Lecture #4
Three-phase Induction Machines

A. Keyhani

Commonly Used Reference Frames.

- q-d axis voltage equations can be written by inspection of Fig. 1

$$v_{qs} = r_s i_{qs} + \omega \lambda_{ds} + p\lambda_{qs}$$

$$v_{ds} = r_s i_{ds} - \omega \lambda_{qs} + p\lambda_{ds}$$

$$v_{os} = r_s i_{os} + p\lambda_{os}$$

$$v'_{qr} = r'_r i'_{qr} + (\omega - \omega_r)\lambda'_{dr} + p\lambda'_{qr}$$

$$v'_{dr} = r'_r i'_{dr} - (\omega - \omega_r)\lambda'_{qr} + p\lambda'_{dr}$$

$$v'_{or} = r'_r i'_{or} + p\lambda'_{or}$$

and flux linkage equations are

$$\lambda_{qs} = L_{ls} i_{qs} + L_m (i_{qs} + i'_{qr})$$

$$\lambda_{ds} = L_{ls} i_{ds} + L_m (i_{ds} + i'_{dr})$$

$$\lambda_{os} = L_{ls} i_{os}$$

$$\lambda'_{qr} = L'_{lr} i'_{qr} + L_m (i_{qs} + i'_{qr})$$

$$\lambda'_{dr} = L'_{lr} i'_{dr} + L_m (i_{ds} + i'_{dr})$$

$$\lambda'_{or} = L'_{lr} i'_{or}$$

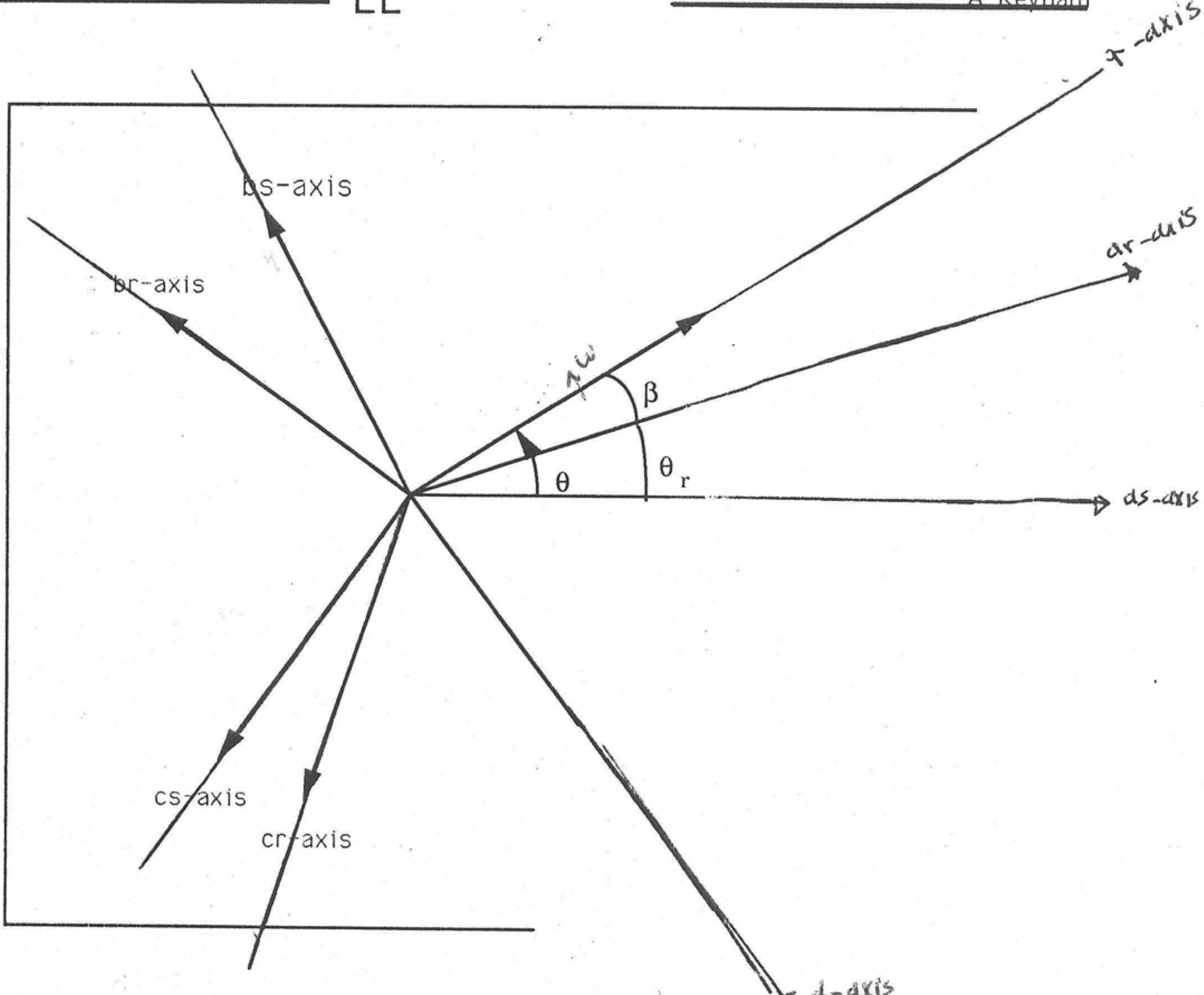

Fig. 1 Axis of 2-pole, 3-phase symmetrical machine

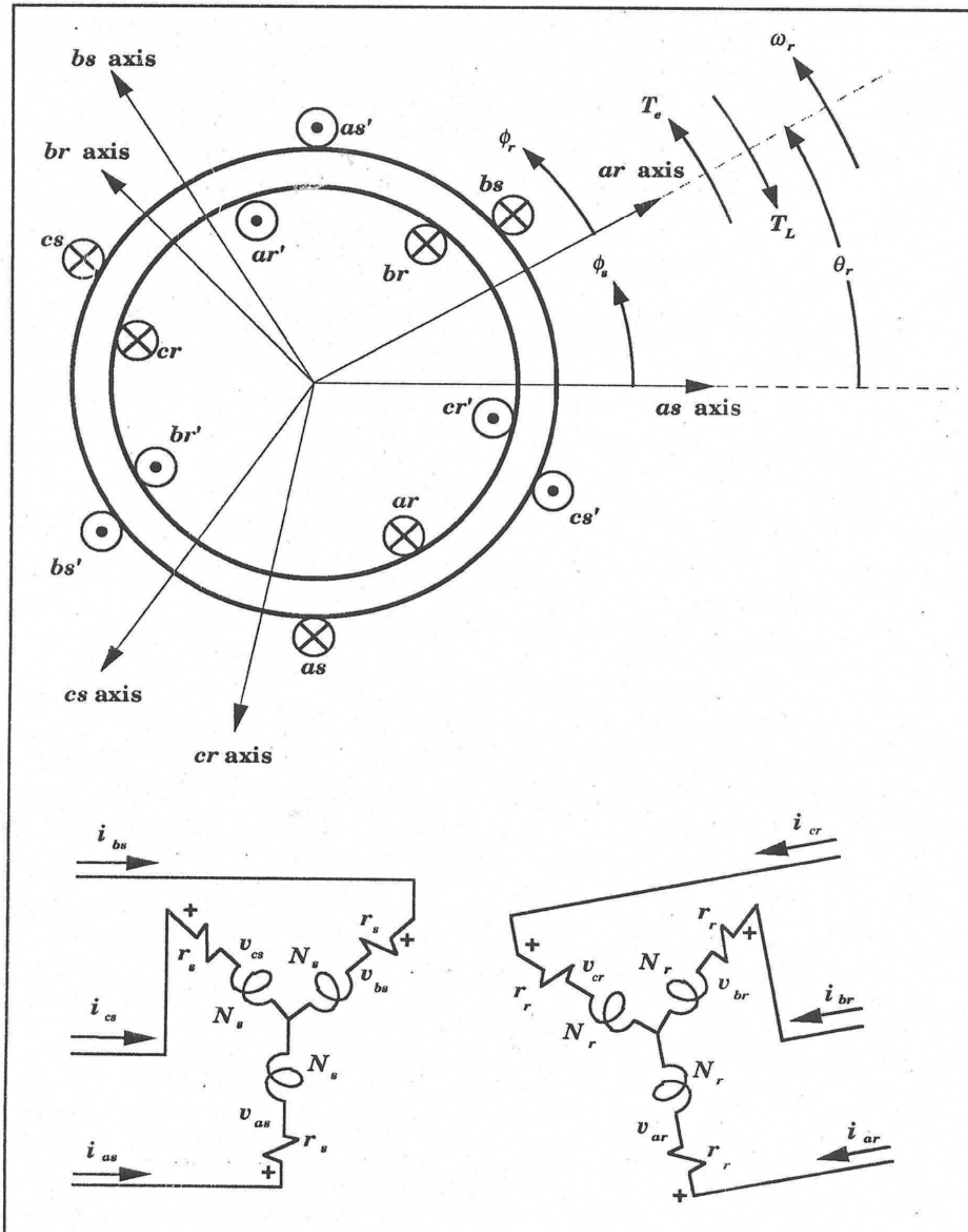

FIGURE 1
A two-pole three-phase symmetrical induction machine.

