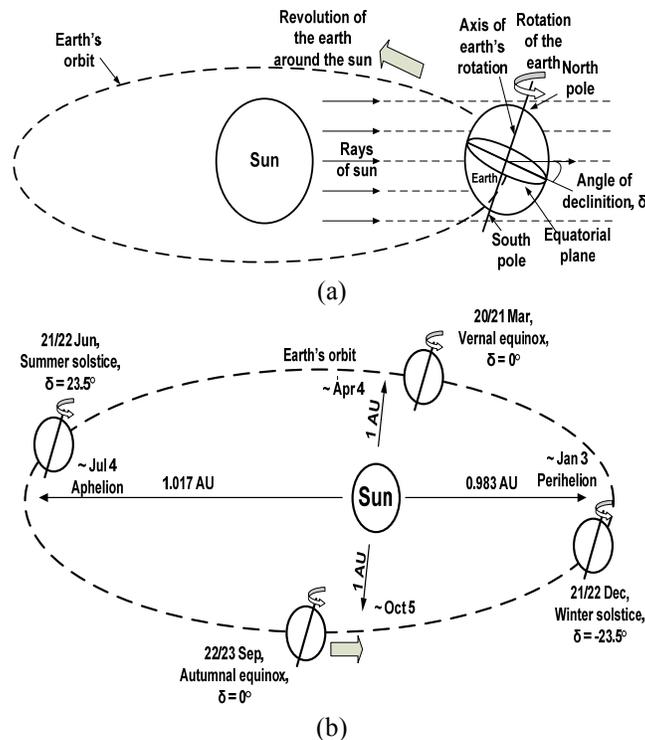


Fig. 3. The Orbit of the earth around the sun over a year (a) defining the angles (b) detailing the orbit of the earth [5]

A location on the surface of the earth is specified by *latitude* and *longitude* at that location. The *latitude* of the location,  $\phi$ , is the angle subtended at the center of the earth by the location and the equatorial plane. *Longitudes* are imaginary lines running from north-pole to south-pole on the surface of the earth.

The angles on the surface of the earth are detailed in Fig. 4. The *declination angle*,  $\delta$ , is the angle which the sun makes with the plane of the equator. The *hour angle*,  $\omega$ , is the angle which the longitude of the location makes with the sun at a particular time of the day. The *zenith angle*,  $\theta_z$ , is the angle which the sun makes with the normal,  $\vec{n}$  (the zenith line) at the location.

For analysis, three orthogonal unit vectors  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$  have been defined with the origin is defined as the center of the earth as shown in Fig. 4.  $\hat{i}$  lies on the equatorial plane, pointing in the direction of the sun,  $\hat{j}$  lies on the equatorial plane, perpendicular to  $\hat{i}$  and pointing towards east, while  $\hat{k}$  is coincidental with the axis of the earth pointing towards the north-pole.

Two other unit vectors are shown in Fig. 4: the unit vector pointing towards the sun is  $\vec{s}$  and the unit normal, towards the zenith, at the location of the panel,  $\vec{n}$ . The angle between the two vectors is the zenith angle. To define these two unit vectors, they are resolved in the  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$  directions to give relations (3) and (4).

$$\vec{s} = \hat{i} \cos \delta + \hat{k} \sin \delta \quad (3)$$

$$\vec{n} = \hat{i} \cos \phi \cdot \cos \omega + \hat{j} \cos \phi \cdot \sin \omega + \hat{k} \sin \phi \quad (4)$$

From the dot product of  $\vec{s}$  and  $\vec{n}$ , the cosine of the angle between the two can be found as shown in (5)

$$\cos \theta_z = \cos \delta \cdot \cos \phi \cdot \cos \omega + \sin \delta \cdot \sin \phi \quad (5)$$

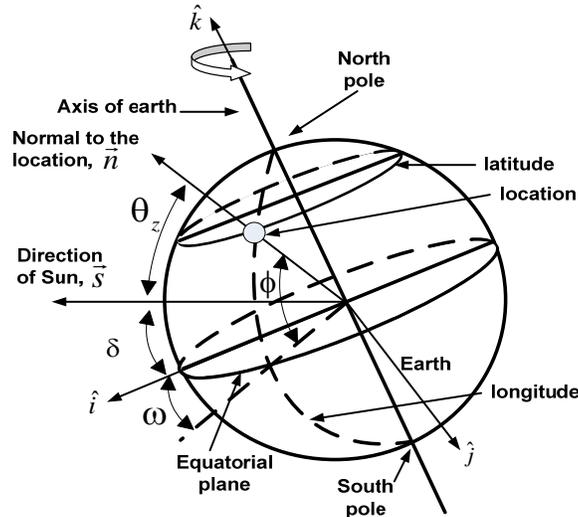


Fig. 4. The location of the panel on the globe showing the angles

### III. CALCULATION OF IRRADIANCE

From Fig. 1, it can be seen that the zenith angle at sunset (also sunrise) is  $\pi/2$ . The hour angle  $\omega$  at sunset is called *sunset angle* and is designated as  $\omega_s$  in Fig 1. From (5) [5],

$$\cos(\pi/2) = \cos \delta \cdot \cos \phi \cdot \cos \omega_s + \sin \delta \cdot \sin \phi \quad (6.a)$$

$$\begin{aligned} \omega_s &= \cos^{-1}[-\tan \delta \cdot \tan \phi] \quad \text{for } |\tan \delta \cdot \tan \phi| \leq 1 \\ &= \pi \quad \quad \quad |\tan \delta \cdot \tan \phi| > 1 \end{aligned} \quad (6.b)$$

As the earth revolves around the sun in an elliptical orbit its distance with sun changes. As seen in Fig. 3(b), the distance between the earth and the sun is maximum around July 4 and this position is called *aphelion*, while, the distance is minimum around January 3 which is known as *perihelion*. The energy received from the sun obeys inverse square law. This is represented by *eccentricity correction factor* of the earth's orbit,  $E_o$  [6].

$$E_o = (r_o / r)^2 = 1 + 0.033 \cos(2\pi d_n / 365) \quad (7)$$

where  $r_o$  is the mean distance of the earth and sun = 1 AU =  $1.496 \times 10^8$  km,  $r$  is the distance of earth and the sun,  $d_n$  is the day number of the year (1 for January 1 through 365 for December 31).

A constant called *solar constant*,  $S$ , is defined as rate of energy at all wavelengths received by a unit area perpendicular to the rays at a distance of one astronomical unit. The value of the solar constant is  $1367 \text{ W m}^{-2}$  (or  $4921 \text{ kJ/m}^2/\text{hour}$ ) as suggested by [7]. *Irradiance* (rate of energy received) on a surface should be adjusted for the variation in the distance of the earth and the sun as shown in (8).

$$I_{dn} = S.E_o \quad (8)$$

where  $I_{dn}$  is the irradiance on the day  $d_n$ .

The irradiance on a horizontal surface is received at an angle  $\theta_z$ . Therefore, the irradiance on the surface horizontal,  $I_{hor}$  is:

$$I_{hor} = I_{dn} \cos \theta_z \quad (9)$$

Using (5) and (8) in (9),

$$I_{hor} = S.E_o \cdot \{\cos \delta \cdot \cos \phi \cdot \cos \omega + \sin \delta \cdot \sin \phi\} \quad (10)$$

The beam energy, or *irradiation*,  $B_o$ , received during the day is found by integrating  $I_{hor}$  over time from sunrise to sunset as shown in (12). It takes the earth 24 hours to make a  $2\pi$  rad rotation. Therefore,

$$\frac{2\pi}{24} dt = d\omega \quad (11)$$

$$B_o = \int I_{hor} dt = \int S.E_o \cdot \{\cos \delta \cdot \cos \phi \cdot \cos \omega + \sin \delta \cdot \sin \phi\} dt \quad (12)$$

Using the relation in (11) in (12),

$$\begin{aligned} B_o &= \frac{24}{\pi} \int_0^{\omega_s} S.E_o \cdot \{\cos \delta \cdot \cos \phi \cdot \cos \omega + \sin \delta \cdot \sin \phi\} d\omega \\ &= \frac{24}{\pi} \cdot S.E_o \left[ (\cos \delta \cdot \cos \phi \cdot \sin \omega_s) + \omega_s (\sin \delta \cdot \sin \phi) \right] \end{aligned} \quad (13)$$

In (13),  $G$  is in  $\text{kJ/m}^2/\text{day}$  if  $S$  is in  $\text{kJ/m}^2/\text{hour}$  or in  $\text{kWh/m}^2/\text{day}$  if  $S$  is in  $\text{kW/m}^2$ .

