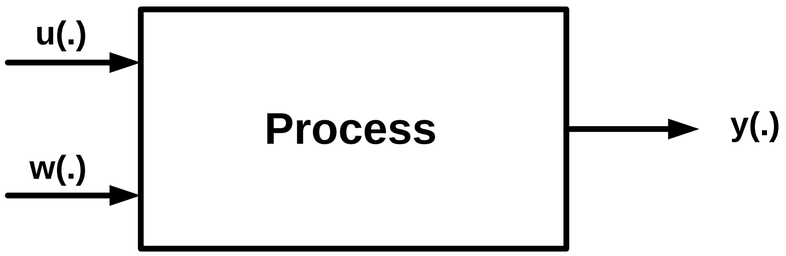
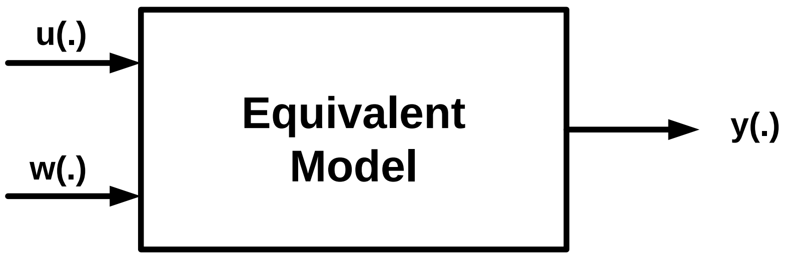
Lecture #3 Identification

ECE 842

Ali Keyhani

Identification is the determination on the basis of input and output, of a process (model), within a specified class of processes (models), to which the process under test is equivalent.





The equivalent model is a model which has captured the underlying mechanism which has generated the original process.

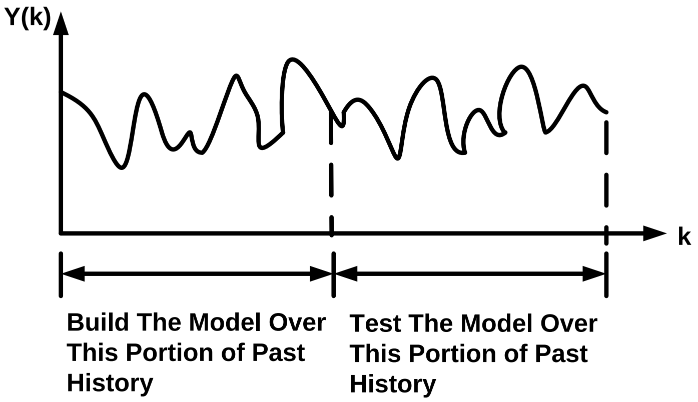
Three – Step Modeling Procedure for Process {y(.)} excited by white noise

1. Model Identification—Means the determination of n and m for process.
2. Parameter Estimation; The determination of $\{ \hat{a}(k) \}$ from { Y(j), J < k }
3. Diagnostic Checking