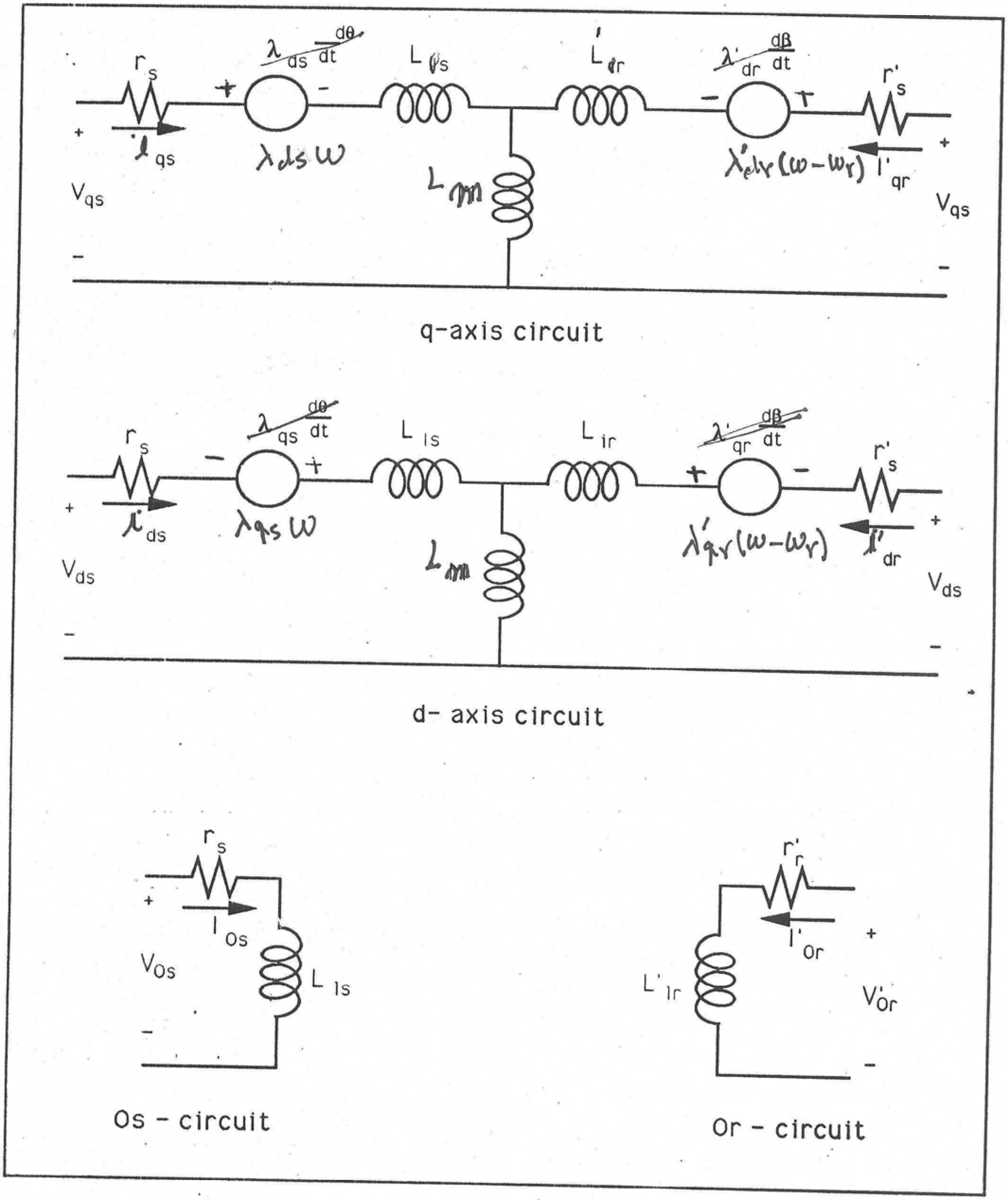

Fig. 3 Equivalent circuits of a 3-phase, symmetrical induction machine with rotating d-q axis at speed of ω