The angle of declination can be calculated from the empirical formula [8]:

$$\delta = 23.45 \sin \left[ \frac{360(d_n + 284)}{365} \right] \quad (14)$$

For maximum collection of energy by the panels, the panels are tilted towards the equator. For a tilt angle  $\beta$ , the angle made by the panel to the equatorial plane is  $\phi - \beta$  instead of  $\phi$ . Hence the angle which the rays of sun make with the panel,  $\theta_{ilt}$  is given by (15):

$$\cos \theta_{ilt} = \cos \delta \cdot \cos(\phi - \beta) \cdot \cos \omega + \sin \delta \cdot \sin(\phi - \beta) \quad (15)$$

The sunset angle as seen by the panel is now given by (16):

$$\omega_{s,tilt} = \cos^{-1}[-\tan\delta.\tan(\phi-\beta)] \quad \text{for } |\tan\delta.\tan\phi| \leq 1 \quad (16)$$

$$= \pi \quad |\tan\delta.\tan\phi| > 1$$

But in practice,  $\omega_{s,tilt}$  cannot be greater than  $\omega_s$ . Therefore, the sunset angle is now the minimum of the two:

$$\omega_s' = \min(\omega_{s,tilt}, \omega_s) \quad (17)$$

Hence, (13) now get modified to (18) using (15) and (17):

$$B_\beta = \frac{24}{\pi} . S . E_o \left[ \begin{array}{l} \{\cos \delta . \cos(\phi - \beta) . \sin \omega_s'\} \\ + \omega_s' \{\sin \delta . \sin(\phi - \beta)\} \end{array} \right] \quad (18)$$

As the rays from the sun pass through the atmosphere, they get scattered and a portion of reaches the earth as diffused irradiance [9]. It is generally assumed that this diffused rays are isotropic, which means that they appear to come from all directions with equal intensity as shown in Fig. 5. Amount of diffused radiation the surface is receives is directly proportional to the cosine of the angle of incidence,  $\theta$ , shown in Fig. 5. The total irradiance for tilt angle  $\beta$  is given by (19).

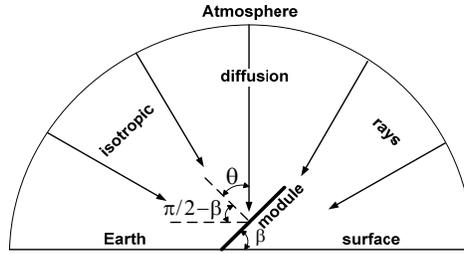


Fig. 5. Isotropic diffusion on the panel

$$D(\beta) \propto \int_{-\pi/2}^{\pi/2-\beta} \cos \theta d\theta = (1 + \cos \beta) \quad (19)$$

The amount of irradiance that reaches the surface of the earth is a function of the air mass and the clarity of the atmosphere. From the meteorological data recorded at the location, a fraction called clearness index,  $K_T$  can be calculated:

$$K_T = G / B_o \quad (20)$$

where  $G$  is global daily irradiation for a typical day of the month.

Form the clearness index, the diffusion component,  $D$ , of the irradiation can be calculated using the empirical formula [9]:

$$D / G = 1 - 1.13 K_T \quad (21)$$

For horizontally placed panels, the diffusion irradiance,  $D$  is obtained by substituting zero for  $\beta$  in (19). Using this fact, we get a relation between  $D(\beta)$  and  $D$  as given in (22):

$$D(\beta) = \frac{1}{2} (1 + \cos \beta) . D \quad (22)$$

Similarly, if ground reflection is considered isotropic, then we get (23).

$$R(\beta) \propto \int_{\pi/2-\beta}^{\pi/2} \cos \theta d\theta = (1 - \cos \beta) \quad (23)$$

As the reflection from the ground, a *reflection coefficient*,  $\rho$ , is defined. And the *reflected irradiation* or *albedo* on a tilted surface is defined by:

$$R(\beta) = \frac{1}{2}(1 - \cos \beta)\rho.D \quad (24)$$

The geographical data like the latitude and the global irradiation data for each month collected at that location must be known.

#### IV. ALGORITHM FOR ESTIMATION OF OUTPUT POWER OF PV STATION

The algorithm to estimate the global irradiation and output power is given by Fig. 6.

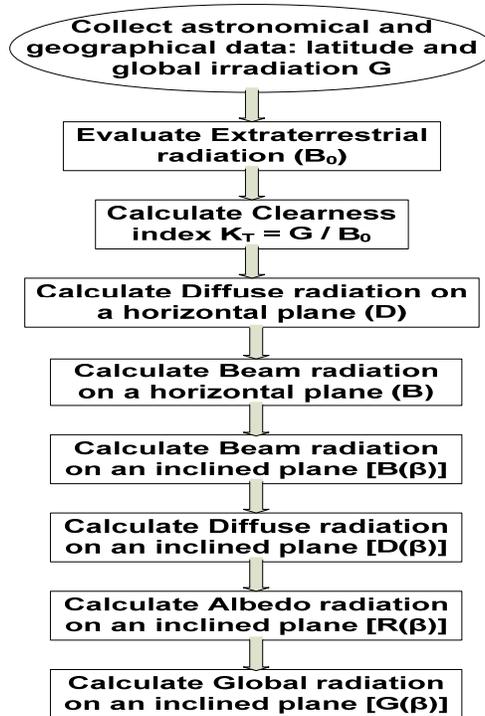
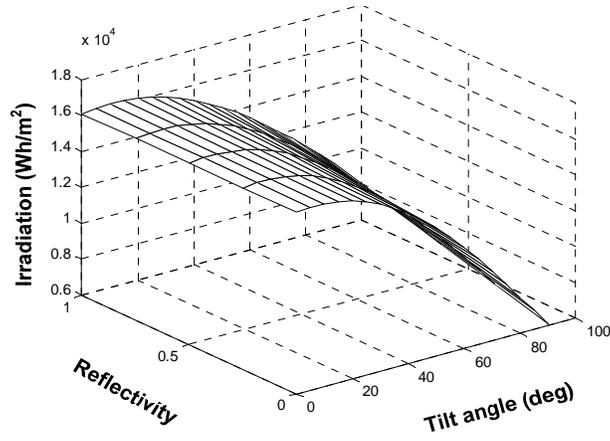


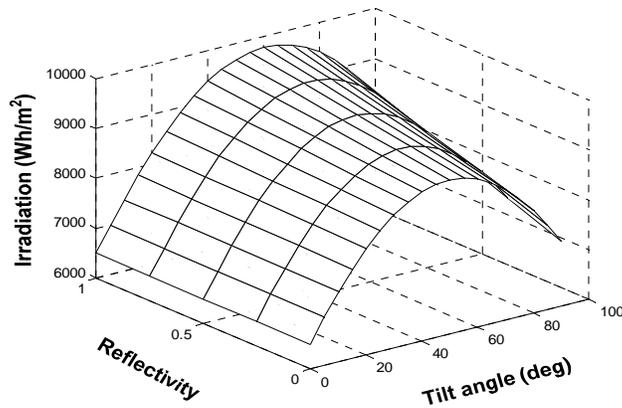
Fig. 6. The algorithm for calculation of global radiation on tilted PV panels

Although the problem of PV station output power can be formulated on daily, weekly, monthly and quarterly basis, in this paper, the quarterly models of expected irradiation ( $\text{Wh/m}^2$ ) are presented. Note that irradiation is computed in  $\text{Wh/m}^2$ . Thus the size of PV stations is a function of surface area of PV panels. The output power of PV stations is computed from location of stations and the irradiance.

As seen from the above analysis, irradiation received on a tilted surface is a non-linear function of the latitude of the location, the tilt angle, the ground reflectivity, clearness index and the time of the year. The irradiation of a tilted surface at Columbus, Ohio, is plotted as a function of reflectivity and tilt angle for various times of the year in Fig. 7. In Fig. 8, the irradiation is plotted as a function of latitude and reflectivity. These plots illustrate the non-linear relationship between the irradiation and tilt angle, latitude and the ground reflectivity.



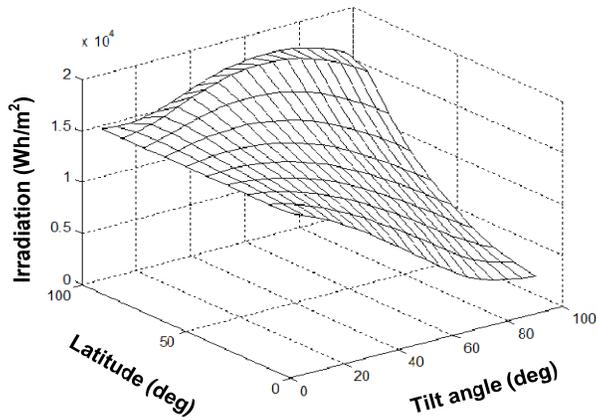
(a)



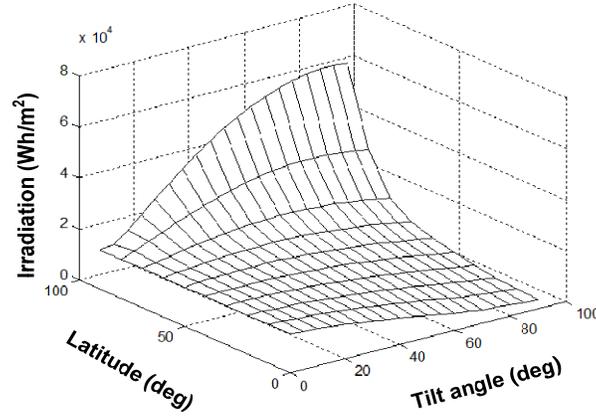
(b)

Fig. 7. Irradiation as a function of tilt angle and reflectivity for Columbus, Ohio from (a) April to June, and (b) October through December

The reflected component from the ground that reaches the panel is a function of ground reflectivity. If the ground reflectivity is high, then more is the reflected component and the PV receives more irradiance as seen in Fig. 7 (a) and (b).