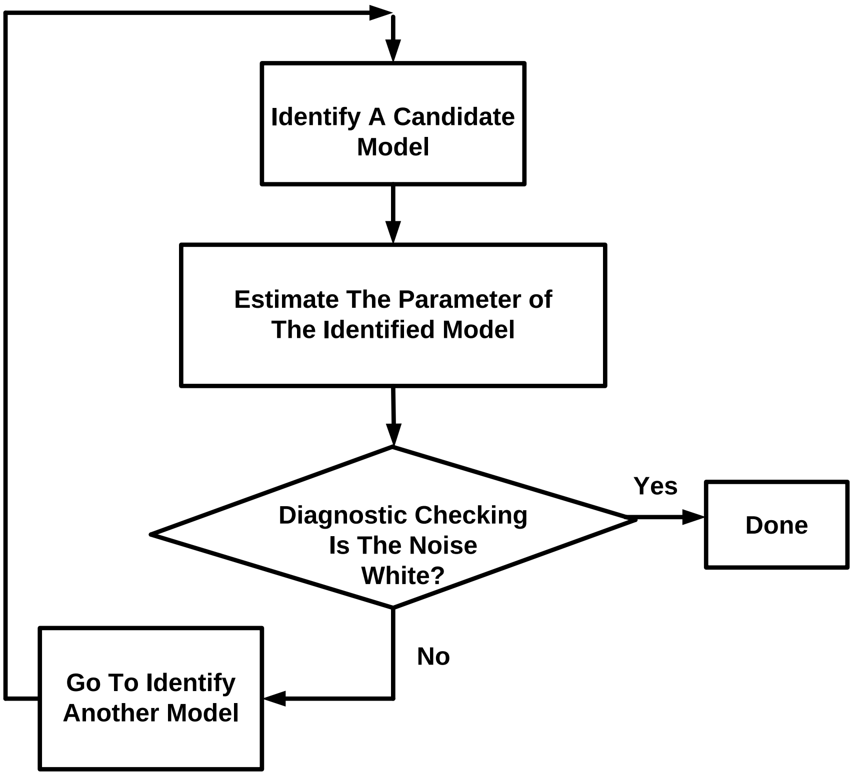
   Checking the {w(.)} sequence to establish that the noise indeed is white.

Classical Approach to Identification and Modeling

Given



A 3 – Step Procedure for Modeling IS:



Covariance: A partial indication of the degree to which one variable is related to another is given by the covariance, which is the expectation of the product of the deviation of two random variables from their means.

{Y(.)}	random process Y(.)
{x(.)}	random process x(.)

$$\text{Cov}\,\{x(k)\,Y(k)\} = E\,[(x(k) - u_x)\,(Y(k) - u_y)]$$

$$= E\,[x(k)\,Y(k)] - u_x\,u_y$$

Where

$$u_x = E\,[x(k)]$$

$$u_y = E\,[Y(k)]$$

Cross – Correlation Coefficient

$$\rho = \frac{E[x(k)Y(k)] - u_x u_y}{\sigma_x \sigma_y}$$

Variance of x $\quad \sigma_x^2 = E[(x(k) - u_x)^2]$

Variance of y $\quad \sigma_y^2 = E[(Y(k) - u_y)^2]$

Notes:

1) The correlation coefficient is a measure of the degree of linear dependence between x and y.
2) If x and y are independent, $\rho = 0$ (the inverse is not true).
3) If Y is a linear function of x, $\rho = \pm 1$

Zero Mean Process: Given process {Y(.)}, if the mean of the process is estimated and then subtracted from each Y(k), we obtain a zero mean process.

Ex.

$$\{Y_1(k)\}_{k=1}^{N}$$

$$\hat{u}_{y1} = \frac{1}{N}\sum Y_1(k)$$

$$Y(1) = Y_1(1) - \hat{u}_{y1}$$

$$Y(2) = Y_1(2) - \hat{u}_{y1}$$

.

. $\quad \rightarrow \{Y(k)\}_{k=1}^{N} \quad is \quad a \quad zero \quad mean \quad process$

.

$$Y(k) = Y_1(k) - \hat{u}_{y1}$$

For zero mean processes {x(.)} and {Y(.)}

Cov { x(k) Y(k) } = E [x(k) Y(k)] = γ_{xy}

γ_{xy} is called cross covariance or correlation between x and Y variables

The basic cross correlation or covariance equation

E [x(k) Y(k+j)] = γ_{xy} (j)

E [Y(k) x(k+j)] = γ_{yx} (-j); $\quad$ i.e. $\quad \gamma_{xy}$ (j) = γ_{yx} (-j)

For j = 0

E [x(k) Y(k)] = $\gamma_{xy} = \gamma_{xy}(0)$

The sample estimate of cross – correlation for covariance,

$$\hat{\gamma}_{xy}(j) = \frac{1}{N-j}\sum_{k=1}^{N-j} x(k)Y(k+j) \qquad j = 0,1,.........m$$

$$\hat{\gamma}_{yx}(-j) = \frac{1}{N-j}\sum_{k=1}^{N-j} Y(k)x(k+j) \qquad j = 0,1,.........m$$