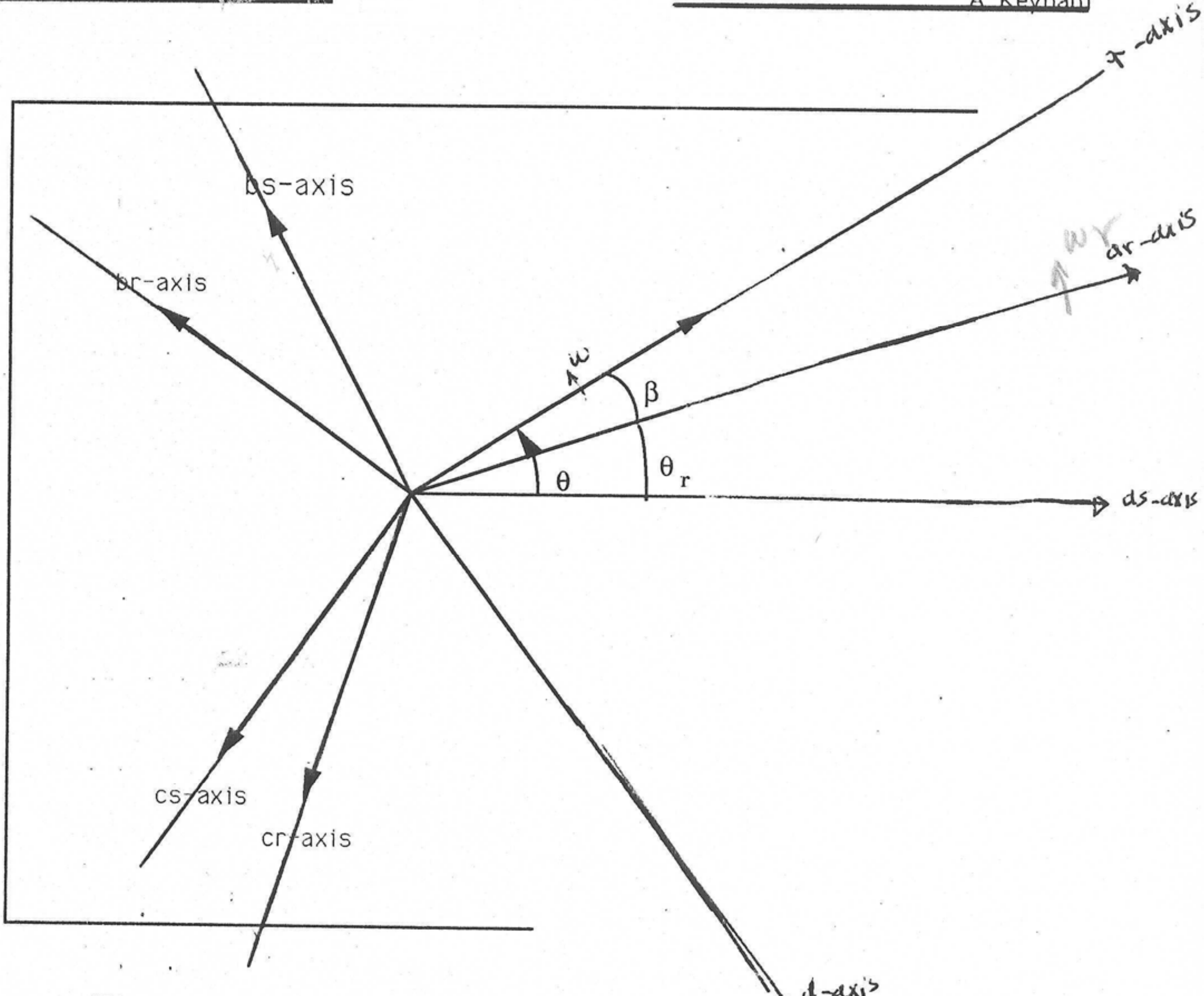

Fig. 1 Axis of 2-pole, 3-phase symmetrical machine (set w = 0)
$\theta = 0$

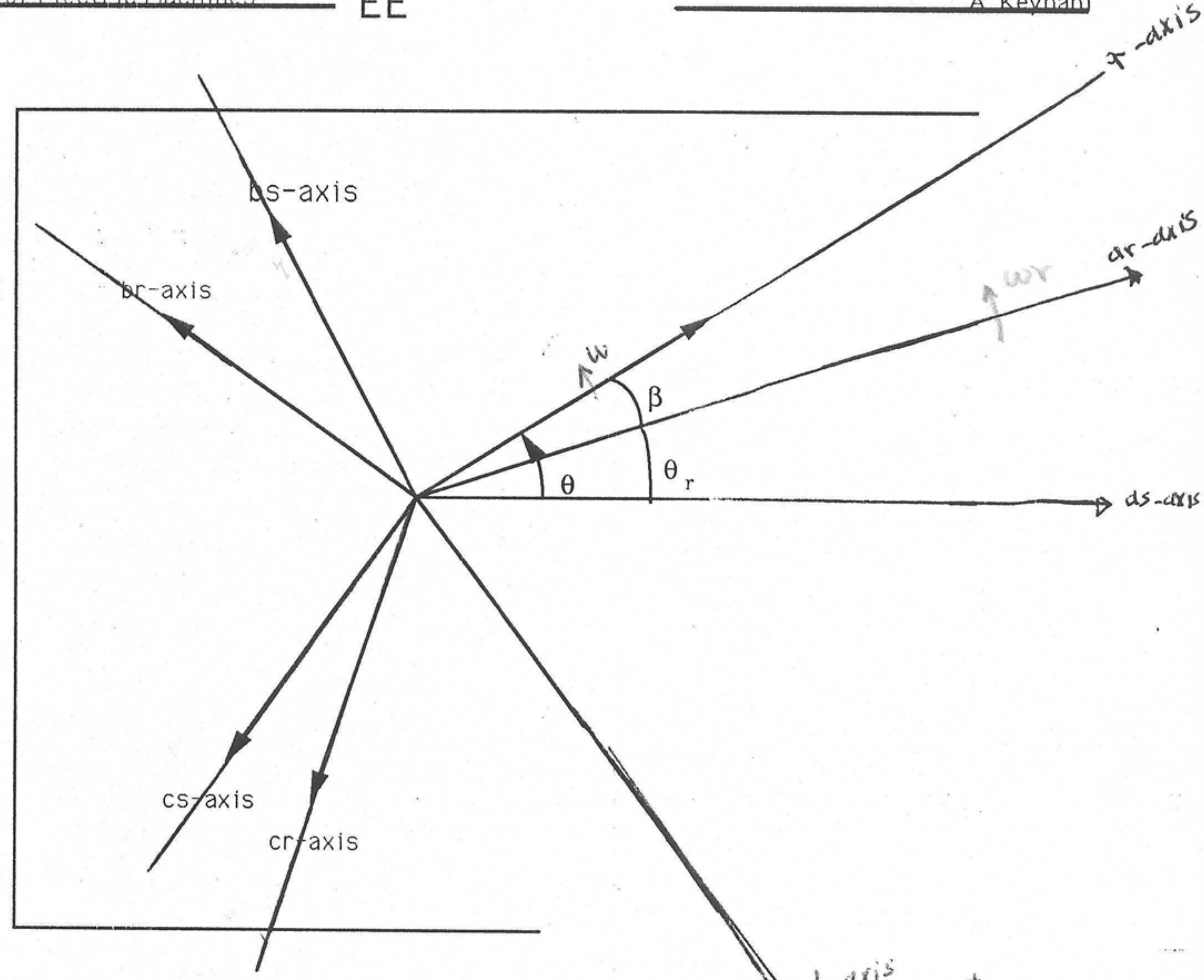

Fig. 1 Axis of 2-pole, 3-phase symmetrical machine

set
(ω = ωr)
β = 0

when $\omega = 0$ (stationary reference frame),
voltage equations became

$$v_{qs} = r_s i_{qs} + p\lambda_{qs}$$
$$v_{ds} = r_s i_{ds} + p\lambda_{ds}$$
$$v_{os} = r_s i_{os} + p\lambda_{os}$$

$$v'_{qr} = r'_r i'_{qr} - \omega_r \lambda'_{dr} + p\lambda'_{qr}$$
$$v'_{dr} = r'_r i'_{dr} + \omega_r \lambda'_{qr} + p\lambda'_{dr}$$
$$v'_{or} = r'_r i'_{or} + p\lambda'_{or}$$

when $\omega = \omega_r$ (rotor reference frame)
which is also referred to as Park's transformation,
voltage equations become

$$v_{qs} = r_s i_{qs} + \omega_r \lambda_{ds} + p\lambda_{qs}$$
$$v_{ds} = r_s i_{ds} - \omega_r \lambda_{qs} + p\lambda_{ds}$$
$$v_{os} = r_s i_{os} + p\lambda_{os}$$

$$v'_{qr} = r'_r i'_{qr} + p\lambda'_{qr}$$
$$v'_{dr} = r'_r i'_{dr} + p\lambda'_{dr}$$
$$v'_{or} = r'_r i'_{or} + p\lambda'_{or}$$