(a)



(b)

Fig. 8. Irradiation as a function of tilt angle and latitude for from (a) April to June, and (b) July through September

It is seen that the irradiation is more sensitive to the tilt angle during the autumn (October to December) when the sun makes a narrower angle with the PV station. However, during spring (April to June), when the sun is makes a larger angle with the PV the optimal angle is closed to zero as the sun apparently moves through a course which is almost overhead in the sky. Therefore, the optimal tilt angle is very close to zero during summer months at the given location.

The non-linear relationship between the tilt angle and the amount of irradiation received on the PV panels makes the computation of the optimal tilt angle and the irradiation difficult to estimate. Therefore, a NN is proposed for the estimation of the optimal tilt angle for each quarter of the year and the total amount of energy that will be available from the PVs when the PVs are installed at the respective optimal angles for each quarter.

## V. NEURAL NETWORK FOR ESTIMATION OF OPTIMAL ANGLE AND IRRADIATION

The amount of irradiation a PV panel receives has a highly non-linear relationship with the tilt angle, reflectivity and the latitude. It also depends on the meteorological data. The algorithm presented in Fig. 6 is used to determine the total irradiation.

A multi layer perceptron is used to estimate the value of a non-linear function. Multiple layers are formed out of basic computational units called processing elements. A typical processing element accepts weighted sum of its inputs and transforms that through a non-linear function to form the output [10]. The multilayer perceptron in this paper consists of perceptrons arranged in four distinct layers: input layer, two hidden layers and output layer. The data presented at the input layer of the network is used to calculate the inputs for the hidden layer. The data from the inputs propagate to the hidden layers and then finally to the output layer. Training is a process in which known patterns of inputs and outputs are presented to the network and the weights between the layers are adjusted iteratively to match the output until the error between the desired output and the output from the NN is below an acceptable value for all the training sets. In this work, the relationship between the input and output are as given by (25) and (26).

$$\beta_i^{opt} = f_{1,i}(G_{month}, \phi, \rho) \quad (25)$$

$$G_{year} = f_2(G_{month}, \phi, \rho) \quad (26)$$

where  $\beta_i^{opt}$  is the optimal tilt angle for the quarter  $i$  of the year and  $G_{year}$  is the total irradiation that the panels receive over the year if the tilt angle is optimized for each quarter of the year.

Fig. 9 shows the structure of the NN which is used to estimate the optimal tilt angle. The NN shown in Fig. 9 estimates the optimal tilt angle for each quarter of the year and also the total irradiation the location receives if the tilt angle is optimized on a quarterly basis over the year as given by (25) and (26). The multilayer perceptron used

has 14 input processing elements corresponding to the irradiation of each month, the latitude and the ground reflectivity of the location as given by (25) and (26). There are five output processing elements corresponding to the optimum tilt angle for each quarter if the year and the energy that is available from the installed PV if optimal tilt angle is used. The number of hidden layers and the number of elements in each layer is chosen arbitrarily depending on the complexity of the mapping. The transfer functions at the hidden layer hyperbolic tangents which introduce the non-linearity and whereas the transfer functions at the input and output layers have linear transfer functions. Levenberg-Marquardt [11] algorithm for training is used for training the NN so that the sum of error squares,  $E$ , between the actual outputs,  $O_A$ , and the desired outputs  $O_D$  is minimized for training over all patterns  $\mu$  and is below acceptable level.

$$E = \sum_{\mu} (O_{D,\mu} - O_{A,\mu})^2 \quad (27)$$

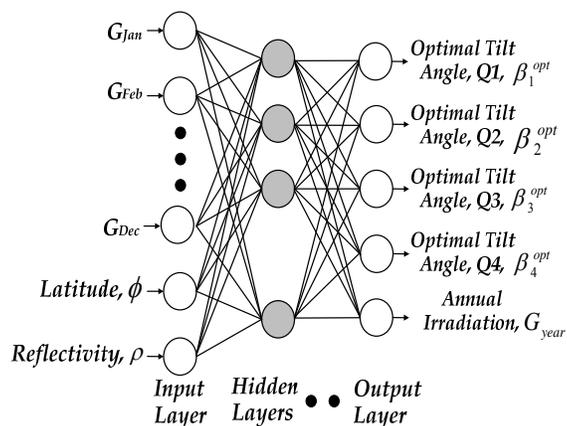


Fig. 9. Structure of NN for estimation of optimal tilt angle

As seen in Fig. 9, the NN takes in the meteorological data in the form of irradiation on a horizontal surface in each month, the latitude of the location and the ground reflectivity of the location to estimate the optimal tilt angle at that location to receive maximum irradiation.

The estimate of the nonlinear mappings  $f_{1,i}$  and  $f_2$  representing the optimal tilt angle for quarter  $i$  and the total irradiation can be expressed using weight vectors  $W$ , bias vectors  $B$  and the inputs.

$$\begin{bmatrix} \beta_1^{opt} \\ \beta_2^{opt} \\ \beta_3^{opt} \\ \beta_4^{opt} \\ G_{year} \end{bmatrix} = W_3 \tanh(W_2 \cdot \tanh(W_1 \cdot I + B_1) + B_2) + B_3 \quad (28)$$

where  $W_1$  is  $40 \times 14$  matrix connecting the 14 inputs with the 40 layer first hidden layer,  $W_2$  is  $40 \times 40$  matrix connecting the first and second hidden layer,  $W_3$  is a  $5 \times 40$  matrix connecting the second hidden layer and the output layer of five neurons.  $B_1$ ,  $B_2$  and  $B_3$  are  $40 \times 1$ ,  $40 \times 1$  and  $5 \times 1$  bias matrices for the first hidden layer, second hidden layer and the output layer respectively. The training patterns required to train the NN are obtained from the rigorous calculations shown in analytical section. The model presented by (28) has 40 perceptrons in the hidden layer and 14 input and 5 outputs. The program for training the neural network is given in APPENDIX A. The weight and bias matrices are given in the APPENDIX B.

## VI. NEURAL NETWORK MODEL VALIDATION

The NN of Fig. 9 is trained from the available geographical and meteorological data. In the NN model (see eq.

28), the optimal tilt angle for each quarter and the total annual irradiance received at different location has been estimated as shown in Table I. It is observed from Table I that the NN is able to estimate the optimal tilt angle and the total irradiation of the location.

TABLE I  
ESTIMATED OPTIMUM TILT ANGLE FOR EACH QUARTER AND ANNUAL IRRADIATION

Location	Latitude (°)	Optimum tilt angle Quarter 1 (°)		Optimum tilt angle Quarter 2 (°)		Optimum tilt angle Quarter 3 (°)		Optimum tilt angle Quarter 4 (°)		Annual total irradiation (kWh/m <sup>2</sup> )	
		Analytical	NN	Analytical	NN	Analytical	NN	Analytical	NN	Analytical	NN
Kolkata, India	22.00	35	35.28	0	0.45	0	-0.57	40	40.00	58.65	58.22
Los Angeles, California	33.93	45	45.30	5	4.29	10	10.15	50	50.54	67.69	67.26
Columbus, Ohio	40.00	50	48.17	10	12.35	15	15.84	55	49.18	52.01	51.40
Barrow, Alaska	71.30	80	89.46	40	32.52	35	24.68	80	86.39	33.29	35.29
Daggett, California	34.87	50	49.99	5	1.32	10	13.11	55	53.81	81.65	78.86

## VII. CONCLUSION

The integration of PV in power grids requires the estimation of the output power of the PV stations. The planning and operation control of grids need the maximum output power of PV stations for power flow studies and operation control for daily scheduling. In this paper, NN method to estimate the optimal tilt angle to estimate the amount of energy available from the sun at the given location has been presented.

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## APPENDIX A

The Matlab code to train the artificial neural network:

```

clc;
clear all;
% Solar irradiation data on horizontal surface in kWh/m^2/day
% latitudes of each location in order

%% Constants
S = 1.367; % kW/m2 global irradiance
deg = 5; % tilt angle adjustment degree

%% Variations
g = 0:.25:1; % reflectivity varied

month = zeros(1,12); % initialising
betha = zeros(1,floor(90/deg)+1); % initialising
for i=1:12 % finding the day number for the middle of each month
    month(i)=15+(i-1)*30;
end
for i=1:floor(90/deg)+1 % Varying the tilt angle
    betha(i)=(i-1)*deg;
end

%% Initializations
Ir = zeros(floor(90/deg)+1,13); % initialising irradiation matrix containing the
value for each month and year
Irradiation = betha'; % Col. 1 = betha, other columns for Irradiation for each
month, year, reflectivity and location

optimal_angle_yr = zeros(size(GI,1),size(g,2)); % optimal angle for year
optimal_angle_q = zeros(size(GI,1),size(g,2),4); % optimal angle for each quarter
Ir_yr_ch_q = zeros(size(GI,1),size(g,2)); % total annual irradiation
(kWh/m^2/day), angle changed quarterly

%% Calculation of irradiation, optimal angles
for l = 1:size(GI,1) % vary location
for k = 1:size(g,2) % vary reflectivity
    Ir(:,13) = 0; % initialize yearly irradiation to zero
for i = 1:floor(90/deg)+1 % vary tilt angle
    for j = 1:12 % vary month
        delta=23.45*sind(360*(284+month(j))/365); % Calculating declination angle
        % Calculate the sunset angle
        x = -tand(L(l))*tand(delta);
        if abs(x) > 1
            if x > 0
                x = 1;
            else
                x = -1;
            end
        end
        ws = acos(x); % Finding sunset angle
        % Calculate the sunset angle for tilted panel
        y = -tand(L(l)-betha(i))*tand(delta);
        if abs(y) > 1
            if y > 0
                y = 1;
            else
                y = -1;
            end
        end
    end
end
end

