Lecture # 3
Three-phase Induction machines

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Voltage Equations in Arbitrary Reference-Frame Variables

$$\bar{V}_{abcs} = \bar{r}_s \, i_{abcs} + p\lambda_{abcs}$$
$$V'_{abcr} = r'_r \, i'_{abcr} + p\lambda'_{abcr}$$

$$\lambda_{abcs} = (\bar{L}_s)\, i_{abcs} + [L'_{sr}]\, i'_{abcr}$$
$$\lambda'_{abcr} = (\bar{L}'_{sr})^T i_{abcs} + (L'_r)\, i'_{abcr}$$

$$\bar{V}_{abcs} = K_s \bar{V}_{qdos} \qquad i_{abcs} = K_s \bar{i}_{qdos}$$
$$V'_{abcr} = K_r \bar{V}'_{qdor} \qquad i'_{abcr} = K_r \bar{i}'_{qdor}$$

- using the above transformation equations, we can transfor the voltage equations to an arbitrary reference frame rotating at speed of ω (see Fig1)

$$V_{qdos} = r_s \, i_{qdos} + \omega \lambda_{dqs} + p\,\lambda_{qdos}$$
$$V'_{qdor} = r'_r \, i_{qdor} + (\omega - \omega_r)\lambda'_{qdr} + p\,\lambda'_{qdor}$$

where

$$(\lambda_{qds})^T = [\lambda_{ds} \quad -\lambda_{qs} \quad 0]$$

$$(\lambda'_{dqr})^T = [\lambda'_{dr} \quad -\lambda'_{qr} \quad 0]$$

- Flux linkage equations in abc reference frame can be transformed to qd axes using K_s and K_r transformation matrices.