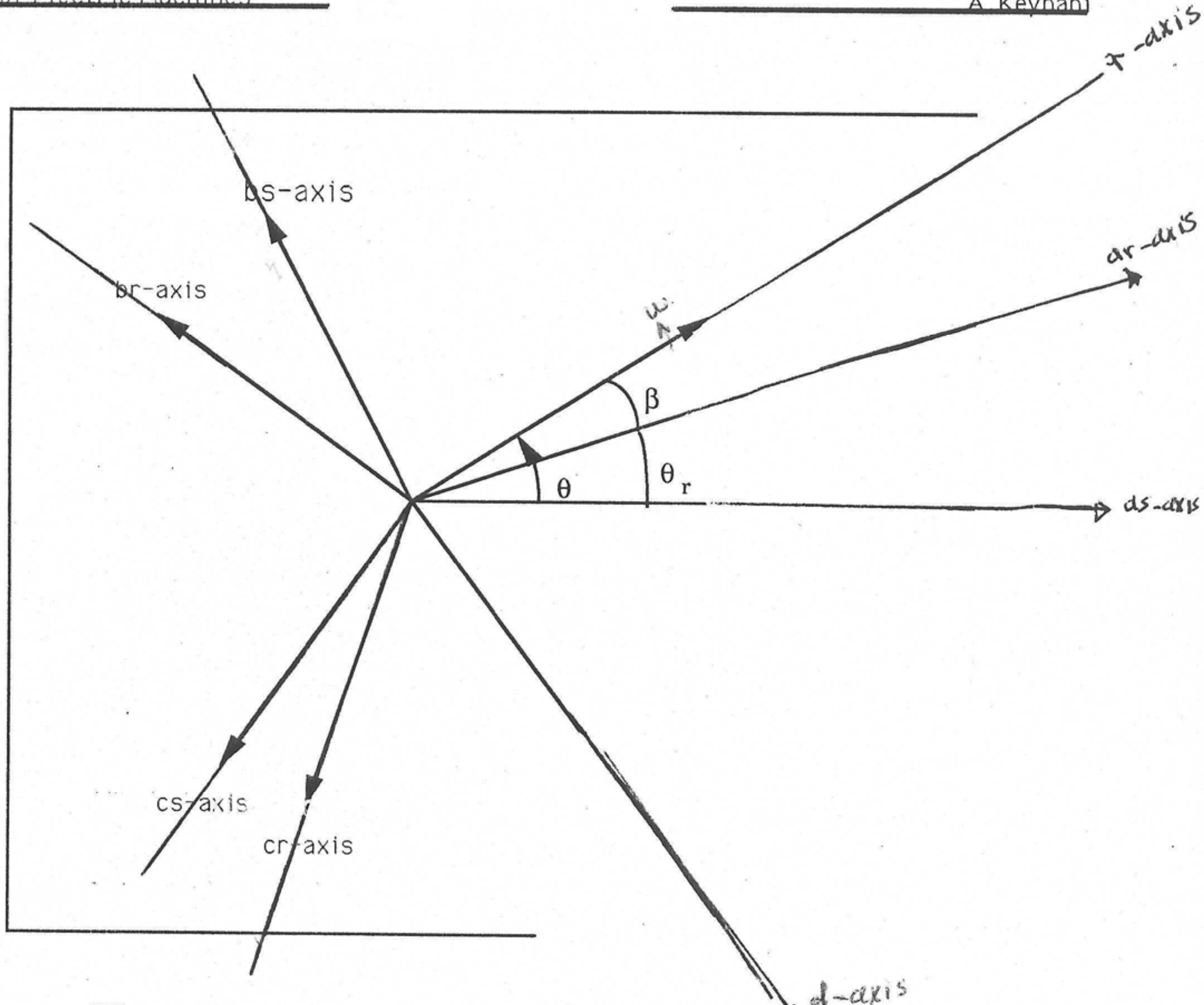

Fig. 1 Axis of 2-pole, 3-phase symmetrical machine (set ω = ωe)

when $\omega = \omega_e$ (synchronously rotating reference frame) the voltage equations become,

$$v_{qs} = r_s i_{qs} + \omega_e \lambda_{ds} + p\lambda_{qs}$$
$$v_{ds} = r_s i_{ds} - \omega_e \lambda_{qs} + p\lambda_{ds}$$
$$v_{os} = r_s i_{os} + p\lambda_{os}$$

$$v'_{qr} = r'_r i'_{qr} + (\omega_e - \omega_r)\lambda'_{dr} + p\lambda'_{qr}$$

$$v'_{dr} = r'_r i'_{dr} - (\omega_e - \omega_r)\lambda'_{qr} + p\lambda'_{dr}$$
$$v'_{or} = r'_r i'_{or} + p\lambda'_{or}$$

- synchronously rotating reference frame is used when incorporating the dynamic characteristics of an induction machine into a digital computer program used to study the transient and dynamic stability of large power systems.