```

```

wsl=acos(y);    % Finding sunset angle for tilted surface
wo=min(ws,wsl);
% Calculating extraterrestrial irradiation

Bo=(24/pi)*S*(1+.0333*cos(2*pi*month(j)/365))*(cosd(L(l))*cosd(delta)*sin(ws)+ws*sind(
L(l))*sind(delta));
if Bo == 0          % no sunrise
    DD = GI(l,j);   % radiation on surface is all diffusion
    B = 0;
else
    KT=GI(l,j)/Bo;   % clearness index
    DD=GI(l,j)*(1-1.13*KT); % diffuse irradiation
    BB=GI(l,j)-DD;   % beam(direct irradiation)
    B=BB*(cosd(L(l)-betha(i))*cosd(delta)*sin(wo)+wo*sind(L(l)-
betha(i))*sind(delta))...
    /(cosd(L(l))*cosd(delta)*sin(ws)+ws*sind(L(l))*sind(delta));
end
D=DD*0.5*(1+cosd(betha(i))); % diffused irradiation on tilted surface
R=DD*0.5*g(k)*(1-cosd(betha(i))); % reflection irradiation on tilted surface
G=B+D+R; % total global irradiation for the month
Ir(i,13)=Ir(i,13)+G; % total global irradiation for a year
Ir(i,j)=G; % irradiation for each month at different inclination angles

end
end
%% Irradiance for the entire year with angle fixed
[Ir_yr_max nn_yr] = max(Ir(:,13)); % finding the optimal yearly value
optimal_angle_yr(l,k) = betha(nn_yr); % optimum angle for the entire year

%% Irradiation for each quarter
Ir_q = zeros(floor(90/deg)+1,4); % Irradiation for each quarter in each column the
sum in fifth column
for q = 1:4 % quarter 1 to 4
    for mo = 3*(q-1)+1:3*(q-1)+3 % months of each quarter
        Ir_q(:,q) = Ir_q(:,q) + Ir(:,mo);
    end
end

Ir_q_max = zeros(1,4); % quarterly maximum irradiances
nn = zeros(1,4); % quarterly maximum index for betha

for q = 1:4 % calculation of optimum quarterly
    [Ir_q_max(q) nn(q)] = max(Ir_q(:,q));
    Ir_yr_ch_q(l,k) = Ir_yr_ch_q(l,k) + Ir_q_max(q); % annual maximum irradiation
end
for angle changed quarterly
    optimal_angle_q(l,k,q) = betha(nn(q)); % optimum quarterly angle
end

%% Storing Irradiation matrix (each month and year) for changing location and
reflectivity
Irradiation = [Irradiation Ir];

end
end
%% Input/Target data structure formation
% rearranging
optimal_angle_q1 = optimal_angle_q(:, :, 1)';
optimal_angle_q2 = optimal_angle_q(:, :, 2)';
optimal_angle_q3 = optimal_angle_q(:, :, 3)';
optimal_angle_q4 = optimal_angle_q(:, :, 4)';
Ir_yr_ch_q = Ir_yr_ch_q';

```

```

targets = [ optimal_angle_q1(:)';      % Optimal angles for quarter 1
            optimal_angle_q2(:)';      % Optimal angles for quarter 2
            optimal_angle_q3(:)';      % Optimal angles for quarter 3
            optimal_angle_q4(:)';      % Optimal angles for quarter 4
            Ir_yr_ch_q(:)'];           % Total yearly irradiation (kWh/m^2/day) with
angles changed quarterly

inputs = [GI(1,:)'; L(1); g(1)];      % Initialization [monthly irradiation; latitude;
reflectivity]
for i = 1:size(GI,1) % Inputs: Monthly irradiation; latitude; reflectivity
    for j = 1:size(g,2)
        if (i == 1 && j == 1) % 1st column already stored during initialization
            continue;
        end
        inputs = [inputs [GI(i,:)'; L(i); g(j)]];
    end
end
Irradiation;
inputs;
targets;
net_optimal_quarter = create_fit_net(inputs,targets);

```

The function to create and train the neural network, “create\_fit\_net()” is given below:

```

function net = create_fit_net(inputs,targets)
%CREATE_FIT_NET Creates and trains a fitting neural network.
%
% NET = CREATE_FIT_NET(INPUTS,TARGETS) takes these arguments:
% INPUTS - RxQ matrix of Q R-element input samples
% TARGETS - SxQ matrix of Q S-element associated target samples
% arranged as columns, and returns these results:
% NET - The trained neural network
%
% For example, to solve the Simple Fit dataset problem with this function:
%
% load simplefit_dataset
% net = create_fit_net(simplefitInputs,simplefitTargets);
% simplefitOutputs = sim(net,simplefitInputs);
%
% To reproduce the results you obtained in NFTOOL:
% load house_dataset
% inputs = houseInputs;
% targets = houseTargets;
% net = create_fit_net(houseInputs',houseTargets');
% Create Network
numHiddenNeurons = [40 40]; % Adjust as desired
net = newfit(inputs,targets,numHiddenNeurons);
net.divideParam.trainRatio = 70/100; % Adjust as desired
net.divideParam.valRatio = 15/100; % Adjust as desired
net.divideParam.testRatio = 15/100; % Adjust as desired

% Train and Apply Network
[net,tr] = train(net,inputs,targets);
outputs = sim(net,inputs);

% Plot
plotperf(tr)
%plotfit(net,inputs,targets)
%plotregression(targets,outputs)

```

## APPENDIX B

The weight and bias matrices in (28) are as follows:

$W_1$  is a  $40 \times 14$  matrix partitioned into 4 matrices  $20 \times 7$  each as given by:

$$W_1 = \begin{bmatrix} W_1^{11} & W_1^{12} \\ W_1^{21} & W_1^{22} \end{bmatrix} \text{ where,}$$

$$W_1^{11} = \begin{bmatrix} -0.4489 & -0.2819 & 0.1822 & 0.1734 & 0.4418 & -0.5700 & 0.7216 \\ 0.4608 & -0.3925 & 0.6693 & 0.2858 & -0.2045 & 0.3337 & 1.2026 \\ 0.6594 & -0.1343 & 0.7357 & -0.3592 & 0.6008 & -0.4021 & 0.3238 \\ 0.6007 & -0.1579 & 0.7803 & -0.6354 & -0.1343 & -0.5919 & -0.2769 \\ 0.0877 & 0.0693 & -0.5515 & 0.7593 & -0.6140 & -0.9595 & 0.6961 \\ 0.1059 & 0.0358 & -0.4539 & -0.8530 & 0.6406 & -0.5954 & -0.1706 \\ -0.5158 & -0.1412 & 0.5623 & 0.5064 & 0.1553 & -0.4736 & 0.6529 \\ 0.8081 & -0.1491 & 0.0368 & -0.9381 & -0.0985 & -0.1050 & -0.7023 \\ -0.2854 & -0.1117 & -0.0968 & 0.1869 & 0.9729 & -0.1471 & 0.6872 \\ -0.6812 & 0.4069 & 0.3031 & 0.0150 & 0.4250 & -0.7578 & 0.6244 \\ 0.5963 & 0.7623 & 0.5372 & -0.4622 & 0.0034 & 0.2539 & -0.4773 \\ 0.3710 & 0.2708 & -0.1135 & -0.0237 & 0.4513 & 0.6148 & -0.4234 \\ 0.6469 & -0.2199 & -0.5502 & -0.5918 & 0.0780 & 0.7975 & -0.4412 \\ -0.5656 & 0.8123 & -0.0599 & 0.3109 & 0.1535 & -0.5405 & -0.2287 \\ 0.4133 & -0.6813 & -0.0600 & 0.3093 & 0.2349 & -0.3456 & -0.4091 \\ 0.2327 & -0.5271 & 0.7112 & -0.6375 & -0.0282 & 0.1222 & 0.5781 \\ -0.6252 & -0.6008 & 0.2592 & -0.5400 & -0.2523 & 0.1659 & -0.4498 \\ -0.2523 & -0.8098 & 0.4334 & -1.0570 & 0.0142 & -0.6845 & 0.1182 \\ -0.3333 & -0.2784 & 0.2579 & -0.9731 & 0.3618 & -0.8603 & -0.5761 \\ 0.5295 & -0.4226 & -0.2311 & 0.3433 & 0.1726 & 0.1288 & -0.1047 \end{bmatrix}$$