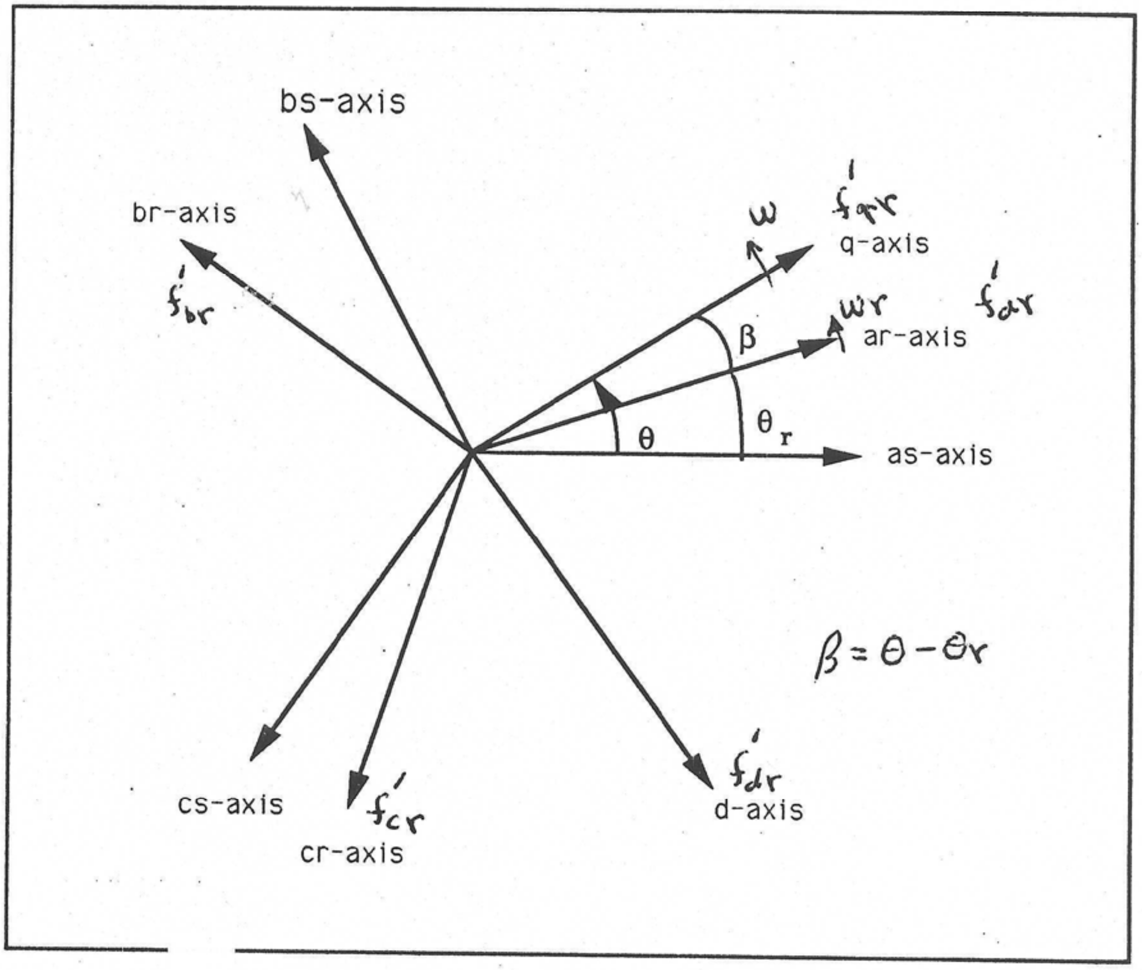

Fig 1 Axis of 2-pole, 3-phase symmetrical machine

$$\begin{bmatrix} \lambda_{qdos} \\ \lambda'_{qdor} \end{bmatrix} = \begin{bmatrix} K_s L_s (K_s)^{-1} & K_s L'_{sr} (K_r)^{-1} \\ K_r (L'_{sr})(K_s)^{-1} & K_r L'_r (K_r)^{-1} \end{bmatrix} \begin{bmatrix} i_{qdos} \\ i_{qdor} \end{bmatrix}$$

where

$$K_s L_s (K_s)^{-1} = \begin{bmatrix} L_{ls}+M & 0 & 0 \\ 0 & L_{ls}+M & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

$$M = \frac{3}{2} L_{ms}$$

$$K_r L'_r (K_r)^{-1} = \begin{bmatrix} L'_{lr}+M & 0 & 0 \\ 0 & L'_{lr}+M & 0 \\ 0 & 0 & L'_{lr} \end{bmatrix}$$

And

$$K_s L'_{sr} (K_r)^{-1} = K_r (L'_{sr})^T (K_s)^{-1} = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Voltage equations written in expanded form can be expressed as

$$v_{qs} = r_s i_{qs} + \omega \lambda_{ds} + p \lambda_{qs}$$

$$v_{ds} = r_s i_{ds} - \omega \lambda_{qs} + p \lambda_{ds}$$

$$v_{os} = r_s i_{os} + p \lambda_{os}$$

$$v'_{qr} = r'_r i'_{qr} + (\omega - \omega_r) \lambda'_{dr} + p \lambda'_{qr}$$

$$v'_{dr} = r'_r i'_{dr} - (\omega - \omega_r) \lambda'_{qr} + p \lambda'_{dr}$$

$$v'_{or} = r'_r i'_{or} + p \lambda'_{or}$$

Flux linkage equations are

$$\lambda_{qs} = L_{ls} i_{qs} + M(i_{qs} + i'_{qr})$$

$$\lambda_{ds} = L_{ls} i_{ds} + M(i_{ds} + i'_{dr})$$

$$\lambda_{os} = L_{ls} i_{os}$$

$$\lambda'_{qr} = L'_{lr} i'_{qr} + M(i_{qs} + i'_{qr})$$

$$\lambda'_{dr} = L'_{lr} i'_{dr} + M(i_{ds} + i'_{dr})$$

$$\lambda'_{or} = L'_{lr} i'_{or}$$

- since machine and power system parameters are nearly always given in ohms or percent or per unit of a base impedance, it is convenient to express the voltage and flux linkage equations in terms of reactances rather then inductances.

Let

$$\psi = \lambda \omega_b$$

then

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b} \psi_{ds} + \frac{p}{\omega_b} \psi_{qs}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b} \psi_{qs} + \frac{p}{\omega_b} \psi_{ds}$$

$$v_{os} = r_s i_{os} + \frac{p}{\omega_b} \psi_{os}$$

$$v'_{qr} = r'_r i'_{qr} + \frac{(\omega - \omega_r)}{\omega_b} \psi'_{dr} + \frac{p}{\omega_b} \psi'_{qr}$$

$$v'_{dr} = r'_r i'_{dr} - \frac{(\omega - \omega_r)}{\omega_b} \psi'_{qr} + \frac{p}{\omega_b} \psi'_{dr}$$

$$v'_{or} = r'_r i'_{or} + \frac{p}{\omega_b} \psi'_{or}$$

and flux linkages become flux linkages per second with the units of volts.

$$\psi_{qs} = X_{ls} i_{qs} + X_m (i_{qs} + i'_{qr})$$
$$\psi_{ds} = X_{ls} i_{ds} + X_m (i_{ds} + i'_{dr})$$
$$\psi_{os} = X_{ls} i_{os}$$

$$\psi'_{qr} = X'_{lr} i'_{qr} + X_m (i_{qs} + i'_{qr})$$
$$\psi'_{dr} = X'_{lr} i'_{dr} + X_m (i_{ds} + i'_{dr})$$
$$\psi_{or} = X'_{lr} i'_{or}$$