- synchronously rotating reference frame is also used in variable frequeny study of induction machines

A. Keyhani

Per unit system.

Machine Data

Stator line frequency f (Hz)

output horse power HP (hp)

Line-to-Line voltage V (rms V)

Pole number P

Base values.

$$P_b = 746\ (hp) \quad \text{watts}$$

$$V_b = \frac{\sqrt{2}\,V}{\sqrt{3}} \quad \text{(volts peak)}$$

$$I_b = \frac{2P_b}{3V_b} \quad \text{(amp peak)}$$

$$Z_b = \frac{V_b}{I_b} = \frac{3}{2}\frac{V_b^2}{P_b} \quad \text{(ohms)}$$

Base electrical angular velocity $= \omega_b = 2\pi f_b$ rad/sec.

Base mechanical angular velocity $= \omega_{brm} = 2\pi f_b/(P/2)$

$$T_b = \frac{P_b}{\omega_{brm}} = \frac{P}{2}\frac{P_b}{\omega_b} \quad (N\text{-}m)$$

$$P_b = \frac{3}{2} V_b I_b = 3\, V_b(\text{rms})\,(I_b\,\text{rms})$$

$$\psi = \lambda \omega_b$$

$$v_{qs} = r_s i_{qs} + \frac{p}{\omega_b} \psi_{qs} + \psi_{ds} \frac{\omega}{\omega_b}$$

$$v_{ds} = r_s i_{ds} + \frac{p}{\omega_b} \psi_{ds} - \psi_{ds} \frac{\omega}{\omega_b}$$

$$v_{os} = \frac{p}{\omega_b} \psi_{os} + r_s i_{os}$$

$$v'_{qr} = r'_r i'_{qr} + \frac{p}{\omega_b} \psi'_{qr} + \psi'_{dr} \left(\frac{\omega - \omega_r}{\omega_b}\right)$$

$$v'_{dr} = r'_r i'_{dr} + \frac{p}{\omega_b} \psi'_{dr} - \psi'_{qr} \left(\frac{\omega - \omega_r}{\omega_b}\right)$$

$$v'_{os} = r'_r i'_{os} + \frac{p}{\omega_b} \psi'_{os}$$

Define

$$v_{qs\,p.u} = \bar{v}_{qs} = \frac{v_{qs}}{v_b} \qquad i_{qs\,p.u} = \bar{i}_{qs} = \frac{i_{qs}}{I_b}$$

$$\psi_{ds\,p.u} = \bar{\psi}_{ds} = \frac{\psi_{ds}}{V_b}$$

Machine voltage equations in p.u can be written as

$$\frac{v_{qs}}{V_b} = \underbrace{\frac{r_s}{Z_b} \frac{i_{qs}}{I_b}}_{V_b} + \frac{p}{\omega_b} \frac{\psi_{qs}}{v_b} + \frac{\psi_{ds}}{V_b} \frac{\omega}{\omega_b}$$

$$\bar{v}_{qs} = \bar{r}_s \bar{i}_q + \frac{p}{\omega_b} \bar{\psi}_{qs} + \bar{\psi}_{ds} \frac{\omega}{\omega_b}$$

similarly

$$\bar{v}_{ds} = \bar{r}_s \bar{i}_{ds} + \frac{p}{\omega_b} \bar{\psi}_{ds} - \bar{\psi}_{qs} \frac{\omega}{\omega_b}$$

$$\bar{v}_{os} = \frac{p}{\omega_b} \bar{\psi}_{os}$$

$$\bar{v}_{qr}' = \bar{r}_r' \bar{i}_{qr} + \frac{p}{\omega_b} \bar{\psi}_{qr} + \bar{\psi}_{dr} \left(\frac{\omega - \omega_r}{\omega_b} \right)$$

$$\bar{v}_{dr}' = \bar{r}_r' \bar{i}_{dr} + \frac{p}{\omega_b} \bar{\psi}_{dr} - \bar{\psi}_{qr} \left(\frac{\omega - \omega_r}{\omega_b} \right)$$

$$\bar{v}_{os} = \frac{p}{\omega_b} \bar{\psi}_{oR}$$

Torque base

$$T_b = \frac{P}{2} \frac{P_b}{\omega_b} = \frac{P}{2\omega_b} \left(\frac{3}{2} V_b I_b \right)$$

$$T_e = \frac{3}{2} \frac{P}{2} \frac{1}{\omega_b} \left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds} \right)$$

$$T_{e\,p.u} = \frac{T_e}{T_b} = \frac{\frac{3}{2} \frac{P}{2} \frac{1}{\omega_b} \left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds} \right)}{\frac{P}{2} \frac{1}{\omega_b} \left(\frac{3}{2} \right) V_b I_b}$$