$$W_1^{12} = \begin{bmatrix} 0.4660 & 0.7244 & -0.0929 & -0.4890 & -0.7315 & 0.4971 & -0.1000 \\ 0.4789 & -0.1190 & 0.7544 & -0.2859 & 0.1572 & -0.3583 & 1.2124 \\ -0.4998 & -0.3113 & -0.5309 & -0.4837 & 0.7050 & 0.0430 & 0.7797 \\ 0.1551 & -0.5123 & -0.2669 & 0.2156 & -0.6982 & -0.4267 & -1.2887 \\ -0.2251 & -0.1442 & 0.0875 & 0.1998 & -0.1264 & -0.2395 & 0.1807 \\ -0.7425 & -0.2758 & -0.7948 & -0.5869 & 0.3585 & 0.3213 & -0.0662 \\ -0.4746 & -0.4881 & 0.3756 & -0.5311 & -0.6975 & -0.5120 & -0.1373 \\ -0.3860 & 0.1266 & 0.3069 & -0.0240 & 0.8155 & -0.7409 & -1.1605 \\ 0.5576 & -0.7347 & -0.2115 & 0.4436 & 0.2353 & -0.1955 & -1.0959 \\ -0.0850 & 0.2959 & 0.7838 & 0.1134 & 0.1338 & -0.7010 & 0.1164 \\ 0.3610 & -0.6786 & 0.6014 & -0.7566 & -0.5384 & -0.2394 & -0.3237 \\ -0.5135 & 0.1220 & 0.4932 & -0.8939 & -0.1195 & 0.6950 & 0.2872 \\ -0.8148 & 0.4934 & -0.3109 & 0.5101 & 0.0492 & -0.0302 & -0.2837 \\ 0.2576 & -0.5086 & -0.0951 & -0.1779 & 1.0097 & -0.1544 & -0.5604 \\ -0.5992 & -0.3111 & 0.4854 & -0.2740 & 0.8108 & -0.6322 & -1.5306 \\ 0.9302 & 0.2200 & -0.5554 & 0.3836 & 0.6117 & 0.7062 & -0.2068 \\ 0.7048 & -0.6870 & -0.6382 & -0.2938 & 0.1887 & 0.0338 & -0.8624 \\ -0.1993 & 0.0605 & -0.5195 & 0.0629 & -0.3000 & 0.7609 & -0.4012 \\ -0.7924 & 0.3396 & 0.1767 & 0.3915 & 0.6977 & -0.0299 & -0.8533 \\ 0.4764 & -0.5598 & 0.9395 & 0.9189 & 0.1587 & -0.0751 & 0.3884 \end{bmatrix}$$

$W_1^{21} =$ 

-0.3659	-0.7851	-0.0301	0.4102	-0.5471	-0.3423	-0.2494
0.6133	0.0373	0.0920	0.3197	0.0453	-0.6496	-0.5168
0.4338	-0.7632	-0.6549	0.0965	-0.4830	0.4344	0.5874
-0.0892	-0.5839	0.2880	1.0350	0.1006	-0.0013	0.9842
0.0024	0.2085	0.2441	-0.8139	0.3872	0.6607	0.6248
0.2196	0.5207	-0.3762	0.5114	-0.3592	0.4770	0.4768
0.5810	0.6840	0.5419	-0.3673	0.2345	-0.3516	-0.3563
0.2590	0.1678	0.5172	-0.0801	-0.3325	0.4153	-0.3086
0.0167	0.7367	-0.2922	-0.7332	-0.7074	0.2322	-0.5062
-0.2645	0.1593	-0.0299	0.2557	-0.3980	0.3034	-1.0497
-0.0067	0.1819	-0.0416	-0.7761	-0.7601	-0.0620	-0.2497
0.4738	-0.1579	0.5665	0.3911	-0.1593	-0.5443	0.2467
0.6712	-0.1474	0.2576	0.3718	-0.7724	0.2045	0.0903
0.4578	0.0139	-0.0996	0.8367	-0.2847	-0.4094	0.1076
-0.7561	-0.6620	0.3314	0.7841	-0.0812	-0.1672	-0.1161
0.2475	0.6607	0.8335	0.1332	-0.7769	0.4249	-0.6886
0.0414	-0.6587	0.6760	-0.3264	0.1352	0.7353	-0.0761
-0.1365	-0.1229	0.4842	-0.7695	0.4098	-0.3014	0.5032
-0.6867	0.5682	-0.6983	-0.0075	-0.4658	-0.2524	0.5709
0.6941	-0.2623	-0.2812	0.1481	0.5856	0.1791	0.0826

 $W_1^{22} =$ 

0.6166	0.6088	0.3094	-0.2906	-0.4750	0.5936	0.8916
0.7325	0.7381	-0.3022	0.2333	-0.1550	0.5355	0.5996
-0.7741	0.4429	-0.3538	0.7612	0.3113	-0.0323	0.0164
-0.0556	0.0034	0.6390	-0.6102	0.3059	0.6790	-1.0041
0.2335	-0.3031	0.7163	0.1671	0.4352	0.5868	-0.8968
0.3369	0.4411	0.1086	-0.3469	0.8062	-0.0569	0.0980
0.2446	-0.4560	-0.4594	-0.2586	0.8156	-0.1414	-0.0314
0.2879	0.9459	-1.0694	-0.6110	0.0715	0.2637	-0.2153
0.3573	0.1446	0.4533	0.3115	0.6227	0.7168	0.1253
-0.3189	0.1471	0.2045	1.0259	-0.6745	0.3812	0.0136
0.2583	-0.4549	0.8340	-0.3931	-0.6809	-0.2657	0.5338
-0.6501	-0.6331	-0.6469	0.5767	-0.2314	0.2659	-0.6296
-0.7083	-0.2480	0.3195	-0.4410	0.1236	0.5466	0.3776
0.8795	0.6731	-0.3499	0.9375	0.1163	0.0452	0.1593
-0.3457	0.6766	0.5498	0.4881	0.6508	0.0569	-0.2092
-0.0648	0.1261	-0.6061	-0.1972	0.0251	-0.2075	1.0635
0.7997	0.4142	-0.0499	0.5324	0.0279	-0.8765	-0.7710
-0.6886	-0.5990	-0.1915	0.0191	0.1006	-0.6804	0.0051
-0.3870	0.2846	0.6758	0.5822	0.3229	-0.7786	0.4112
-0.4095	0.6830	0.3907	-0.3329	-1.1056	-0.0924	0.1757

$W_2$  is a  $40 \times 40$  matrix partitioned into 8 matrices  $20 \times 10$  each as given by:

$$W_2 = \begin{bmatrix} W_2^{11} & W_2^{12} & W_2^{13} & W_2^{14} \\ W_2^{21} & W_2^{22} & W_2^{23} & W_2^{24} \end{bmatrix} \text{ where,}$$

$$W_2^{11} = \begin{bmatrix} -0.1351 & 0.0965 & 0.2926 & 0.1228 & 0.0193 & -0.1166 & 0.2693 & -0.2152 & 0.1583 & 0.3366 \\ 0.0381 & 0.5027 & 0.3862 & -0.1868 & 0.2629 & 0.1919 & 0.3305 & 0.5767 & -0.2319 & -0.0261 \\ 0.4184 & 0.3188 & 0.0440 & 0.2229 & 0.2381 & -0.3597 & 0.1514 & 0.0116 & 0.3692 & -0.1761 \\ 0.2355 & -0.1370 & -0.1786 & -0.2645 & -0.0003 & 0.3402 & -0.2574 & 0.0313 & -0.4238 & 0.0775 \\ -0.6883 & 0.1675 & 0.2491 & -0.1485 & -0.3604 & 0.4338 & -0.5646 & 0.0444 & 0.3033 & 0.3069 \\ -0.1869 & 0.0556 & -0.3188 & 0.1878 & -0.2467 & -0.2457 & 0.1553 & -0.4891 & -0.0014 & 0.0533 \\ -0.3005 & 0.6170 & -0.1667 & 0.3750 & 0.1106 & 0.2685 & 0.1448 & -0.1605 & 0.2044 & -0.0016 \\ 0.1848 & 0.2922 & -0.0162 & 0.0039 & 0.2143 & -0.4875 & 0.3735 & 0.1111 & -0.1294 & 0.0262 \\ -0.3574 & 0.4395 & -0.0482 & 0.2992 & -0.1772 & 0.1576 & 0.2818 & -0.5714 & -0.3548 & -0.0870 \\ 0.2048 & -0.0930 & 0.4009 & 0.7529 & 0.0471 & -0.1418 & 0.2208 & -0.5271 & 0.0263 & -0.1079 \\ 0.5775 & 0.1653 & 0.2119 & -0.1475 & 0.2652 & -0.1565 & 0.0745 & 0.1383 & -0.0488 & 0.2876 \\ -0.0620 & 0.3199 & 0.3392 & -0.2058 & 0.2375 & 0.2907 & -0.0426 & 0.0630 & 0.3957 & 0.1899 \\ -0.0709 & -0.3956 & -0.3495 & -0.3424 & 0.3702 & -0.2894 & 0.4734 & 0.2987 & -0.0873 & 0.2937 \\ 0.5662 & 0.7144 & -0.1731 & 0.0794 & -0.0782 & -0.2414 & 0.0184 & 0.6080 & 0.0575 & -0.2156 \\ -0.1082 & 0.0303 & -0.6121 & -0.2101 & 0.2615 & -0.2641 & -0.3189 & 0.1434 & -0.0998 & 0.2903 \\ 0.1283 & -0.0674 & 0.1400 & 0.2028 & -0.0280 & -0.0465 & 0.0437 & -0.5513 & 0.5604 & -0.2691 \\ -0.2138 & 0.3699 & -0.1169 & -0.6623 & -0.4551 & 0.3783 & 0.1182 & -0.5016 & -0.0862 & -0.4655 \\ -0.0911 & -0.1034 & 0.3778 & -0.1493 & -0.1105 & 0.3527 & -0.3846 & -0.2128 & -0.3165 & -0.2054 \\ 0.1125 & -0.3660 & 0.4332 & -0.5569 & -0.2293 & -0.3290 & 0.2790 & -0.1157 & -0.5638 & -0.1953 \\ -0.3960 & -0.1620 & 0.4025 & 0.2107 & -0.0196 & -0.2377 & -0.2373 & 0.1547 & 0.0977 & 0.0800 \end{bmatrix}$$