- Fig. 2 presents the arbitrary reference-frame equivalent circuits for a three-phase symmetrical induction machine

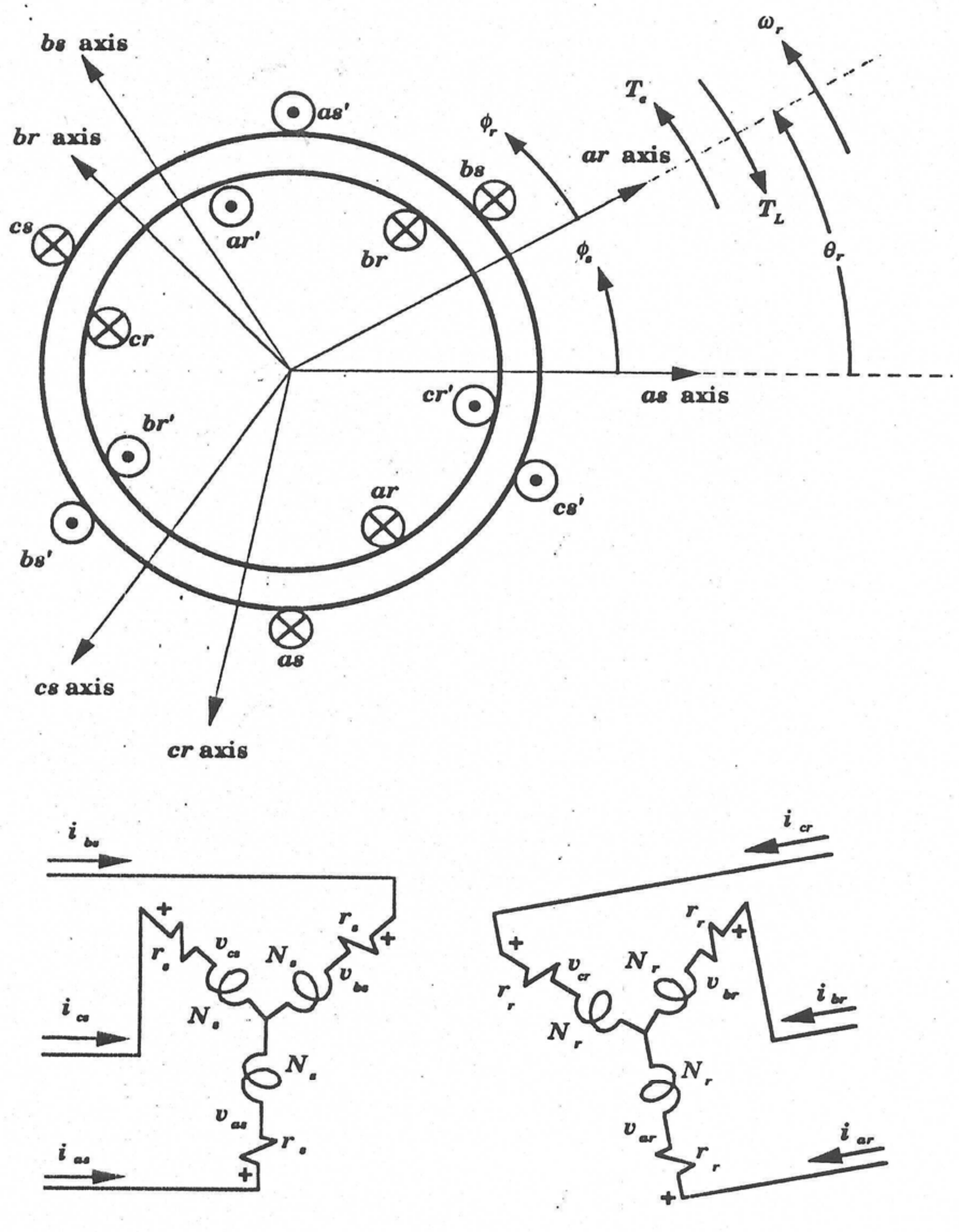

FIGURE 1
A two-pole three-phase symmetrical induction machine.