$$T_{e\,p.u} = \frac{\psi_{ds}}{V_b} \frac{i_{qs}}{I_b} - \frac{\psi_{qs}}{V_b} \frac{i_{ds}}{I_b}$$

$$= \bar{\psi}_{ds} \bar{i}_{qs} - \bar{\psi}_{qs} \bar{i}_{ds}$$

Per-unit equation of motion is

$$\frac{T_e}{T_b} - \frac{T_L}{T_b} = \frac{\frac{\omega_b J}{P/2} \frac{d}{dt} (\omega_r / \omega_b)}{\frac{P}{2} \frac{1}{\omega_b} P_b}$$

$$T_{e\,p.u} - T_{L\,p.u} = \frac{\omega_b^2 J}{(\frac{P}{2})^2 P_b} \frac{d}{dt} \left(\frac{\omega_r}{\omega_b}\right)$$

Refine quantity $H = \dfrac{\text{Stored Kinetic Energy at } \omega_b}{\text{base Power}}$

$$H = \frac{\frac{1}{2} J \omega_{brm}^2}{P_b} = \frac{\frac{1}{2} J \omega_b^2}{(\frac{P}{2})^2 P_b}$$

Therefore

$$\frac{\omega_b^2 J}{(\frac{P}{2})^2 P_b} = 2H$$

J: $Kg \cdot m^2$

P_b: watts

ω_b: rad/sec.

$$T_{e\,p.u} - T_{L\,p.u} = 2H \frac{d}{dt} \left(\frac{\omega_r}{\omega_b}\right)$$

t: in sec. $\quad \omega_{r\,p.u} = \dfrac{\omega_r}{\omega_b}$

The quantity H is called the inertia constant and has units of seconds.

$$H = \frac{5.48 \times 10^{-6}\, n^2 J}{KVA}$$

$$H = \frac{0.237 \times 10^{-6}\, n^2 (WK^2)}{KVA}$$

n: rpm

KVA : base KVA

J: $Kg - m^2$

WK^2: $Lb - ft^2$

Example.

$HP = 115$ $\quad r_s = .016\ \Omega$

Poles $= 4$ $\quad r'_r = 0.001\ \Omega$

$V_{rated} = 210$ rms/phase $\quad X_{ls} = 0.0706\ \Omega$

$f = 50$ HZ (rated) $\quad X'_{lr} = 0.0903\ \Omega$

$X_m = 2.8413\ \Omega$

$J = 100$ Lb$-$ft^2

Base values

$$P_b = (746)(115) = 85{,}800 \text{ Watts}$$

$$V_b = (210)\sqrt{2} = 297 \text{ Volts peak}$$

$$I_b = \frac{2}{3}\frac{P_b}{V_b} = 193 \text{ Amp. peak}$$

$$Z_b = \frac{V_b}{I_b} = \frac{297}{193} = 1.54\ \Omega$$

Base angular freq. $= \omega_b = 2\pi(50) = 314$ rad/sec

Torque base $= T_b = \dfrac{P_b\,(P/2)}{\omega_b} = 547$ N-m.

$$\bar{r}_s = \frac{r_s}{Z_b} = \frac{0.016}{1.54} = 0.0103$$

$$\bar{r}'_r = \frac{r'_r}{Z_b} = \frac{0.031}{1.54} = 0.02$$

$$\bar{X}_{ls} = \frac{X_{ls}}{Z_b} = \frac{.0706}{1.54} = .045$$

$$\bar{X}'_{\ell r} = \frac{X'_{\ell r}}{Z_b} = \frac{0.0903}{1.54} = 0.0587$$

$$\bar{X}_M = \frac{X_M}{Z_b} = \frac{2.8413}{1.54} = 1.845$$

$$H = \frac{(0.231)(10^{-6})(1500)^2(100)}{85.8} = 0.607 \text{ sec.}$$

Induction machine parameters

Machine rating			T_B	$I_{B(abc)}$	r_s	X_{ls}	X_M	X'_{lr}	r'_r	J
hp	volts	rpm	N·m	amps	ohms	ohms	ohms	ohms	ohms	kg·m²
3	220	1710	11.9	5.8	0.435	0.754	26.13	0.754	0.816	0.089
50	460	1705	198	46.8	0.087	0.302	13.08	0.302	0.228	1.662
500	2300	1773	1.98×10^3	93.6	0.262	1.206	54.02	1.206	0.187	11.06
2250	2300	1786	8.9×10^3	42.1	0.029	0.226	13.04	0.226	0.022	63.87