$$W_2^{12} = \begin{bmatrix} 0.0855 & 0.3288 & 0.4348 & 0.1713 & -0.7663 & 0.4023 & 0.0472 & 0.2536 & -0.2844 & 0.0634 \\ 0.2362 & -0.2277 & -0.1034 & -0.1139 & 0.2007 & -0.1714 & -0.3679 & 0.2167 & 0.4784 & 0.0172 \\ 0.0098 & 0.0028 & -0.3283 & -0.3462 & -0.1171 & 0.2414 & -0.4129 & 0.2509 & 0.2076 & 0.1460 \\ 0.1788 & -0.0425 & -0.1651 & 0.2857 & -0.1349 & -0.3521 & -0.2929 & 0.1954 & -0.2158 & 0.1347 \\ -0.2895 & -0.2624 & 0.0420 & 0.0224 & -0.3483 & -0.1684 & -0.1876 & -0.2866 & 0.2445 & -0.3989 \\ 0.0028 & 0.1618 & -0.1029 & 0.2442 & 0.3043 & -0.0102 & 0.0411 & -0.0067 & -0.4081 & 0.3252 \\ -0.4912 & 0.1473 & -0.1270 & -0.1304 & -0.2958 & -0.1945 & -0.0984 & 0.4077 & 0.3075 & 0.1208 \\ 0.4084 & 0.1503 & 0.1868 & 0.1525 & -0.0501 & -0.4943 & 0.2435 & 0.1552 & 0.5385 & 0.1638 \\ -0.3186 & -0.1130 & -0.3254 & 0.3556 & -0.4369 & 0.2125 & 0.2202 & 0.2060 & -0.5930 & 0.0469 \\ 0.4463 & -0.0621 & -0.1145 & -0.5093 & 0.4363 & -0.0597 & -0.5306 & 0.0928 & 0.3449 & -0.0427 \\ -0.1797 & 0.3613 & -0.1784 & -0.2012 & -0.2043 & -0.0646 & -0.0222 & -0.7456 & 0.4086 & -0.0414 \\ 0.1402 & -0.0392 & -0.2896 & 0.2315 & 0.2370 & -0.4078 & 0.6521 & 0.4905 & 0.0513 & -0.7994 \\ 0.3223 & -0.0632 & -0.4047 & 0.0908 & -0.0013 & 0.0552 & -0.1916 & -0.1391 & 0.2050 & 0.0147 \\ -0.3742 & 0.3820 & 0.1005 & -0.4215 & -0.0651 & -0.5167 & -0.1472 & -0.0169 & 0.1853 & 0.3494 \\ -0.3807 & -0.3590 & -0.2277 & 0.6399 & 0.4686 & 0.3626 & 0.0473 & 0.3613 & 0.1095 & -0.2742 \\ -0.3553 & 0.2018 & -0.1267 & 0.0768 & 0.2757 & -0.2150 & 0.1554 & -0.1614 & 0.2219 & 0.0741 \\ 0.0017 & 0.2709 & -0.5388 & 0.1162 & 0.3433 & -0.1298 & -0.9974 & 0.0752 & -0.4647 & -0.4545 \\ 0.0375 & -0.1886 & -0.2157 & 0.0573 & -0.0877 & 0.2418 & -0.0452 & -0.2511 & -0.3599 & 0.0318 \\ -0.2263 & -0.3769 & -0.0720 & 0.0747 & 0.7048 & 0.3255 & -0.0795 & -0.1142 & 0.1158 & -0.2198 \\ 0.2421 & 0.3883 & 0.4520 & 0.0986 & -0.2329 & -0.0671 & -0.2648 & 0.0817 & -0.1258 & -0.4031 \end{bmatrix}$$

$W_2^{13} =$ 

-0.0699	-0.2786	0.3132	-0.2311	0.5311	-0.1795	0.0057	-0.2071	0.3105	-0.4129
0.0565	0.1539	0.0917	0.0739	-0.7087	-0.6055	0.0042	-0.3921	-0.2475	0.4675
-0.1259	0.3038	0.2270	-0.2245	0.3844	-0.0706	0.0638	0.2025	-0.0540	0.0590
0.0234	0.3972	0.0877	-0.2316	0.1997	0.0419	-0.1502	-0.4275	0.1955	-0.0859
0.1570	-0.0526	0.2742	0.1754	-0.3259	-0.2482	0.5287	0.0457	-0.4347	0.1205
-0.5087	0.0760	-0.4257	-0.0077	-0.0218	0.0869	0.1427	0.1312	0.3157	-0.4198
-0.2217	0.0502	-0.0438	0.0208	-0.1968	-0.5540	-0.4089	0.2934	0.0951	0.3542
-0.1708	-0.3482	-0.4960	0.2410	0.0183	-0.2412	-0.1680	0.3166	0.2250	-0.0398
0.3718	0.3422	0.3975	-0.0863	-0.2030	0.1528	0.4068	-0.0251	0.0492	0.0078
0.4028	0.4615	0.1090	0.1156	-0.3568	0.1864	-0.6357	-0.2302	-0.1253	0.1117
-0.0740	-0.2827	-0.1467	0.4021	0.6949	-0.3080	0.3198	0.2780	0.4309	0.1276
0.1860	-0.4350	-0.4312	-0.1531	-0.3239	-0.0520	-0.2465	0.3619	0.2496	-0.2090
0.0809	0.0668	0.0812	-0.4424	-0.1067	0.3422	-0.0881	0.0520	-0.0215	-0.5115
0.5172	0.3760	0.0809	-0.7620	-0.5001	-0.2588	0.2844	-0.2620	-0.3053	-0.3415
0.2698	-0.4239	-0.3730	0.3059	0.3518	-0.0906	-0.0910	0.1359	0.3771	-0.2021
-0.2298	0.0521	-0.2623	0.0135	0.1225	0.1188	-0.1409	-0.2274	0.2514	0.2313
0.3798	-0.1568	-0.1508	0.4521	0.6071	0.2561	-0.2438	0.1073	-0.1280	-0.3986
0.3585	-0.2401	-0.0743	-0.2903	-0.0078	-0.3241	0.3105	-0.2390	-0.3685	0.4331
0.3053	-0.3773	-0.4161	0.5002	0.0346	0.3143	0.0227	-0.1977	0.2639	0.2110
0.3394	0.5016	0.0109	-0.0589	-0.3949	0.0496	0.2183	0.1665	0.0097	0.4370

 $W_2^{14} =$ 

-0.1003	-0.0370	0.1705	0.3757	-0.1106	-0.1055	0.1950	-0.3201	0.1198	0.0471
0.8243	-0.2554	0.2566	0.2446	-0.0916	-0.0474	-0.0571	0.1449	0.3796	0.2932
0.1677	0.2768	-0.2300	-0.2296	0.0278	0.2110	0.2686	0.3173	0.2900	0.0330
-0.1758	-0.3101	-0.1948	-0.3727	0.1891	0.2768	-0.3649	-0.2959	-0.1486	-0.1727
0.0948	0.1103	0.3648	-0.1276	-0.0519	0.1044	0.4905	-0.0977	0.6066	0.0605
-0.1687	-0.1795	0.0353	-0.3307	0.1032	0.2648	-0.2426	-0.4467	-0.4056	-0.1887
-0.1688	-0.4098	0.4083	-0.0327	0.2358	-0.0436	-0.4492	0.0195	0.1293	-0.1402
-0.5279	-0.2405	0.1281	0.4206	-0.3480	-0.0856	0.3304	0.0740	0.1964	-0.0651
0.0886	0.1714	-0.1295	-0.2579	-0.2522	-0.0181	0.2505	0.0739	0.0330	0.2276
0.3226	-0.3105	-0.3374	-0.0559	0.1096	-0.7143	0.9042	0.0479	-0.5671	0.3158
-0.2547	-0.4316	-0.2945	0.0212	0.2152	0.2699	-0.2377	0.0360	-0.0811	-0.2340
-0.4630	0.7763	0.1918	0.3619	-0.2316	0.0408	0.0469	0.0926	-0.1993	-0.1528
-0.0490	-0.0244	0.2857	0.3686	-0.2645	0.3602	-0.0298	0.2185	-0.0616	-0.1071
0.1774	-0.6624	-0.1245	-0.0147	0.0436	0.1799	0.0019	0.2456	0.2739	0.2525
0.1808	0.5746	-0.2382	0.1134	0.1665	-0.1328	0.3476	-0.1855	0.1078	-0.0039
-0.0026	-0.0836	0.4504	0.3571	-0.0301	-0.1980	0.3939	-0.1130	0.3491	0.2164
0.1046	-0.3057	0.3528	0.1972	-0.3366	0.0102	-0.1456	-0.2228	-0.0856	0.5815
0.1595	-0.6471	-0.0674	0.2524	0.1703	0.3629	0.1667	0.3425	0.4522	-0.0763
0.0179	0.2020	-0.1834	0.3564	0.0269	0.3620	-0.1230	-0.1272	-0.4097	-0.3363
-0.2228	0.2022	0.1241	0.0302	-0.0539	0.3272	0.1068	-0.2106	0.0155	0.1196