Sample cross – correlation coefficient of zero mean {Y(.)} and {x(.)} processes.

$$\hat{\rho}_{xy}(0) = \frac{\hat{E}[x(k)Y(k)]}{\hat{\sigma}_x \hat{\sigma}_y}$$

$$\hat{\gamma}_{xy}(0) = \hat{E}[x(k)Y(k)] = \frac{1}{N}\sum_{k=1}^{N} x(k)Y(k)$$

$$\hat{\rho}_{xy}(0) = \frac{\hat{\gamma}_{xy}(0)}{\hat{\sigma}_x \hat{\sigma}_y}$$

$$\hat{\gamma}_{0x} = \hat{\sigma}_x^2 = \frac{1}{N-1}\sum_{k=1}^{N}(x(k))^2$$

$$\hat{\gamma}_{0y} = \hat{\sigma}_y^2 = \frac{1}{N-1}\sum_{k=1}^{N}(Y(k))^2$$

J – Period apart cross correlation for process {x(.)} and {Y(.)} with zero mean.

γ_{xy} (j) = E [x(k) Y(k+j)] = γ_{xy} (j)

Cross – correlation coeff.

$$\rho_{xy}(j) = \frac{\gamma_{xy}(j)}{\sigma_x \sigma_y}$$

Same cross – correlation coeff.

$$\hat{\rho}_{xy}(0) = \frac{\hat{\gamma}_{xy}(0)}{\hat{\sigma}_x \hat{\sigma}_y}$$

Historically, the correlation function is defined without restriction on mean and without normalization:

Correlation function $\hat{R}_{xy}(j) = \frac{1}{N-j}\sum_{k=1}^{N-1} x(k)Y(k+j) \qquad j = 0,1,..........m$

$$\hat{R}_{xy}(j) = \hat{E}[x(k)Y(k+j)]$$

The covariance function has the mean value removed

$$\gamma_{xy}(j) = \hat{C}_{xy}(j) = \frac{1}{N-j}\sum_{k=1}^{N-j}[x(k)-u_x][Y(k+j)-u_y] \qquad j = 0,1,..........m$$

In practice, the mean value usually has been subtracted from the data.

The correlation coefficient function $\rho_{xy}(j) = \frac{\gamma_{xy}(j)}{\sigma_x \sigma_y}$, $\sigma_x = \sqrt{\gamma_x(0)}$, $\sigma_y = \sqrt{\gamma_y(0)}$

j – Period Apart, auto-covariance for process {Y(.)} with zero mean E [Y(k)] = 0

$$\gamma(j) = e\,[\,(Y(k)\;Y(k+j)\,]$$

Autocorrelation coefficient

$$\rho(j) = \frac{\gamma(j)}{\gamma_0}$$

Sample correlation coefficient

$$\hat{\rho}(j) = \frac{\hat{E}[Y(k)Y(k+j)]}{\hat{\gamma}_0} = \frac{\hat{\gamma}(j)}{\hat{\gamma}_0}$$

Where

$$\hat{\gamma}_{yy}(j) = \hat{E}[Y(j)Y(k+j)] = \frac{1}{N-j}\sum_{k=1}^{N-j} Y(k)Y(k+j) \qquad j = 0,1,2..........$$

$$\hat{\gamma}_0 = \frac{1}{N-1}\sum_{k=1}^{N}(Y(k))^2$$

Note that j period apart autocorrelation is a partial indication of the degree to which present is dependent to the past. i.e.

1) The autocorrelation is a measure of the degree of linear dependence between the present and the past.

2) If Y(k) and Y(k+j) are independent, then $\hat{\rho}_{yy}(y) = 0$.

3) $R_{YY}(0) = {\gamma_0}^2$ i.e the mean-square value of the random process can always be obtained by setting the j = 0.

4) $R_{YY}(j) = R_{YY}(-j)$

5) $|R_{YY}(j)| \leq R_{YY}(0)$ the largest value of the autocorrelation always occurs at j = 0.