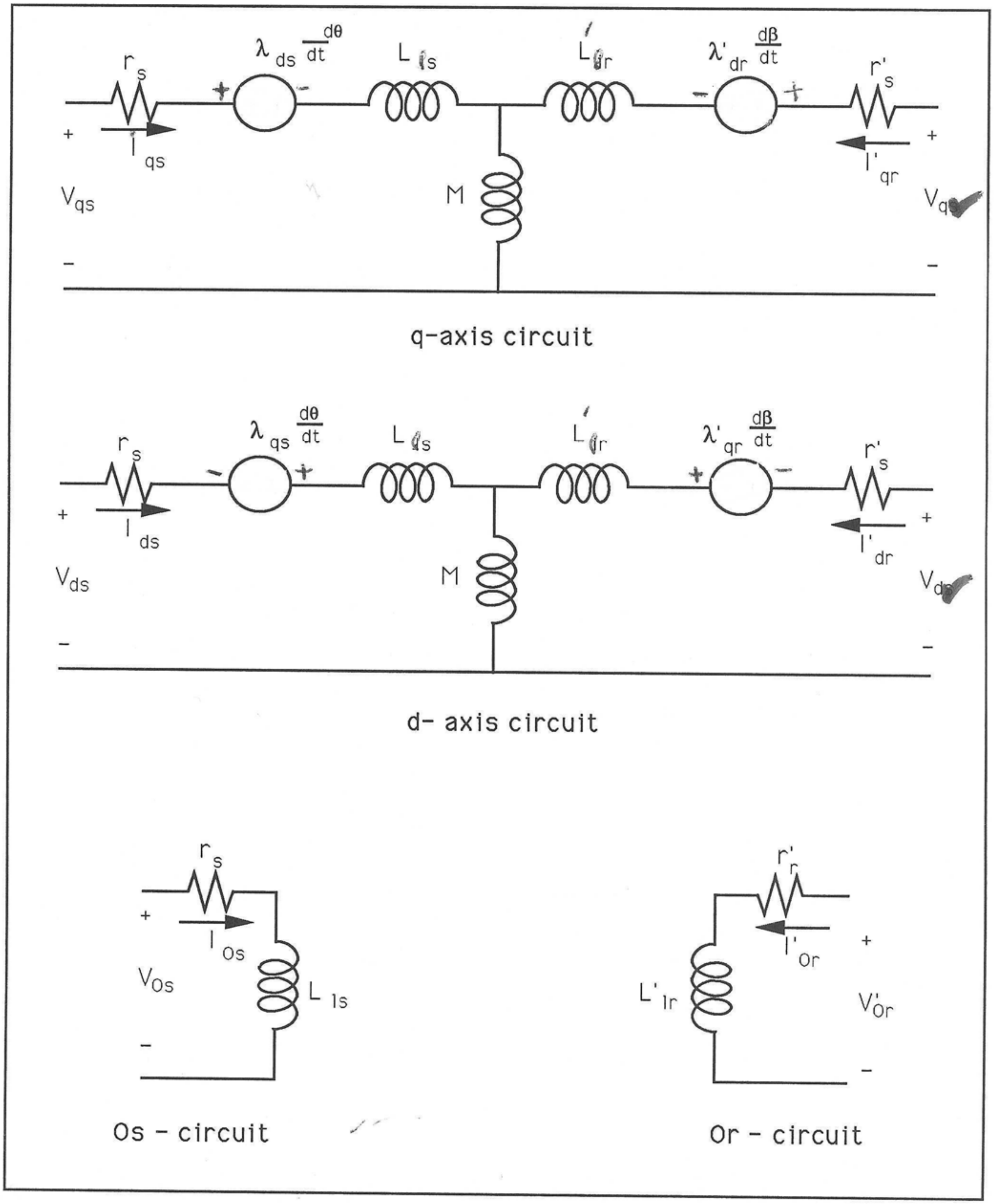

Fig. 3 Equivalent circuits of a 3-phase, symmetrical induction machine with rotating d-q axis at speed of ω

Torque Equation in Arbitrary Reference-Frame Variables

- Electromagnetic torque in terms of arbitrary reference-frame variables may be obtained by substituting the equations of transformation in

$$T_e = \frac{P}{2} (i_{abc})^T \frac{\partial}{\partial \theta_r} (L'_{sr}) i'_{abcr}$$

$$T_e = \frac{P}{2} \left[(K_s)^{-1} i_{qd0s} \right]^T \frac{\partial}{\partial \theta_r} [L'_{sr}] (K_r)^{-1} i'_{qd0r}$$

After some work, we will have the following:

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right) M \left(i_{qs} i'_{dr} - i_{ds} i'_{qr} \right)$$

where T_e is positive for motor action. other expressions for the electromagnetic torque of an induction machine are

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right) \left(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr} \right)$$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right) \left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds} \right)$$

$$T_e = \frac{3}{2} \left(\frac{P}{2}\right) \left(\frac{1}{\omega_b}\right) \left(\psi'_{qr} i'_{dr} - \psi'_{dr} i'_{qr} \right)$$