$W_2^{21} =$ 

-0.0189	-0.3559	-0.0741	-0.6089	0.0876	0.1777	-0.1187	-0.0863	0.2980	-0.2174
0.3759	-0.0926	-0.2456	0.0256	-0.2314	0.1689	-0.4636	0.1050	0.1545	0.1368
-0.1510	-0.5205	0.0328	0.2866	0.3340	0.3341	-0.1192	-0.3083	0.0596	0.0711
-0.4313	-0.0437	-0.2646	-0.3701	0.3201	0.4083	-0.1067	0.3503	0.2574	0.3198
0.0119	-0.2323	-0.1207	-0.5133	-0.2561	0.1361	0.0104	-0.0062	0.0483	0.0712
0.5025	-0.1253	-0.2278	-0.2673	-0.0614	0.1205	-0.3402	-0.1352	0.5025	0.3272
-0.0857	-0.1664	-0.2700	-0.1539	0.1490	-0.1804	-0.2340	-0.2125	-0.6848	0.3771
0.5061	-0.1148	0.1302	0.0095	0.2947	0.2197	-0.2575	-0.2828	-0.1977	0.0102
-0.1756	-0.2382	0.2361	-0.3840	-0.0447	0.3914	0.4104	-0.0495	0.0080	-0.0286
0.2182	-0.1077	-0.0707	0.2086	0.0606	0.0254	-0.0762	-0.0389	0.4900	0.6047
-0.3305	0.4277	-0.1008	0.2097	-0.0915	0.3640	0.2043	0.4577	0.1736	-0.3757
0.0353	-0.2492	0.1788	0.2838	-0.1709	-0.0096	0.2178	-0.5084	-0.1835	0.0145
-0.4902	0.2563	0.2598	-0.0085	0.4083	0.2261	0.0812	0.4909	-0.3496	0.3334
0.3260	0.0120	0.4481	0.0668	0.2303	0.1301	-0.3334	-0.1081	0.0632	-0.0908
-0.1991	0.6020	0.3784	-0.2944	0.1039	0.0745	0.4562	0.5292	0.2551	-0.0992
0.1893	0.2824	-0.1030	0.0579	-0.2134	-0.2259	0.3080	0.1129	-0.1850	-0.3240
0.1700	-0.2224	0.0882	0.3464	0.1270	-0.0438	-0.4009	0.3519	-0.3164	0.3205
-0.3540	-0.1449	-0.0555	-0.2703	0.1353	0.4871	-0.0452	-0.3894	-0.3813	-0.3056
0.3908	-0.0595	0.1040	-0.1584	0.3689	-0.1364	-0.1652	0.2418	0.3963	-0.1291
-0.1178	-0.0581	0.1267	0.2102	-0.4182	0.0832	0.1998	-0.6308	0.3513	0.3788

 $W_2^{22} =$ 

-0.1064	-0.3841	-0.4095	-0.1159	-0.1262	-0.5936	-0.0234	-0.4027	0.2556	0.2845
-0.2954	0.0877	0.4275	-0.3613	0.5907	-0.2714	0.5460	0.0494	0.2191	0.0134
-0.3764	-0.0670	-0.2946	0.2773	-0.1348	0.2450	-0.1244	0.4559	0.0337	-0.5276
-0.5056	-0.6076	0.2775	-0.3907	0.6068	0.6178	-0.2614	-0.3460	-0.2467	-0.8650
0.2859	0.0438	0.4334	0.4666	-0.3860	0.0794	-0.0629	-0.5456	-0.3377	0.0898
-0.3423	0.1440	0.3010	-0.1378	-0.0153	0.3690	-0.3446	0.3756	0.4046	-0.0658
-0.1958	0.0062	-0.3403	0.2095	0.0187	-0.1156	0.1905	-0.3417	0.0469	-0.1865
-0.2055	0.3218	0.0300	-0.2484	0.4139	0.5478	-0.1578	-0.2080	0.3158	0.0704
-0.4374	0.2273	-0.2734	-0.2913	-0.2417	0.1901	-0.2325	0.2288	-0.6703	0.5147
0.2057	-0.2482	-0.3520	0.2485	-0.4204	0.0358	0.2943	-0.3167	0.0861	0.2348
0.1598	0.1660	0.0623	-0.3947	0.1011	-0.0744	0.0467	0.4135	-0.3742	0.2812
0.1253	-0.4092	-0.1109	0.1077	-0.0415	0.0427	0.3464	-0.2287	0.2035	-0.0925
0.2202	0.2659	-0.1385	0.0160	-0.1016	0.1840	-0.2271	-0.1206	0.3917	0.3518
0.0879	0.2289	-0.1279	0.1481	0.0225	0.2134	0.5508	-0.0026	0.1510	-0.1619
-0.1724	0.1933	0.3659	-0.3005	-0.8112	0.2413	-0.1654	0.2837	-0.4141	0.0486
0.1156	0.0045	-0.0181	0.4040	-0.0255	0.2950	-0.2270	0.1511	-0.2691	-0.3426
-0.3383	-0.1429	0.1918	-0.1310	-0.0184	0.2202	-0.3565	0.2190	0.2355	-0.2373
-0.3760	0.0566	-0.0211	-0.2160	0.3045	0.3868	-0.1971	0.2773	0.0891	0.0620
-0.2912	0.4396	-0.4346	-0.2366	-0.3071	0.2145	0.3994	-0.3935	-0.1510	0.4442
0.4361	-0.1668	-0.2130	-0.0621	0.1946	0.3816	-0.2991	0.1040	0.2549	-0.1294

$W_2^{23} =$ 

0.5386	0.3861	0.2537	-0.0729	-0.4541	0.1492	0.1264	-0.0201	-0.3967	0.0201
0.3750	0.0298	-0.1581	-0.0535	-0.0338	-0.2358	0.2651	0.2194	-0.5711	-0.1308
0.0042	-0.1597	-0.3302	0.1142	-0.2719	0.0971	-0.0022	-0.2907	-0.2796	-0.1051
-0.1153	-0.9255	0.3084	0.5829	0.3354	-0.3314	0.2623	0.1988	0.3094	-0.0451
0.3583	0.3957	0.2343	-0.2040	-0.2100	0.0517	0.3108	-0.4511	-0.0850	0.6045
0.1919	0.0144	0.1850	-0.1566	0.2536	0.3763	-0.4543	0.2990	0.2249	-0.2772
-0.6290	-0.0795	-0.4178	-0.3237	0.3724	0.3846	0.1414	0.0420	0.0122	0.2063
-0.3786	-0.2635	-0.2066	0.0591	-0.1819	0.1040	0.2079	0.4008	-0.3700	-0.3601
0.1368	0.1583	-0.4126	0.0466	0.0954	0.2898	0.2852	-0.3216	0.2981	-0.0491
0.0246	0.6651	0.3037	0.0597	-0.3426	0.1625	-0.0135	0.5118	-0.1733	-0.3099
-0.2372	0.3400	-0.1148	-0.3586	-0.0812	0.3540	0.4370	-0.1065	-0.2165	0.2638
0.0651	-0.5943	0.0954	-0.1556	-0.2201	-0.1520	-0.4314	0.2049	0.1470	-0.3857
0.3104	0.0023	0.2604	0.3514	-0.2524	-0.2207	-0.0857	-0.2291	-0.2174	-0.3908
-0.5080	-0.5559	-0.0761	0.7092	0.5336	0.5931	-0.2089	-0.4746	-0.2330	-0.0477
0.0051	-0.1441	0.1236	0.7188	-0.0772	-0.0147	-0.1415	0.1238	0.1085	0.2713
-0.1576	-0.0228	0.3029	0.2816	-0.2522	0.0182	0.2867	-0.3255	0.3534	-0.3012
0.4446	-0.2355	0.1431	-0.3735	-0.0683	0.2183	-0.2633	-0.1799	0.0590	0.0164
0.2997	-0.0411	0.3177	0.2814	0.1454	0.1079	0.3032	0.0421	-0.3335	0.2469
-0.0179	0.2586	-0.0693	0.4446	0.0313	-0.0053	0.3327	0.3915	-0.2748	0.2161
-0.3426	0.0929	-0.2201	0.1425	-0.0802	-0.0944	-0.2678	0.1415	0.0339	-0.1837

 $W_2^{24} =$ 

0.3218	-0.2708	0.1073	0.2842	-0.5635	0.2724	-0.3667	-0.2854	-0.1794	0.2400
-0.2878	-0.3316	0.1478	-0.2865	-0.0112	-0.1144	0.2715	0.3268	0.0847	-0.3523
0.1635	0.1233	-0.2013	0.2906	-0.3202	0.2091	0.3442	0.1910	0.1860	-0.0060
-0.1905	0.4856	-0.1255	-0.1175	-0.3390	-0.0854	-0.1003	0.0978	-0.1527	-0.1875
-0.2900	0.3049	-0.3411	0.1057	0.4482	0.3789	0.2851	-0.4178	0.2706	0.1918
0.1429	-0.1478	0.2090	0.5585	0.1528	0.0189	0.6879	-0.2261	-0.2481	0.1764
0.1096	0.2051	-0.0962	0.0766	-0.3444	0.4642	0.1163	0.3581	0.4291	0.0566
0.4039	-0.1529	-0.1118	-0.1160	-0.0236	-0.2890	-0.0477	-0.0892	-0.2390	-0.3780
0.3384	0.2463	-0.2452	-0.1999	0.1413	0.2809	-0.2616	-0.0939	-0.4245	-0.1690
-0.1533	-0.3465	0.2933	-0.1208	-0.3649	0.1191	-0.0908	-0.3601	0.4313	-0.3863
-0.1503	-0.1916	0.0302	-0.3338	0.2879	-0.3072	-0.2094	-0.3166	-0.2324	0.2507
-0.2219	-0.1566	0.0966	0.0543	-0.2197	0.4083	-0.1294	-0.2628	-0.5295	0.4049
0.3351	0.1945	0.1978	-0.3405	0.0240	0.0936	0.0635	0.4197	-0.1465	-0.4213
0.1389	0.2358	0.0651	-0.1064	-0.3383	0.4611	-0.0396	0.3602	-0.3240	0.2757
0.3163	-0.2507	0.3092	0.3165	0.1626	0.3342	-0.1246	0.0566	0.0942	0.1481
-0.0355	0.2732	-0.2541	0.0051	-0.3797	0.2880	0.3228	-0.3687	0.1341	0.1485
-0.1606	0.1987	0.0652	-0.0341	0.3710	0.0259	-0.2401	-0.1107	0.3152	0.4217
-0.2974	0.2299	0.3309	0.0809	-0.3291	0.2985	-0.1360	-0.1994	0.3282	-0.5106
0.0901	-0.1046	-0.0046	-0.1579	-0.1102	0.3779	0.1151	-0.0956	0.1217	-0.0710
-0.3404	-0.1795	0.2129	0.1587	0.2114	0.0269	0.0766	0.3654	-0.1227	0.1054

$W_3$  is a  $5 \times 40$  matrix partitioned into 4 matrices,  $5 \times 10$  each as given by:

$$W_3 = [W_3^{11} \quad W_3^{12} \quad W_3^{13} \quad W_3^{14}] \text{ where,}$$

$$W_3^{11} = \begin{bmatrix} -0.1976 & -0.0062 & -0.2435 & -0.8133 & 0.7924 & -0.7625 & 0.2489 & -0.2694 & -0.5101 & 0.5667 \\ -1.1264 & -0.0277 & -0.3543 & 0.4852 & 0.5260 & -0.3719 & -0.8596 & 0.2028 & -0.0072 & 0.2359 \\ 0.0299 & -0.3397 & -0.1069 & 0.6634 & -0.3752 & 0.4787 & 0.8010 & -0.3661 & 0.3112 & -0.3577 \\ -0.5161 & 0.2658 & 0.3693 & -0.0138 & -0.2331 & 0.0352 & -0.0203 & -0.7094 & 0.6768 & 1.2261 \\ 0.1829 & -0.8631 & -0.0467 & 0.2779 & -0.8353 & 0.7396 & -0.2017 & -0.6158 & 0.3161 & 0.2648 \end{bmatrix}$$

$$W_3^{12} = \begin{bmatrix} -0.5029 & 0.0508 & 0.8987 & -0.0344 & -0.4517 & 0.4460 & 0.8904 & 0.8185 & 0.4635 & 0.2938 \\ -0.0691 & 0.4455 & 0.3532 & 0.4029 & 0.7242 & -0.3235 & -0.1367 & 0.3664 & -0.4003 & -0.1427 \\ -0.0525 & 0.1615 & -0.5422 & 0.3739 & -0.1576 & -0.9592 & -0.2441 & -0.2462 & 0.8325 & -0.6075 \\ -0.8986 & 0.4823 & 0.6196 & -0.0808 & 0.3072 & -0.0496 & 0.4944 & -0.0840 & -0.5974 & -0.4376 \\ 0.5202 & 0.4840 & -0.0468 & -0.4023 & 0.4079 & -0.3894 & 0.1888 & 0.7512 & -0.6961 & -0.2969 \end{bmatrix}$$

$$W_3^{13} = \begin{bmatrix} 0.8042 & 0.5544 & -0.1341 & 0.2306 & -0.9215 & -0.2144 & -0.6330 & 0.9115 & 0.9118 & 1.0744 \\ -0.1617 & -0.0919 & -0.1262 & -0.2613 & -0.4973 & 0.1349 & -0.3722 & -0.1774 & 0.0213 & 0.2202 \\ -0.3453 & 0.6452 & -0.3988 & 0.0180 & 0.4648 & 0.5072 & -0.0032 & -0.4422 & -0.2103 & -0.4012 \\ 0.4349 & 0.1037 & -0.7849 & 0.1662 & -0.2558 & -0.0158 & 0.8416 & 0.4667 & 0.8623 & -0.0233 \\ -0.6477 & 0.2480 & -0.5190 & 0.1308 & 0.2707 & -1.1011 & 0.4023 & -1.1509 & -0.5225 & -0.8820 \end{bmatrix}$$

$$W_3^{14} = \begin{bmatrix} 0.5290 & 0.3536 & 0.0598 & -0.1588 & -0.2978 & -0.2512 & -0.5830 & -0.5929 & 0.5540 & 0.1788 \\ 0.5081 & 0.5851 & 0.7387 & 0.4778 & 0.3706 & 0.5309 & 0.1142 & 0.3688 & 0.8421 & 0.2399 \\ 0.2232 & -0.3588 & 0.9793 & 0.5041 & -0.3990 & 0.3559 & 0.2512 & 0.4808 & 0.1291 & 0.8196 \\ 0.2073 & -0.3477 & 0.4763 & -0.1402 & -1.0383 & 0.3551 & -0.0641 & -0.4659 & -0.0000 & 0.5758 \\ -0.6448 & -0.5063 & -0.3595 & -0.6040 & -0.1629 & 0.3165 & -0.1749 & -0.2796 & 0.6127 & -0.0615 \end{bmatrix}$$

$B_1$  is a  $40 \times 1$  matrix partitioned into 4 matrices,  $10 \times 1$  each as given by:

$$B_1 = [B_1^{11} \quad B_1^{12} \quad B_1^{13} \quad B_1^{14}]^T \text{ where,}$$

$$B_1^{11} = \begin{bmatrix} 1.9253 & -1.8944 & -1.6577 & -1.0650 & -1.4495 & -1.3627 & 1.3385 & -1.1674 & 1.1614 & 1.0747 \end{bmatrix}$$

$$B_1^{12} = \begin{bmatrix} -0.6676 & -0.7999 & -0.9107 & 0.9741 & -0.7753 & -0.0458 & 0.2860 & 0.6338 & 0.3574 & -0.6184 \end{bmatrix}$$

$$B_1^{13} = \begin{bmatrix} 0.1741 & -0.0946 & 0.0453 & -0.6662 & -0.2802 & 0.5379 & 0.1189 & 0.7994 & 1.0076 & -0.9481 \end{bmatrix}$$

$$B_1^{14} = \begin{bmatrix} -0.4564 & 1.3027 & 1.4972 & 1.1793 & -1.3394 & 1.4328 & -1.5327 & -1.6878 & -1.4817 & 1.8290 \end{bmatrix}$$

$B_2$  is a  $40 \times 1$  matrix partitioned into 4 matrices,  $10 \times 1$  each as given by:

$$B_2 = [B_2^{11} \quad B_2^{12} \quad B_2^{13} \quad B_2^{14}]^T \text{ where,}$$

$$B_2^{11} = \begin{bmatrix} 1.4243 & -1.4975 & -1.3573 & -1.3030 & 1.0483 & 1.2013 & 1.2121 & -1.0728 & 1.0658 & 0.9745 \end{bmatrix}$$

$$B_2^{12} = \begin{bmatrix} -0.6968 & -0.4883 & 0.6918 & 0.5008 & 0.5242 & -0.3790 & 0.5102 & 0.1615 & -0.1662 & 0.0326 \end{bmatrix}$$

$$B_2^{13} = \begin{bmatrix} 0.0830 & -0.1232 & -0.1865 & -0.1685 & 0.1547 & 0.6500 & 0.5327 & 0.5249 & -0.5434 & 0.7434 \end{bmatrix}$$

$$B_2^{14} = \begin{bmatrix} -0.8306 & -0.7240 & -1.1770 & 0.9754 & 1.1814 & 1.2333 & 1.2800 & -1.5098 & 1.3730 & -1.4018 \end{bmatrix}$$

$B_3$  is a  $5 \times 1$  matrix given by:

$$B_3 = [0.1158 \quad 0.7870 \quad 0.4279 \quad -0.6710 \quad -0.5003]^T$$